Local Volatility Calibration in Commodity Markets

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IMPA

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Pricing Problem

Required properties:

- Robustness.
- Reliability.
- Simple calibration.

Desirable property: **implied smile adherence**.

A well-known model in equity markets: *Dupire's Local Volatility.*

Applications: Calendar spread options, path dependent options, ...

Challenges in Commodity Markets

- For each future we have options with only one maturity.
  - WTI oil: three business days before the termination of trading in the underlying futures contract.
  - HH natural gas: the business day immediately preceding the expiration of the underlying futures contract.
  - HO heating oil: three business days before the expiration of the underlying futures contract.
  - RBOB: three business days before the expiration of the underlying futures contract.

Source: CME webpage.

Conclusion: We do not have a surface of option prices on each future.

Challenges in Commodity Markets (cont.)

Important features:

- In commodity markets convenience yield is one important feature.
- Market vanilla option prices are American and then are more expensive than the European ones.
- We need to extract European from American prices.
- The inverse problem associated to American pricing is much harder: There is no framework similar to Dupire's equation for pricing American options. Then, the forward problem should be solved for each strike and maturity.

We pass to the transformation of American in European prices.
This is based on the framework introduced by Black [Bla76].

Black's Framework

Under the risk-neutral measure, with constant coefficients:

- r is the risk-free interest rate,
- σ is Black's volatility,
- d is the convenience yield.
- S_t is the commodity spot price, satisfying

$$dS_t = S_t((r-d)dt + \sigma d\widetilde{W}_t)$$

- $F_{t,T}$ is the commodity future, satisfying

$$dF_{t,T} = \sigma F_{t,T} d\widetilde{W}_t$$

- They are related by $F_{t,T} = e^{(r-d)(T-t)} S_t$

European call options on $F_{t,T}$ satisfy Black's equation:

$$-C_t = \frac{1}{2}\sigma C_{ff}, \text{ for } f \geq 0, \ t > 0,$$

with the terminal condition:

$$C(T, f) = (f-K)^+, \text{ for } f \geq 0.$$

American options under Black-Scholes [WHD95]:

$$x := \log(S/K) \quad \text{and} \quad \tau = (T-t)\frac{1}{2}\sigma^2$$

Then we have the linear complementary problem:

$$\begin{cases} (u_\tau - u_{xx}) \geq 0, & (u(x,\tau) - g(x,\tau)) \geq 0, \\ (u_\tau - u_{xx}) \quad \cdot & (u(x,\tau) - g(x,\tau)) = 0, \end{cases}$$

where, for $\kappa = (r-d)/\left(\frac{1}{2}\sigma^2\right)$,

$$g(x,\tau) = e^{\frac{1}{4}(\kappa+1)^2\tau}\left(e^{\frac{1}{2}(\kappa+1)x} - e^{\frac{1}{2}(\kappa-1)x}\right)^+ \text{ for a call.}$$

The boundary conditions are:

$$u(x,0) = g(x,0) \quad \text{and} \quad \lim_{x\to\pm\infty} u(x,\tau) = \lim_{x\to\pm\infty} g(x,\tau)$$

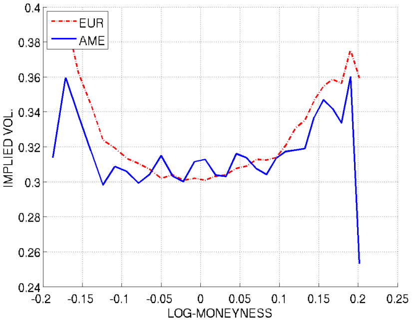
Then, call prices are given by $C(S,t) = Ke^{-\frac{1}{2}(\kappa-1)x+\frac{1}{4}(\kappa+1)^2\tau}u(x,\tau)$.

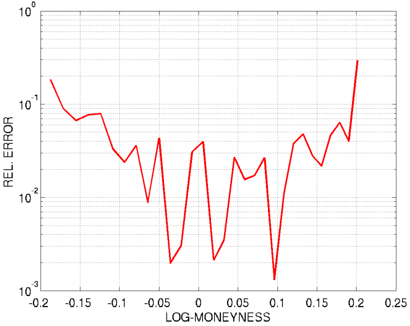
European from American Prices

- Transforming American prices in European ones,
- Then we could use Dupire's framework.
- Another possibility is the following:
  1. Find the American implied vol. from market option prices.
  2. Then use Black's formula to find European prices.

$$C_{\text{AME}} \overset{\text{B-S AME Pricing}}{\longmapsto} \sigma_{\text{AME}} \overset{\text{B-S Formula}}{\longmapsto} C_{\text{EUR}}.$$

HH Nat. Gas Implied Vol. - Mat.:10/28/2013





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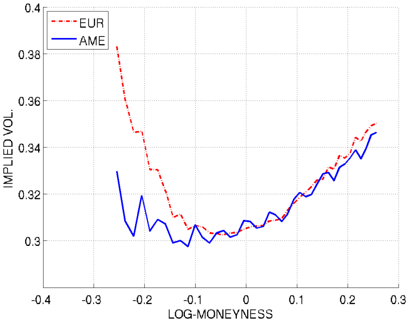

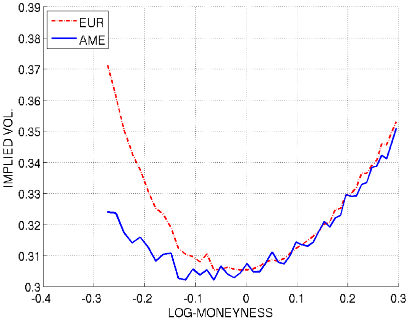
EUR
AME
IMPLIED VOL.
LOG-MONEYNESS
0.39
0.38
0.37
0.36
0.35
0.34
0.33
0.32
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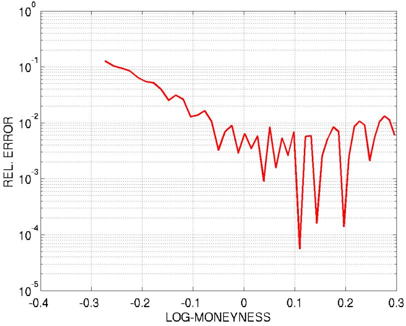
REL. ERROR
LOG-MONEYNESS
10^{0}
10^{-1}
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-0.4
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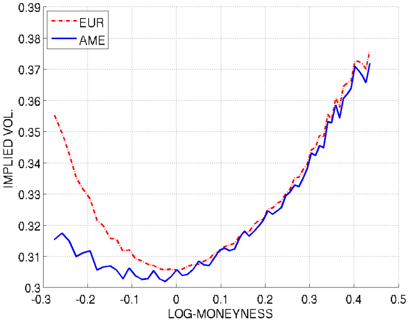

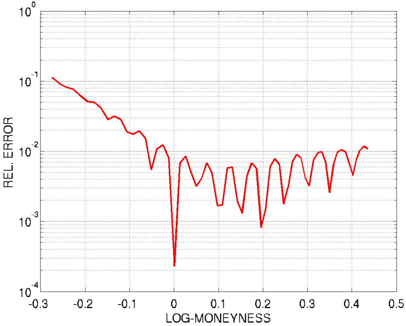

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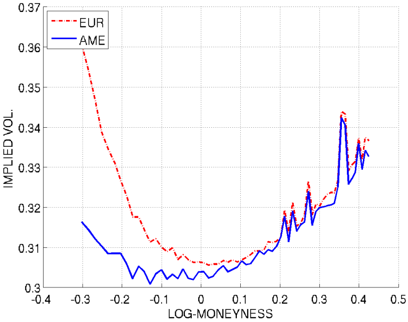

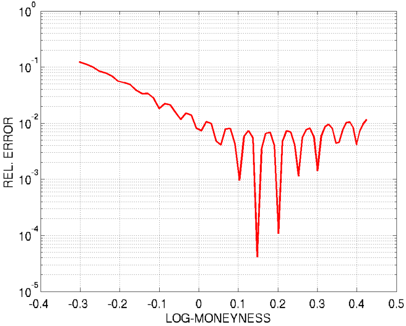

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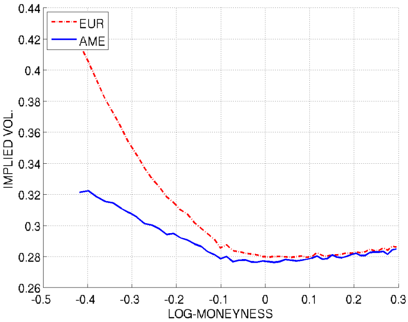
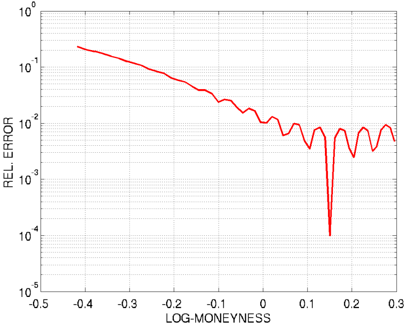
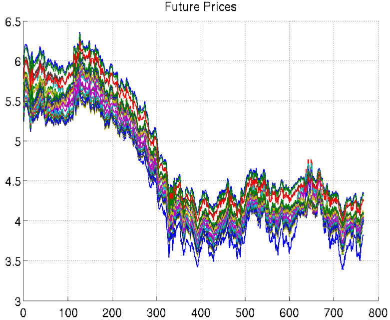




We present also one important feature of Commodity futures.

Correlations

Futures on the same commodity for different maturities are highly correlated.



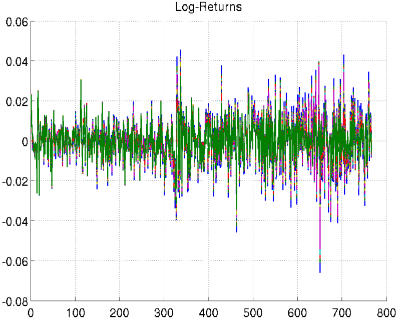


Figure: Example: Future prices and daily log-returns of Henry Hub nat. gas.

Correlations

Example: Future prices and daily log-returns of Henry Hub nat. gas.

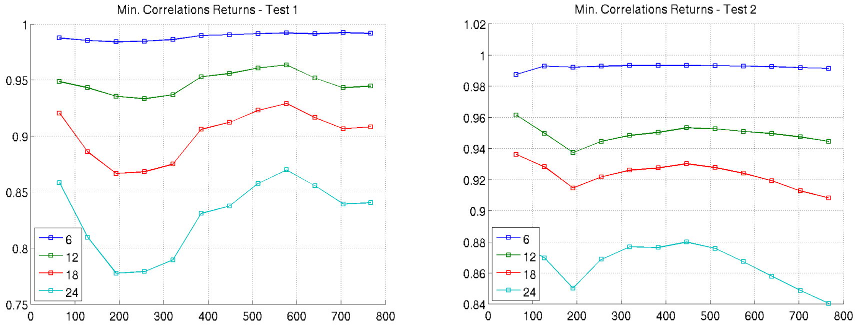


Figure: Minimum of correlations between daily log-returns - first and second tests.

Correlations

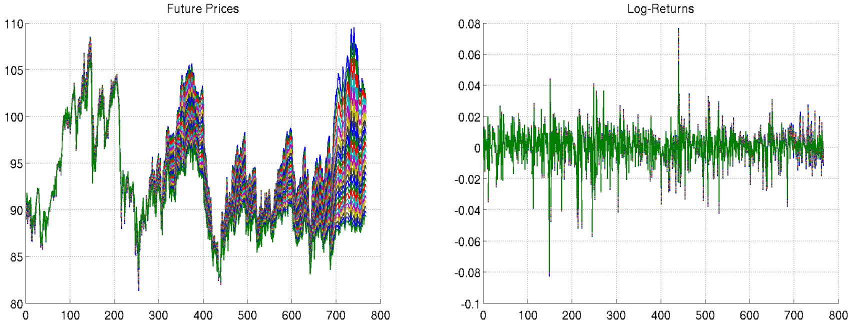


Figure: Example: Future prices and daily log-returns of WTI oil.

Correlations

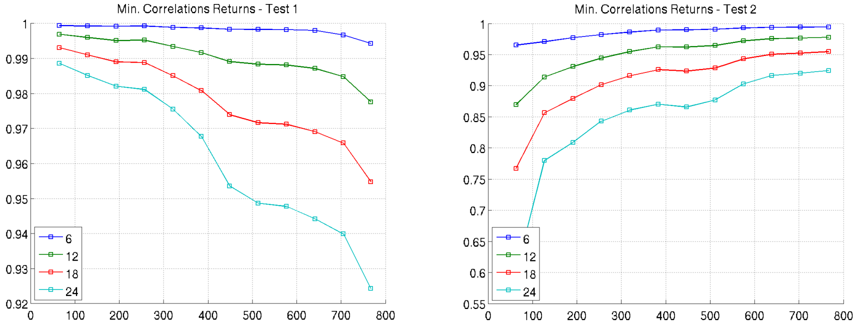


Figure: Minimum of correlations between daily log-returns - first and second tests.

We present now some features of the present model.

Some Features

- Under this framework, the term-structure is given by:
  - The current curve of future prices $F_{0,T}$, for $T > 0$
  - The local volatility surface.
- The model would work fine for short maturity options and a small term-structure curve, since it has only one factor.
- We can form a unique surface of normalized option prices on futures with different maturities.
- Dupire's formula is not stable in practice, since the inverse problem is ill-posed.
- We apply usual calibration procedures, e.g. Tikhonov regularization.

In what follows, we present the theoretical aspects of the model.

Dupire's Local Vol. in Commodity Markets

- $(\Omega, \mathcal{V}, \mathcal{F}, \widetilde{\mathbb{P}})$ - risk neutral filtered probability space.
- Commodity futures are the underlying assets.
- $F_{t,T}$ denotes the future price at time $t \geq 0$ with maturity $T \geq t$.
- S_t denotes the (unknown) spot price at time $t \geq 0$.
- $F_{t,T} = \widetilde{\mathbb{E}}[S_T | \mathcal{F}_t]$, then $\{F_{t,T}\}_{t \in [0,T]}$ is a martingale.

Then, we assume that, $F_{t,T}$ satisfies:

$$\begin{cases} dF_{t,T} & = & \sigma(F_{0,T}, t, F_{t,T}) F_{t,T} d\widetilde{W}_t, \text{ for } 0 \leq t \leq T \\ F_{0,T} & \text{is} & \text{given and } F_{T,T} = S_T. \end{cases}$$

Now, the PDE for pricing call options.

Dupire's Equation

Fix the current time at $t = 0$, European call options satisfy, with $T \leq T'$:

$$\begin{cases} \dfrac{\partial C}{\partial T} = \dfrac{1}{2}\sigma^2(F_{0,T'}, T, K)K^2 \dfrac{\partial^2 C}{\partial K^2}, \; 0 < T < T', \; K \geq 0 \\ \lim\limits_{K \to 0} C(T,K) = F_{0,T'}, \; 0 < T < T', \\ \lim\limits_{K \to +\infty} C(T,K) = 0, \; 0 < T < T', \\ C(T,K) = (F_{0,T'} - K)^+, \text{ for } K > 0. \end{cases} \tag{1}$$

We need some technical adaptations.

Technical Points

Perform the change of variables

$$\tau = T \quad \text{and} \quad y = \log(K/F_{0,T'}).$$

Then define:

$$V(F_{0,T'},\tau,y) := C(F_{0,T'},\tau,F_{0,T'}e^{y}) \text{ and } a(F_{0,T'},\tau,y) := \frac{1}{2}\sigma^2(F_{0,T'},\tau,F_{0,T'}e^{y}).$$

Moreover, normalize the option prices by its underlying futures:

$$V(F_{0,T'},\tau,y) = V(F_{0,T'},\tau,y)/F_{0,T'}.$$

Thus, from the previous PDE we have the following problem:

A Surface of Option Prices

We also assume that

$$V(F_{0,T'},\tau,y) = V(S_0,\tau,y) \quad \text{and } a(F_{0,T'},\tau,y) = a(S_0,\tau,y).$$

Then, V satisfies:

$$\begin{cases} \dfrac{\partial V}{\partial \tau}(\tau,y) &= a(S_0,\tau,y)\left(\dfrac{\partial^2 V}{\partial y^2}(\tau,y) - \dfrac{\partial V}{\partial y}(\tau,y)\right), \; T>0, \; y\in\mathbb{R} \\ \lim\limits_{y\to-\infty} V(\tau,y) &= 1, \; \tau>0, \\ \lim\limits_{y\to+\infty} V(\tau,y) &= 0, \; \tau>0, \\ V(\tau,y) &= (1-e^y)^+, \text{ for } y\in\mathbb{R}. \end{cases} \tag{2}$$

It is independent of $F_{0,T}$!

We present some background properties of the forward operator.

The Forward Operator

Let $a_1, a_2 \in \mathbb{R}$ be such that $0 < a_1 \leq a_2 < +\infty$.
Consider a_0 in $H^{1+\varepsilon}(D)$, with $\varepsilon > 0$ and $a_1 \leq a_0 \leq a_2$.
Define the set

$$Q := \{a \in a_0 + H^{1+\varepsilon}(D) : a_1 \leq a \leq a_2\}. \tag{3}$$

Proposition ([DCSZ12])

If $a \in Q$, then Pricing Call Options on futures by Dupire's Equation is a well-posed problem in $W^{1,2}_{2,loc}(D)$

The Forward Operator (cont.)

Definition

Let $\varepsilon > 0$ *and* $a_0 \in H^{1+\varepsilon}(D)$ *be fixed. Define the* forward operator*:*

$$\begin{array}{rcl} F : Q \subset H^{1+\varepsilon}(D) & \longrightarrow & W_2^{1,2}(D) \\ a \in Q & \to & u(a) - u(a_0) \in L^2(D), \end{array}$$

The Forward Operator (cont.)

Proposition (From [DCSZ12])

We have the following regularity properties for the forward operator F:

(i) *It is continuous and compact.*

(ii) *It is also weakly continuous and weakly closed.*

(iii) *F is differentiable at $a \in Q$ in every direction $h \in H^{1+\varepsilon}(D)$ such that $a+h \in Q$.*

(iv) *$F'(a)$ is extensible to a bounded linear operator on $H^{1+\varepsilon}(D)$.*

(v) *It also satisfies the Lipschitz condition:*

$$\|F'(a) - F'(a+h)\|_{\mathcal{L}(H^{1+\varepsilon}(D),L^2(D))} \leq c\|h\|,$$

for every $h \in H^{1+\varepsilon}(D)$ such that $a+h \in Q$.

Corollary (From [AZ12])

The forward operator F is injective.

The Inverse Problem

The local volatility calibration problem can formulated as follows:

Problem

If $u \in L^2(D)$ is a surface of European call option prices, then find $a^\dagger \in Q$, a local volatility surface, satisfying

$$u - u(a_0) = F(a^\dagger), \tag{4}$$

with $a_0 \in Q$ fixed and known.

Since F is injective, there exists a unique $a \in Q$ satisfying Equation (4).

In practice, we observe only the noisy data u^δ, which is related to u by:

$$u^\delta = u + e \tag{5}$$

e compiles all the uncertainties concerning the measurement of u^δ.

$$\|u - u^\delta\| = \|e\| \leq \delta$$

Problem

Find $a^\dagger \in Q$ satisfying

$$u^\delta - u(a_0) = F(a^\dagger) + e, \tag{6}$$

with $a_0 \in Q$ fixed and known and $e \in L^2(D)$ unknown with $\|e\| \leq \delta$ and $\delta > 0$.

Tikhonov-type Regularization

Problem

Find one minimizer a_α^δ in Q for the Tikhonov functional below:

$$\mathcal{F}_{a_0,\alpha}^{u^\delta} = \|u(a) - u^\delta\|^2 + \alpha f_{a_0}(a) \tag{7}$$

with $\alpha > 0$ appropriately chosen and $f_{a_0} : \mathcal{D}(f_{a_0}) \subset H^{1+\varepsilon}(D) \to [0, +\infty)$ a suitable functional.

Morozov's Discrepancy Principle

The choice of α is based on the relaxed version of Morozov's discrepancy principle below:

Definition

For $1 < \tau_1 \leq \tau_2$ *we choose* $\alpha = \alpha(\delta, u^\delta) > 0$ *such that*

$$\tau_1\delta \leq \|u(a_\alpha^\delta) - u^\delta\| \leq \tau_2\delta \tag{8}$$

holds for some a_α^δ *minimizer of the Tikhonov Functional.*

The Functional f_{a_0}

We assume that:

- f_{a_0} is convex
- $f_{a_0}(a) = 0$ if and only if $a = a_0$.
- It is also coercive, i.e., if $\{a_n\}_{n\in\mathbb{N}}$ satisfy $\|a_n\| \to +\infty$, then $f_{a_0}(a_n) \to +\infty$.
- f_{a_0} is weakly lower semi-continuous, i.e., if $\{a_n\}_{n\in\mathbb{N}}$ converges to $\tilde{a} \in Q$ weakly in $H^{1+\varepsilon}(D)$, then the

$$f_{a_0}(\tilde{a}) \leq \liminf_{n\to\infty} f_{a_0}(a_n)$$

  holds.

Some Examples

Some canonical examples of f_{a_0} are:

1. Standard quadratic:

$$f_{a_0}(a) = \|a - a_0\|^2_{L^2(D)}.$$

2. Smoothing quadratic:

$$f_{a_0}(a) = \beta_1\|a - a_0\|^2_{L^2(D)} + \beta_2\|\partial_x a - \partial_x a_0\|^2_{L^2(D)} + \beta_3\|\partial_\tau a - \partial_\tau a_0\|^2_{L^2(D)}.$$

   β_j can be arbitrarily chosen and should account discretization levels.

3. Kullback-Leibler: denoting $x := (\tau, y) \in D$,

$$f_{a_0}(a) = \int_D [\log(a(x)/a_0(x)) - (a_0(x) - a(x))]dx.$$

4. Total Variation:

$$f_{a_0} = \|\partial_x a - \partial_x a_0\|_{L^1(D)} + \|\partial_\tau a - \partial_\tau a_0\|_{L^1(D)}.$$

Well-Posedness

Proposition

The level sets

$$\mu_\alpha(M) = \left\{ a \in Q \,\middle|\, \mathcal{F}^{u^\delta}_{a_0,\alpha}(a) \leq M \right\}$$

are pre-compact in the weak topology of $H^{1+\varepsilon}(D)$. The restriction of F onto $\mu_\alpha(M)$ is weakly continuous.

Theorem (Existence)

Let $\alpha > 0$ and $a_0 \in Q$ be fixed. Then, the Tikhonov functional has a minimizer in Q.

Definition

A minimizer $a \in Q$ of the Tikhonov functional is said stable if, for small perturbations on the data $u \in L^2(D)$, a minimizer of (7) with the perturbed data is in the neighborhood of a.

Theorem (Stability)

Every minimizer of the Tikhonov functional (7) is stable.

Convergence[1,2]

Theorem ([AZ12])

The regularizing parameter $\alpha = \alpha(\delta, \mathcal{U}^\delta)$ *obtained through Morozov's discrepancy principle satisfies:*

$$\lim_{\delta\to 0+} \alpha(\delta, \mathcal{U}^\delta) = 0 \quad \textit{and} \quad \lim_{\delta\to 0+} \frac{\delta^2}{\alpha(\delta, \mathcal{U}^\delta)} = 0.$$

Theorem ([AZ12])

Let $\{\delta_k\}_{k\in\mathbb{N}}$ *be a sequence of positive numbers converging monotonically to* 0*.*
Let $\{u^{\delta_k}\}_{k\in\mathbb{N}}$ *be the associated sequence of noisy data.*
Then, the sequence of minimizers $\{a^{\delta_k}_{\alpha_k}\}_{k\in\mathbb{N}}$ *converges weakly to* $a^\dagger$*, the true solution.*

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[1] V.A. & J.P. Zubelli, *Online Local Vol. Calib. by Convex Regularization with Morozov's Principle and Conv. Rates. Available on SSRN*

[2] V.A. & J.P. Zubelli, *Local Volatility Models in Commodity Markets and Online Calibration. Working article*

Convergence: The Discrete Case[3]

Definition

Let $\{X_m\}_{m\in\mathbb{N}}$ be a sequence of finite dimensional subspaces of $H^{1+\varepsilon}(D)$, such that

$$X_m \subset X_{m+1} \text{ for } m \in \mathbb{N} \quad \text{and} \quad \overline{\cup_{m\in\mathbb{N}} X_m} = H^{1+\varepsilon}(D).$$

Define also the finite-dimensional domains $Q_m := Q \cap X_m$.

We assume that $Q_m \neq \emptyset$ for every $m \in \mathbb{N}$.

Definition ([ACZ13])

Let $\delta > 0$, u^δ and be fixed. For $1 < \tau \leq \lambda$, then choose $\alpha = \alpha(\delta, u^\delta) > 0$ and $m \in \mathbb{N}$ such that

$$\tau_1 \delta \leq \|F(a^\delta_{m,\alpha}) - u^\delta\| \leq \lambda\delta, \tag{9}$$

holds for $a^\delta_{m,\alpha}$ a minimizer of the Tikhonov functional in Q_m.

[3] V.A., A. De Cezaro & J.P. Zubelli, *Discrepancy Based Choice for Domain Discretization Level and Regularization Parameter. Working article.*

Convergence: The Discrete Case[3] (cont.)

Theorem

Let $\{\delta_k\}_{k\in\mathbb{N}}$ be a sequence of positive numbers converging monotonically to 0. Let $\{u^{\delta_k}\}_{k\in\mathbb{N}}$ be the associated sequence of noisy data. Then, if m_k and α_k are chosen through the discrepancy principle above, the associated finite-dimensional minimizers $\{a^{\delta_k}_{m_k,\alpha_k}\}_{k\in\mathbb{N}}$ converge weakly to $a^\dagger$, the true solution.

[3] V.A., A. De Cezaro & J.P. Zubelli, *Discrepancy Based Choice for Domain Discretization Level and Regularization Parameter. Working article.*

Theorem (Convergence Rates [AZ12])

Assume that $\alpha = \alpha(\delta, u^\delta)$ *is chosen through the Morozov's discrepancy principle.*
Furthermore, assume that $f_{\mathcal{A}_0}(a) = \|a - a_0\|^2_{H^{1+\varepsilon}(D)}$.
Then

$$\|a^\delta_\alpha - a^\dagger\|_{H^{1+\varepsilon}(D)} = \mathcal{O}(\delta^{\frac{1}{2}}) \quad \text{and} \quad \|u(a^\delta_\alpha) - u^\delta\| = \mathcal{O}(\delta),$$

where $a^\delta_\alpha \in Q$ *is the regularized solution.*

[1] V.A. & J.P. Zubelli, *Online Local Vol. Calib. by Convex Regularization with Morozov's Principle and Conv. Rates. Available on SSRN*

Convergence Rates: The Discrete Case[3]

Under the discrete setting, we have the following result.

Theorem (Convergence Rates [ACZ13])

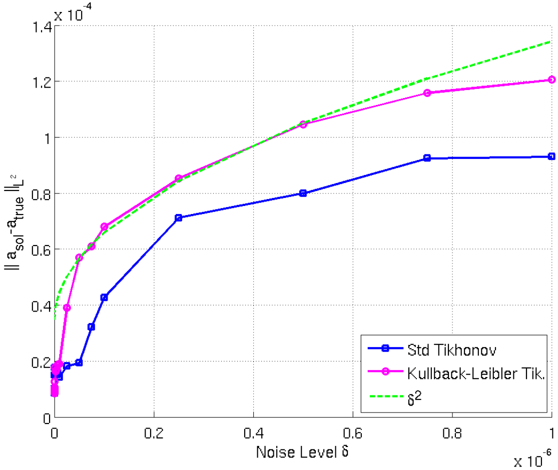
Assume that $\alpha = \alpha(\delta, u^\delta)$ *and the discretization level* $m = m(\delta, u^\delta)$ *are chosen through the discrepancy principle above.*
Furthermore, assume that $f_{\mathcal{A}_0}(a) = \|a - a_0\|^2_{H^{1+\varepsilon}(D)}$ *and there exists* $a \in Q_m$ *such that* $\|u(a) - u^\delta\| \leq \varepsilon\delta$ *and* $f_{a_0}(a) < f_{a_0}(a^\dagger)$*, with* $1 < \varepsilon < \tau$*.*
Then

$$\|a^\delta_{m,\alpha} - a^\dagger\|_{H^{1+\varepsilon}(D)} = \mathcal{O}(\delta^{\frac{1}{2}}) \quad \text{and} \quad \|u(a^\delta_{m,\alpha}) - u^\delta\| = \mathcal{O}(\delta),$$

where $a^\delta_{m,\alpha} \in Q_m$ *is the finite-dimensional regularized solution.*

[3]V.A., A. De Cezaro & J.P. Zubelli, *Discrepancy Based Choice for Domain Discretization Level and Regularization Parameter. Working Article.*

Convergence Rates[4]



[4] V.A., A. De Cezaro & J.P. Zubelli, *Convex Regularization of Local Volatility Estimation in a Discrete Setting. Available on SSRN.*

Online Approach[1,2]

How to improve results even further?

Introducing more information:

$$\mathcal{F}^{\mathcal{U}^\delta}_{\mathcal{A}_0,\alpha}(\mathcal{A}) = \int_{S_{\min}}^{S_{\max}} \|u(a(s)) - u^\delta(s)\|^2 ds + \alpha \mathcal{F}_{\mathcal{A}_0}(\mathcal{A}), \tag{10}$$

In the discrete case:

$$\mathcal{F}^{\mathcal{U}^\delta}_{\mathcal{A}_0,\alpha}(\mathcal{A}) = \sum_{j=1}^{M} \|u(a(s_j)) - u^\delta(s_j)\|^2 + \alpha \mathcal{F}_{\mathcal{A}_0}(\mathcal{A}_M), \tag{11}$$

with $S_{\min} \leq s_j \leq S_{\max}$ for every $j = 1, \ldots, M$.

[1] V.A. & J.P. Zubelli, *Online Local Vol. Calib. by Convex Regularization with Morozov's Principle and Conv. Rates. Available on SSRN*

[2] V.A. & J.P. Zubelli, *Local Volatility Models in Commodity Markets and Online Calibration. Working article*

Online Approach (cont.)

Theorem

There exists a solution for a minimizer for the online Tikhonov functional.
It is stable.
Assume that $\delta \to 0$ and α is chosen through the Morozov's discrepancy principle.
Then the regularized solutions converge weakly to the solution of the noiseless inverse problem $\mathcal{A}^\dagger \in \mathfrak{Q}$.
In addition, when $\mathcal{F}_{\mathcal{A}_0}(\mathcal{A}) = \|\mathcal{A} - \mathcal{A}_0\|^2_{H^l(0,T,H^{1+\varepsilon}(D))}$, these solutions satisfy the convergence rate:

$$\|\mathcal{A}^\delta_\alpha - \mathcal{A}^\dagger\| = O(\sqrt{\delta}).$$

The same holds in the discrete case.

Synthetic Data: Local Volatility.

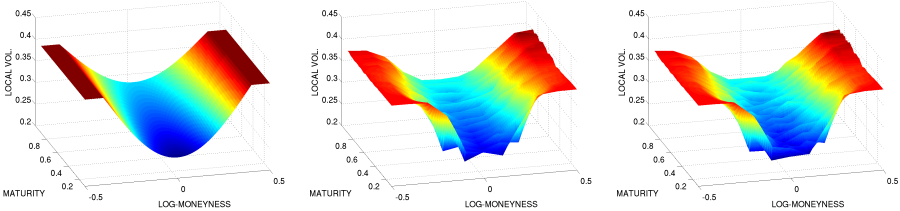


Figure: Left: Original. Center and right.: Reconstructions with noisy data.

Synthetic Data: Local Volatility.

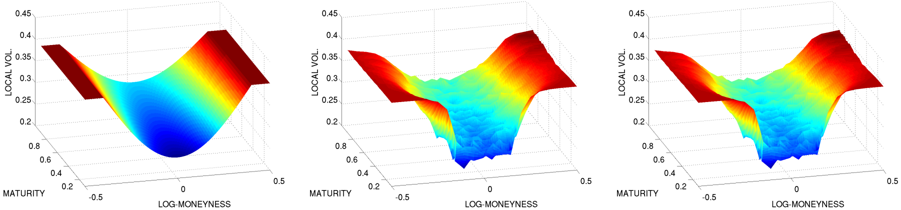


Figure: Left: Original. Center and right.: Reconstructions with noisy data.

Synthetic Data: Residual and Error Evolutions.

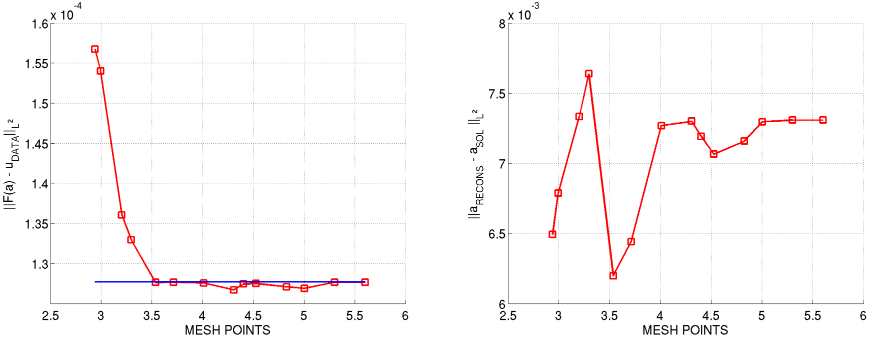


Figure: Left: Residual $\times$ discretization level. Right: Error $\times$ discretization level.

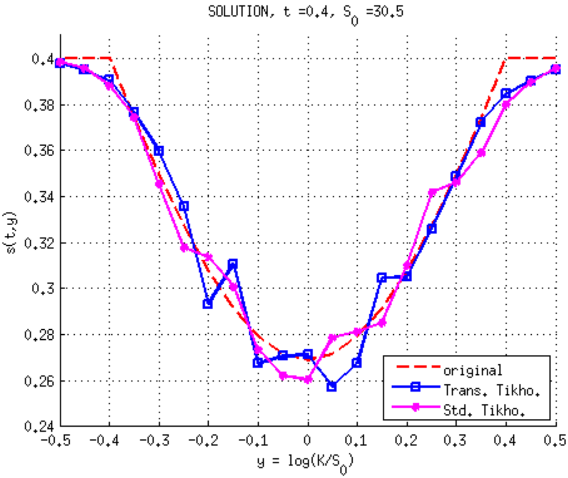


Figure: More data, better results!

Online × Standard Calibration

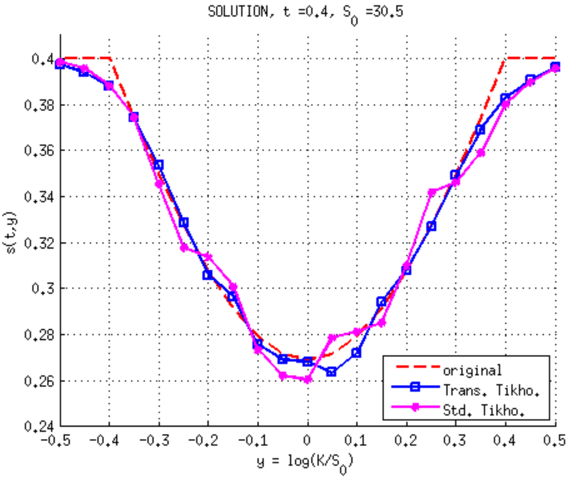
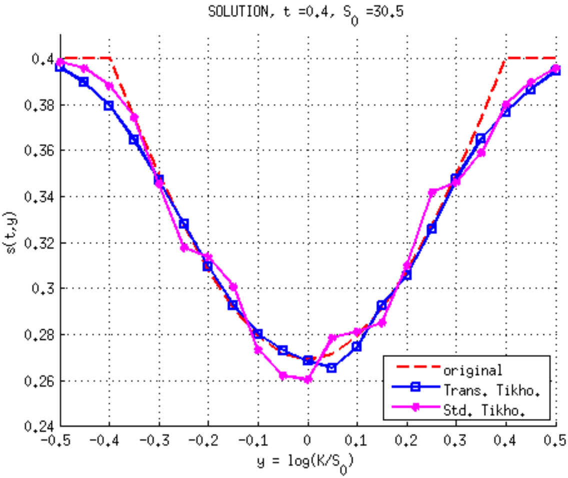


Figure: More data, better results!

Online $\times$ Standard Calibration



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Online × Standard Calibration

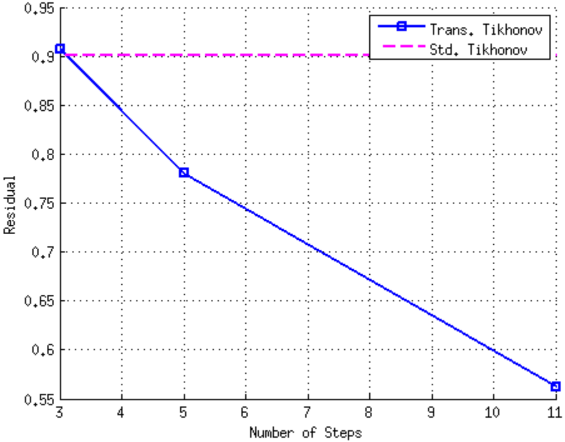


Figure: L^2 distance between original and reconstructed local vol.

WTI Local and Implied Volatilities

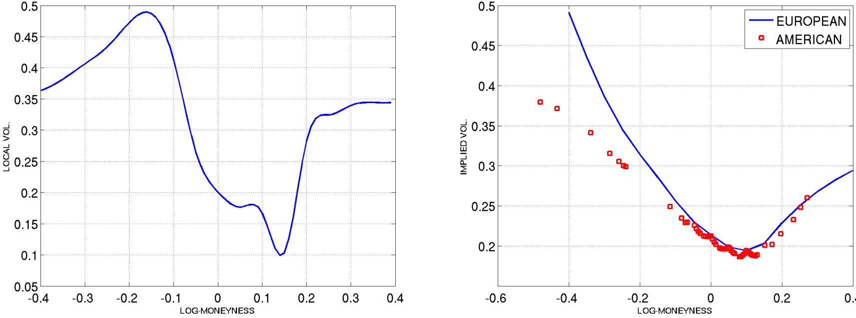
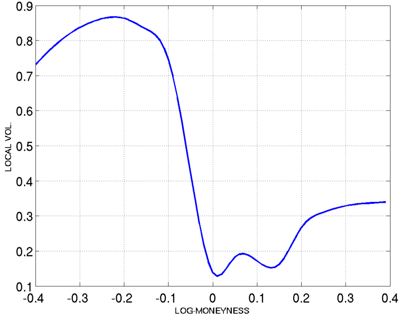


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

WTI Local and Implied Volatilities



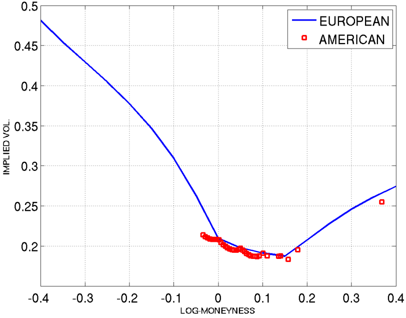


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

WTI Local and Implied Volatilities

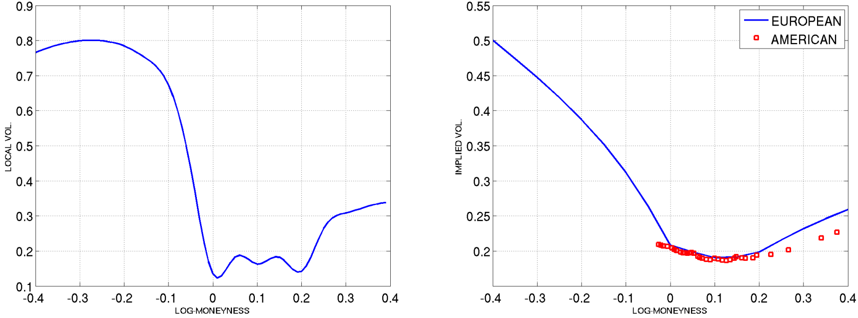


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

Local Vol.: Henry Hub Nat. Gas

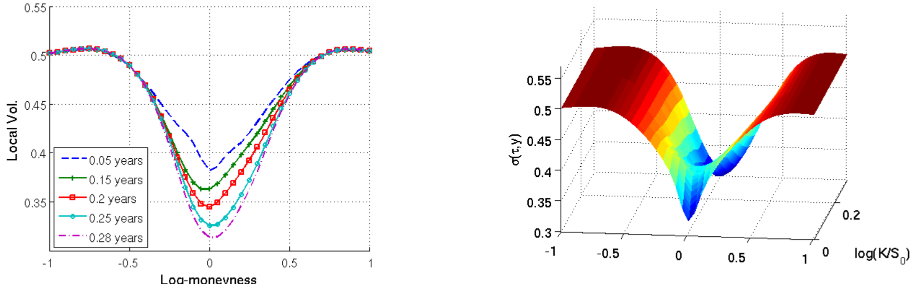


Figure: Left: local vol. reconstructed for some maturities. Right: reconstructed local vol. surface.

Implied Volatility Comparison

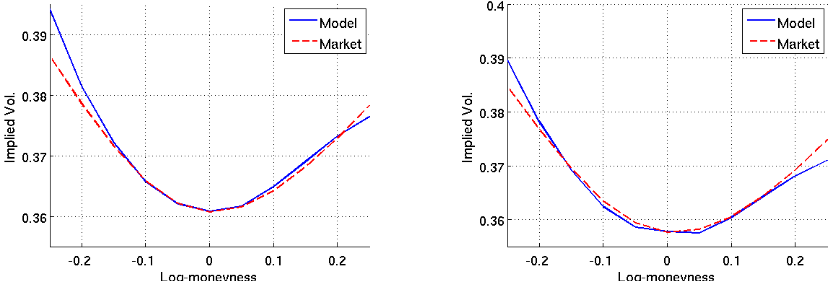


Figure: Implied vol. (Black) for market prices (dashed) and model prices (continuous) for two maturities.

HH Local and Implied Volatilities

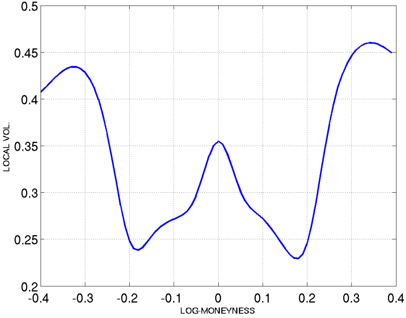
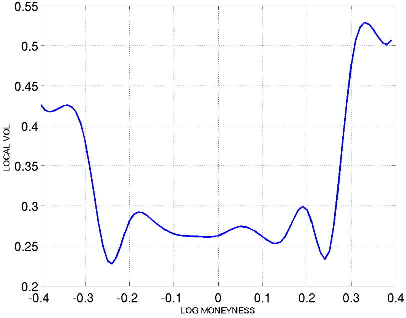


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

HH Local and Implied Volatilities



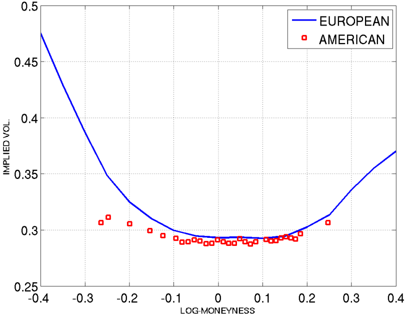


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

HH Local and Implied Volatilities

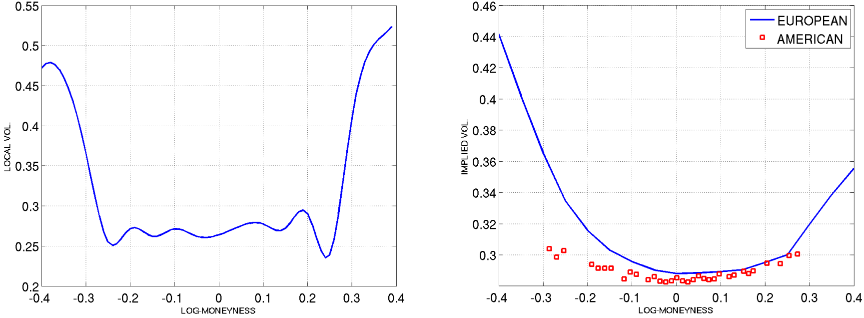


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

HH Local and Implied Volatilities

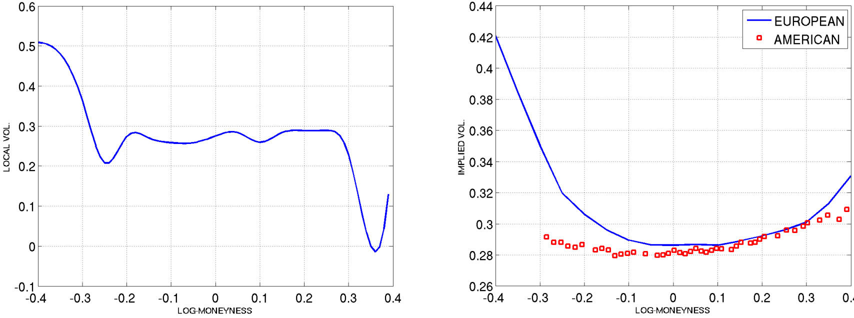


Figure: Left: Local Volatility. Right: Implied Vol. of model (cont.) and market (squares).

Conclusions

- Dupire's local vol. applied to commodity markets.
- Implemented American to European prices transformation.
- Local vol. calibration solved by convex regularization.
- Online approach.
- Morozov's discrepancy principle.
- Numerical tests.

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