



MTM3100 - Pré-cálculo

Gabarito parcial da 10ª lista complementar de exercícios

1. (a) É função. $\text{Im}(f) = \mathbb{N}^*$. (b) Não é função.
(c) (d)
(e) É função. $\text{Im}(f) = \{0, 1, 4, 9, 16, \dots\}$. (f) É função. $\text{Im}(f) = \mathbb{N}^*$.
2. (a) $f(-2) = -12, f(1) = 3, f(0) = 0, f(\frac{1}{3}) = \frac{19}{27}, f(0, 2) = 0, 408 = \frac{51}{125}$.
(b) $f(2) = -\frac{1}{3}, f(-2) = -3, f(\frac{1}{2}) = \frac{1}{3}, f(-x) = \frac{1+x}{1-x}, f(a) = \frac{1-a}{1+a}, f(a-1) = \frac{-a+2}{a}, f(-1)$ não existe pois -1 não pertence ao domínio de f .
(c) $f(1) = 2, f(-1) = -2, f(x) = x + \frac{1}{x}, f(\frac{1}{x}) = \frac{x^2+1}{x}, f(10000) = \frac{100000001}{10000}$.
(d) $f(0) = -4, f(1) = 0, f(\sqrt{2}) = 3\sqrt{2} - 2, f(x+1) = x^2 + 5x, f(-x) = x^2 - 3x - 4, f(x^2) = x^4 + 3x^2 - 4$.
(e) $f(-2) = 6, f(0) = 2, f(\frac{1}{2}) = 1, f(x+1) = 2|x|, f(10) = 18$.
(f) $f(-3) = -1, f(-2) = -1, f(2) = 1, f(3) = 1, f(-2342552) = -1, f(23498755) = 1$.
3. (a) $f : \mathbb{R} \rightarrow \mathbb{R}$ dada por $f(x) = x^2$.
(b) $f : \mathbb{Z} \rightarrow \mathbb{R}$ dada por $f(x) = |x| + 3$.
(c)
(d) $f : \mathbb{R} \rightarrow \mathbb{R}$ dada por $f(x) = (x+3)^2$.
(e)
(f) $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ dada por $f(x) = \frac{\sqrt{x} + 8}{3}$.
4. (a) A função que, a cada número real, eleva ao quadrado e adiciona 1.
(b)
(c) A função que eleva cada número do conjunto $\{-1, 0, 1\}$ eleva ao cubo e subtrai $\frac{3}{4}$.
(d) A função que, a cada número real, subtrai 1, eleva ao quadrado, multiplica por 2 e, em seguida, subtrai 3.
5. (a) Não está bem definida. (b) Está bem definida.
(c) (d)
(e) Não está bem definida.

6. Há mais de uma resposta possível em cada item desse exercício.

(a) Método (A). $f : [\frac{5}{3}, \infty) \rightarrow \mathbb{R}$ dada por $f(x) = \sqrt{3x - 5} - 3$.

(b) Correta.

(c)

(d)

(e) Método (A). $f : \mathbb{R} - \{1\} \rightarrow \mathbb{R}$ dada por $f(x) = \frac{x^2 - 1}{x^2 - 2x + 1}$.

7. (a) $D(f) = [-2, \infty) - \{-1\}$.

(b) $D(f) = [-2, \infty) - \{2\}$.

(c) $D(f) = \mathbb{R}$.

(d) $D(f) = \mathbb{R} - \{-\frac{3}{2}\}$.

(e)

(f) $D(f) = \mathbb{R}$.

(g)

(h)

(i) $D(f) = (-3, -1] \cup [1, 5)$.

(j) $D(f) = [-4, -3) \cup (-1, 2] \cup [3, \infty)$.

(k) $D(f) = [1, 2] \cup [3, 5)$.

8. (a) f e g são iguais.

(b) f e g não possuem relação.

(c)

(d) f e g são iguais.

(e) f é uma extensão e uma correstrrição de g .

(f) f é uma extensão de g .

(g) f e g são iguais.

(h) f é uma coextensão de g .

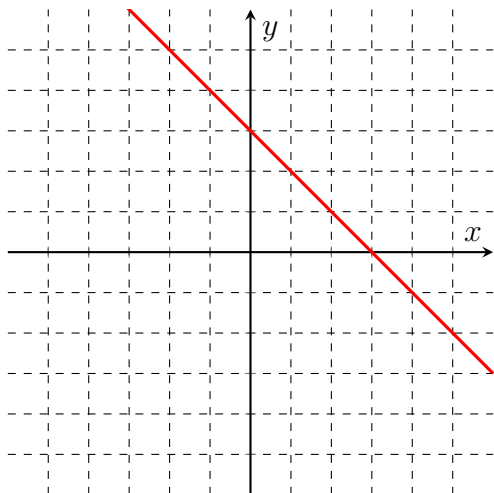
(i) f e g não possuem relação.

(j)

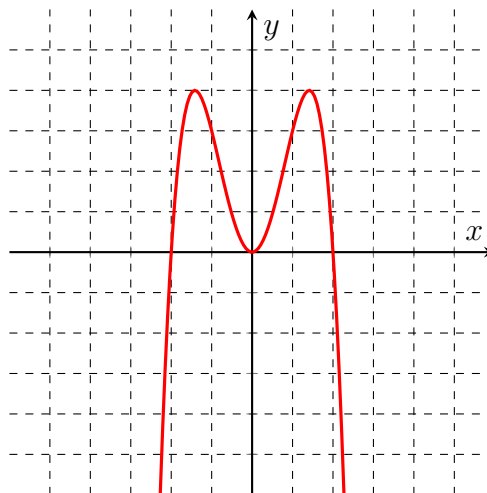
9. Sim. $f : \mathbb{R} \rightarrow \mathbb{R}$ dada por $f(x) = x^2$ e $g : \mathbb{R} \rightarrow \mathbb{R}_+$ dada por $g(x) = x^2$ possuem o mesmo gráfico, mas são funções diferentes.

10. Contradomínio.

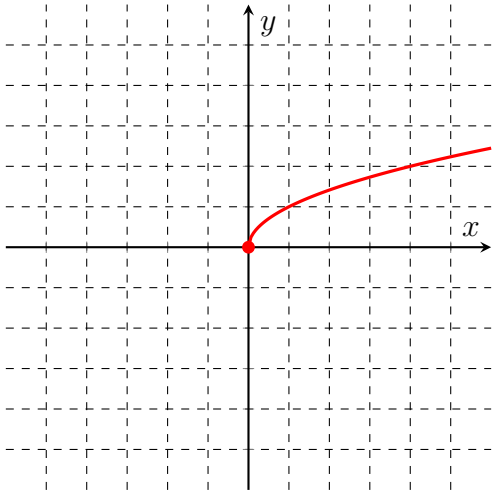
11. (a)



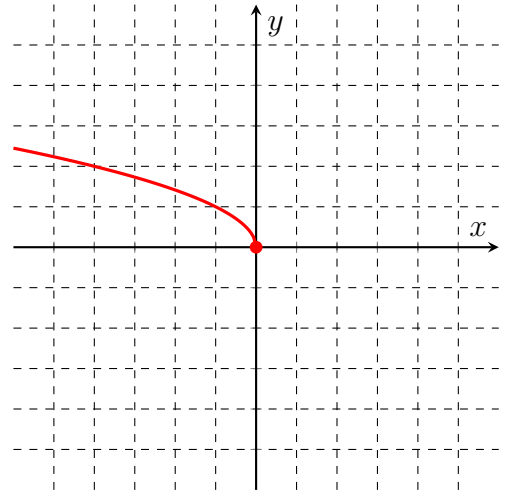
(b)



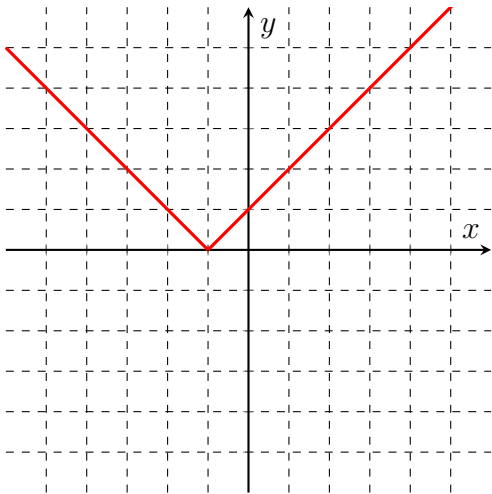
(c)



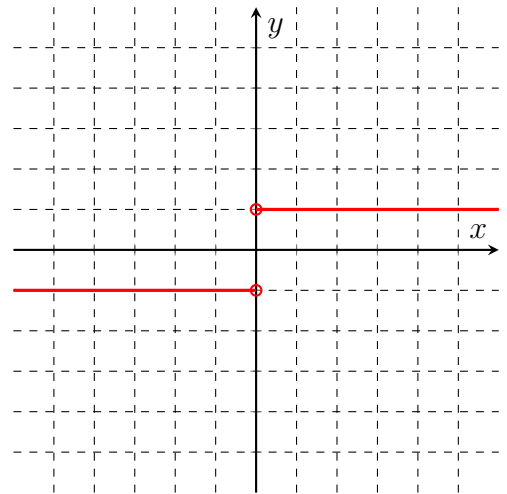
(d)



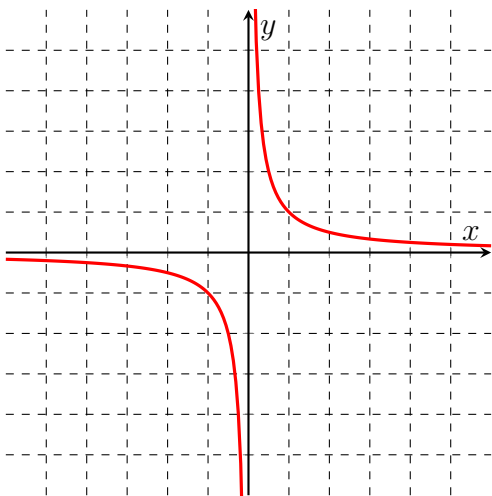
(e)



(f)



(g)



12.

(a) $\frac{f(a+h) - f(a)}{h} = 3.$

(c) $\frac{f(a+h) - f(a)}{h} = 0.$

(e) $\frac{f(a+h) - f(a)}{h} = \frac{1}{(a+h+1)(a+1)}.$

(g) $\frac{f(a+h) - f(a)}{h} = \frac{h}{\sqrt{a+h} + \sqrt{a}}.$

(i) $\frac{f(a+h) - f(a)}{h} = 3a^2 + 3ah + h^2.$

(b) $\frac{f(a+h) - f(a)}{h} = 2a + h.$

(d) $\frac{f(a+h) - f(a)}{h} = -\frac{1}{(a+1)(a+h+1)}.$

(f) $\frac{f(a+h) - f(a)}{h} = -\frac{2}{(a+h-1)(a-1)}.$

(h) $\frac{f(a+h) - f(a)}{h} = -5 + 8a + 4h.$

13.

(a) $\frac{f(x) - f(a)}{x - a} = 3.$

(c) $\frac{f(x) - f(a)}{x - a} = 0.$

(e) $\frac{f(x) - f(a)}{x - a} = \frac{1}{(x+1)(a+1)}.$

(g) $\frac{f(x) - f(a)}{x - a} = \frac{1}{\sqrt{x} + \sqrt{a}}.$

(i) $\frac{f(x) - f(a)}{x - a} = x^2 + xa + a^2.$

(b) $\frac{f(x) - f(a)}{x - a} = x + a.$

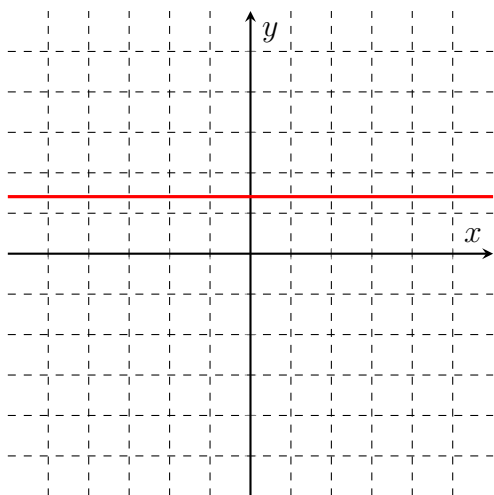
(d) $\frac{f(x) - f(a)}{x - a} = -\frac{1}{(x+1)(a+1)}.$

(f) $\frac{f(x) - f(a)}{x - a} = -\frac{2}{(x-1)(a-1)}.$

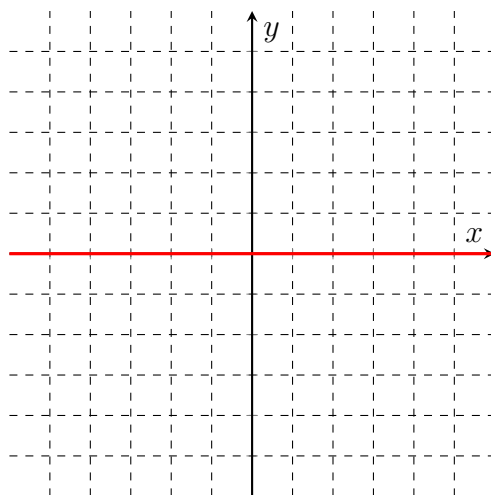
(h) $\frac{f(x) - f(a)}{x - a} = 4x + 4a - 5.$

14.

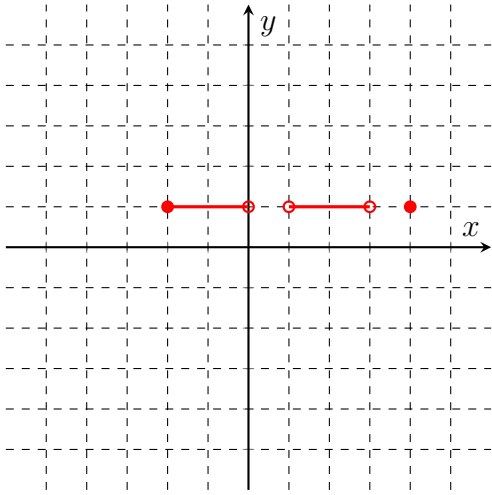
(a)



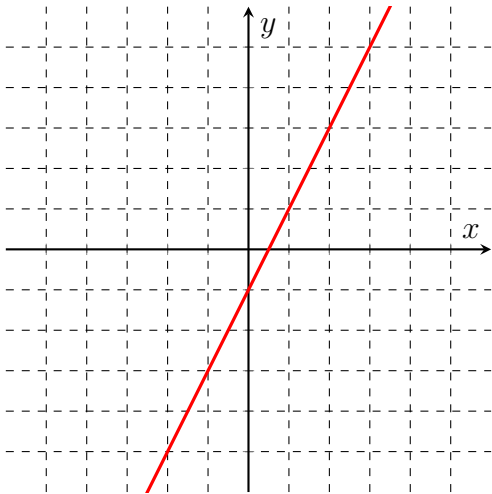
(b)



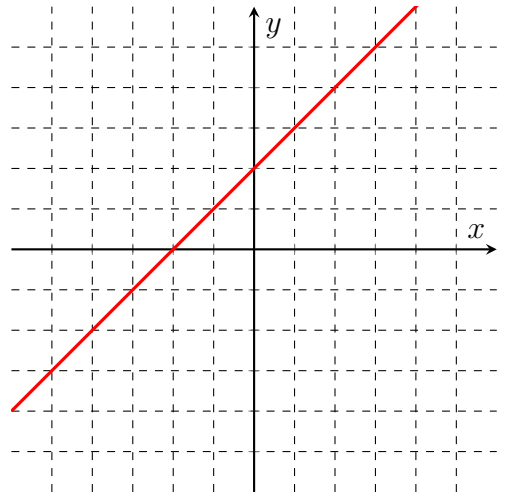
(c)



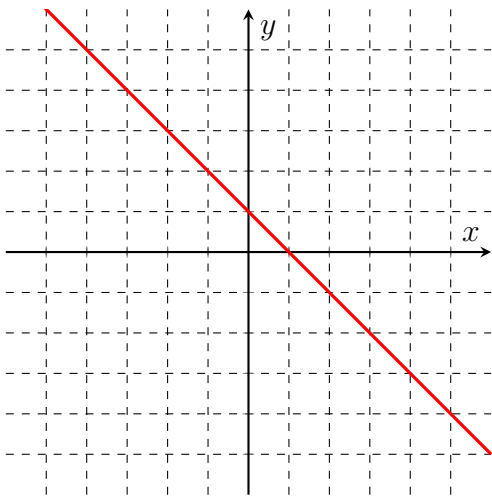
15. (a) $\text{Im}(f) = \mathbb{R}$.



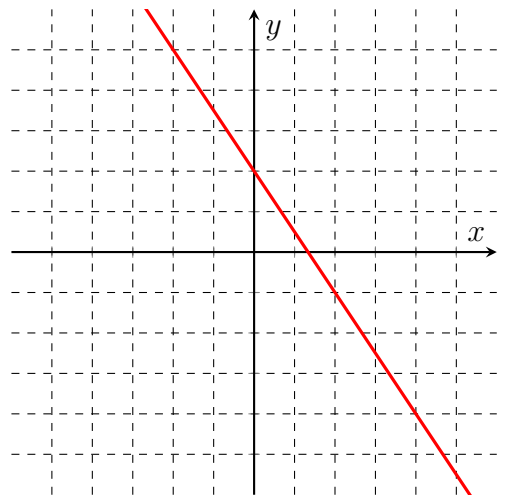
(b) $\text{Im}(f) = \mathbb{R}$.



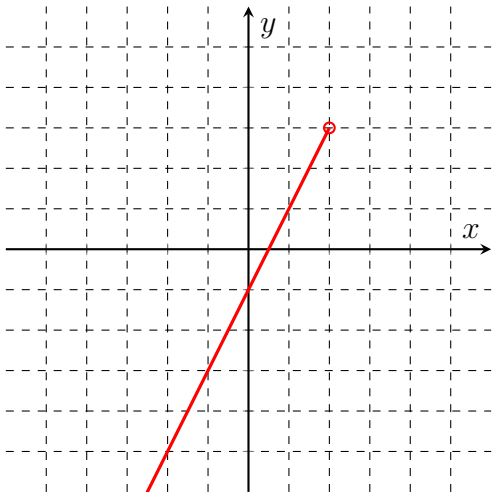
(c) $\text{Im}(f) = \mathbb{R}$.



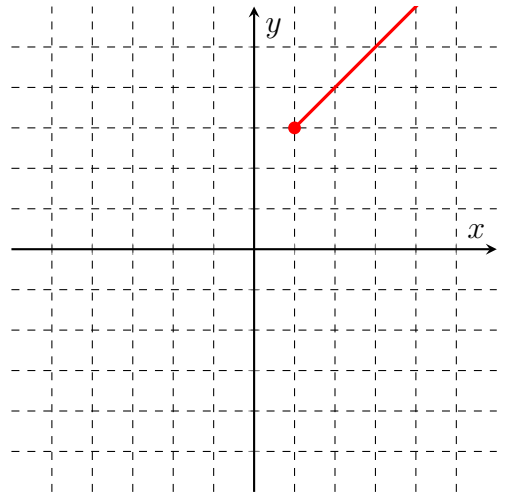
(d) $\text{Im}(f) = \mathbb{R}$.



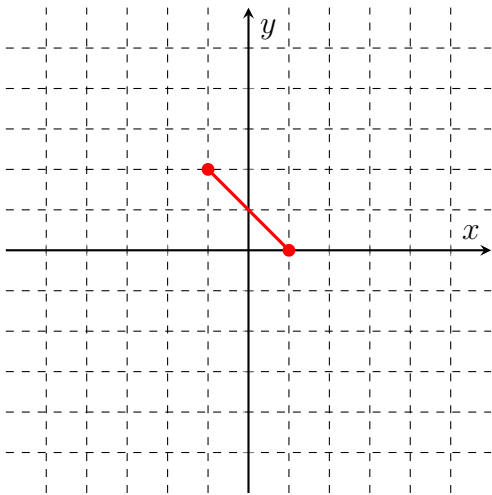
(e) $\text{Im}(f) = (-\infty, 3)$.



(f) $\text{Im}(f) = [3, \infty)$.



(g) $\text{Im}(f) = [0, 2]$.



16. (a) $(0, -1)$ e $(\frac{1}{2}, 0)$.

(c) $(0, 1)$ e $(1, 0)$.

(e)

(g) $(0, 1)$ e $(1, 0)$.

(b) $(0, 2)$ e $(-2, 0)$.

(d)

(f) Não possui intersecções com os eixos coordenados.

17. (a) $y = x - 5$.

(b) $y = 2$.

18. $f(3) = -1$.

19. $y = -\frac{1}{2}x - \frac{1}{2}$.

20. $y = \frac{3}{2}x + 4$.

21. (a) $f(x) = \frac{1}{3}x + \frac{1}{3}$.

(b) $f(x) = -\frac{1}{2}x + 4$.

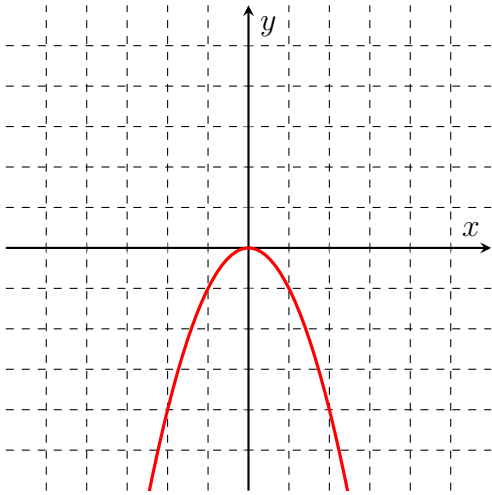
(c)

(d) $f(x) = 2x + 3$.

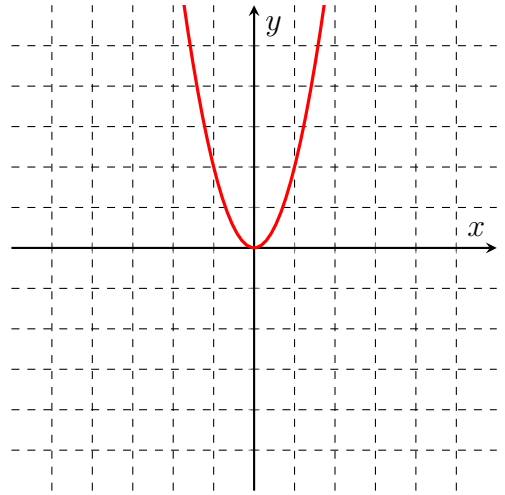
(e)

(f) $f(x) = -3x - 5$.

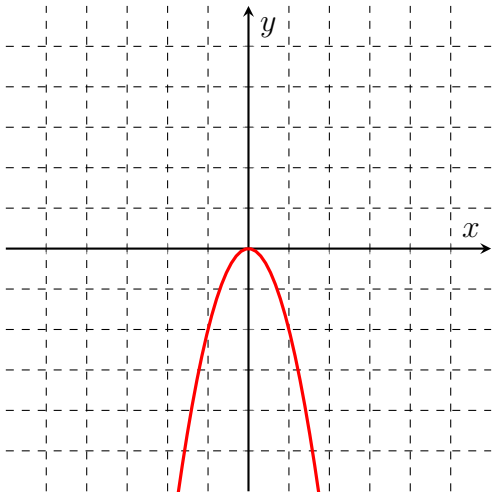
22. (a) $\text{Im}(f)(-\infty, 0]$



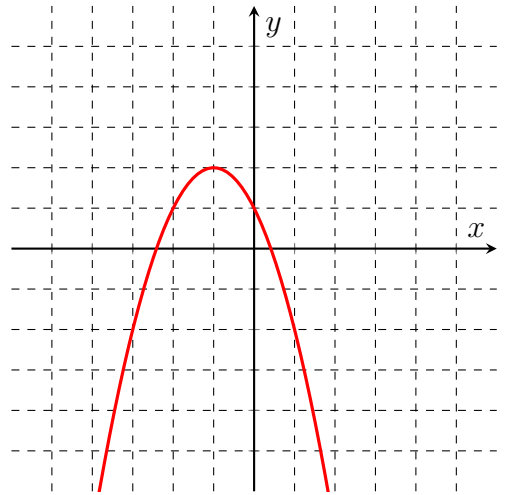
(b) $\text{Im}(f) = [0, \infty)$



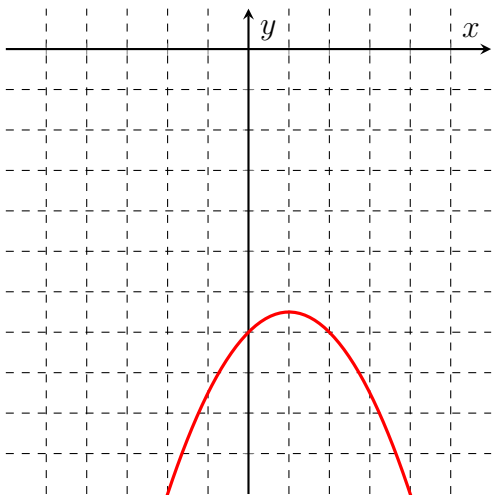
(c) $\text{Im}(f)(-\infty, 0]$



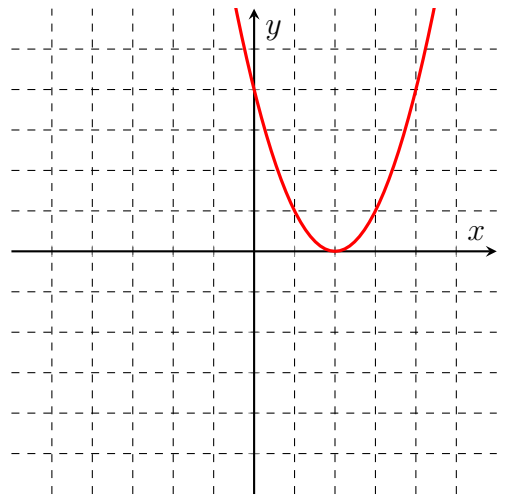
(d) $\text{Im}(f)(-\infty, 2]$



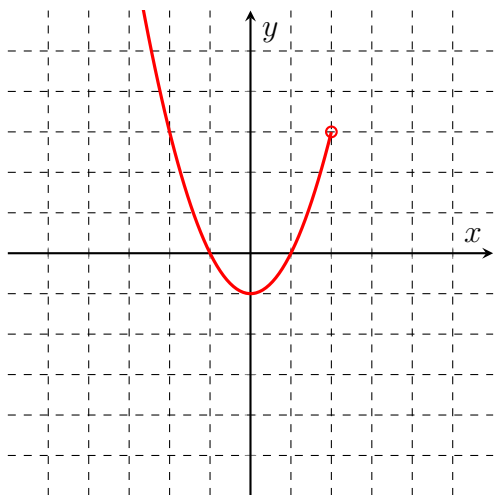
(e) $\text{Im}(f)(-\infty, -13/2]$



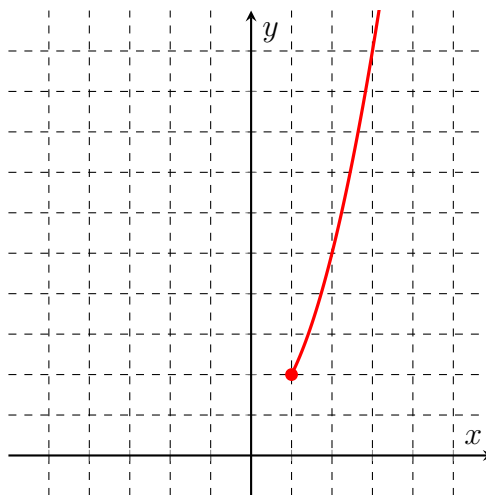
(f) $\text{Im}(f) = [0, \infty)$



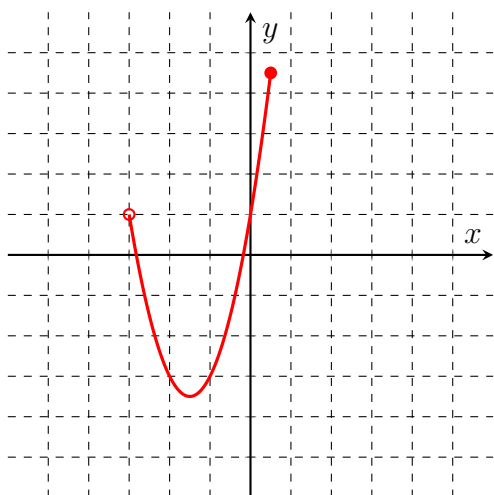
(g) $\text{Im}(f) = [-1, \infty)$



(h) $\text{Im}(f) = [2, \infty)$



(i) $\text{Im}(f) = [-7/2, 9/2]$

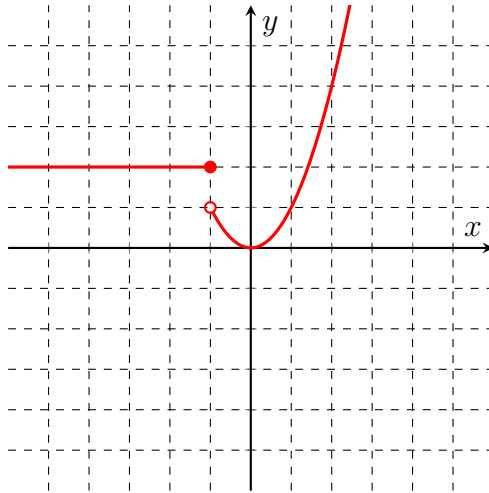


23. $f(x) = -x^2 - 7x + 10$.

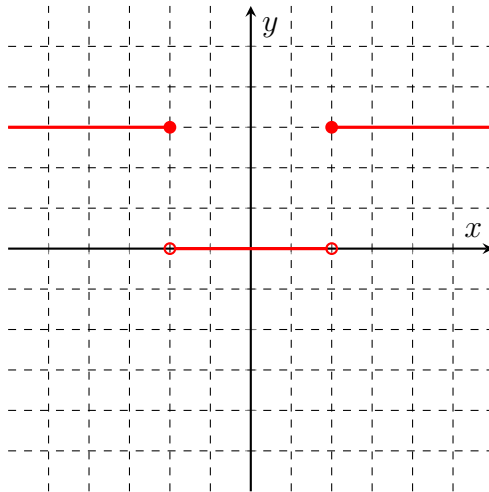
24. $m > -\frac{9}{16}$.

25. $m < -\frac{1}{4}$.

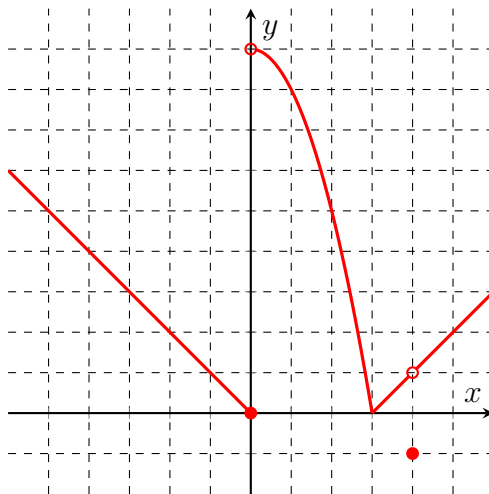
26. (a) $D(f) = \mathbb{R}$, $f(1) = 1$, $\text{Im}(f) = [0, \infty)$.



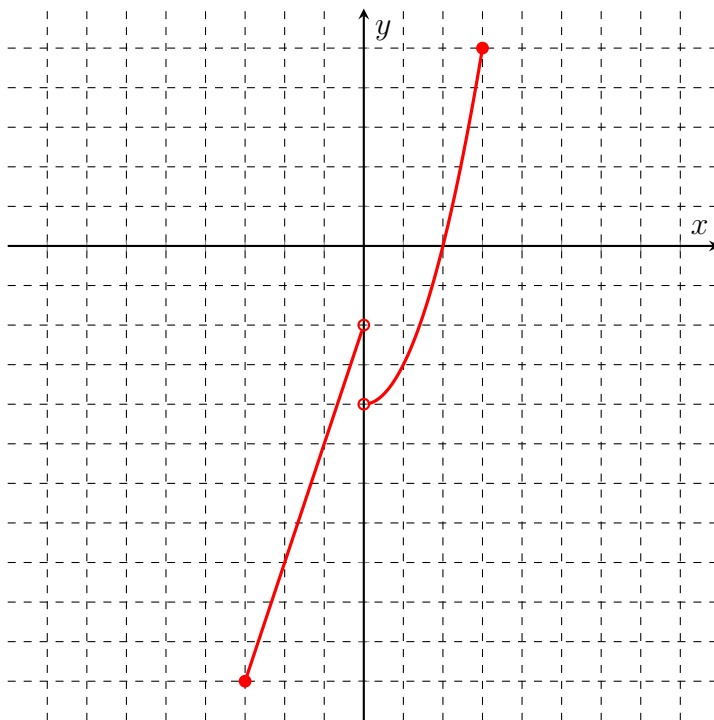
(b) $D(f) = \mathbb{R}$, $f(1) = 0$, $\text{Im}(f) = \{0, 3\}$.



(c) $D(f) = \mathbb{R}$, $f(1) = 8$, $\text{Im}(f) = [0, \infty) \cup \{-1\}$.



(d) $D(f) = [-3, 3] - \{0\}$, $f(1) = -3$, $\text{Im} = [-11, 5]$.



27.

(a) $y = \frac{4 - x^2}{2}$.

(b) $y = \frac{21 - 3x}{7}$.

(c) y não pode ser escrito em função de x .

(d) y não pode ser escrito em função de x .

(e)

(f) $y = 9 - x^2$.

(g)

(h)

(i) $y = -2|x|$.

(j)

(k) $y = \sqrt[3]{x}$.

(l) y não pode ser escrito em função de x .

28.

(a) $(f + g)(x) = 4x^2 + 2x - 1$, $D(f) = \mathbb{R}$.

$(f - g)(x) = -2x^2 + 2x + 1$, $D(f) = \mathbb{R}$.

$(fg)(x) = 3x^4 + 6x^3 - x^2 - 2x$, $D(f) = \mathbb{R}$.

$\left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x}{3x^2 - 1}$, $D(f) = \mathbb{R} - \left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$.

$(2f)(x) = 2x^2 + 4x$, $D(f) = \mathbb{R}$.

(b)

(c)

29. (a) $g(f(-2)) = -119$; (b) (c) $g(g(2)) = -2$;
 (d) $f(2 - x^2) = -3x^2 + 1$; (e) $g(f(x)) = -9x^2 + 30x - 23$; (f)
 (g) $g(g(x)) = -x^4 + 4x^2 - 2$.

30. (a) $(f \circ g)(x) = 3x - 5$, $\text{Dom}(f \circ g) = \mathbb{R}$;
 $(g \circ f)(x) = 3x - \frac{5}{2}$, $\text{Dom}(g \circ f) = \mathbb{R}$;
 $(f \circ f)(x) = 36x - 35$, $\text{Dom}(f \circ f) = \mathbb{R}$;
 $(g \circ g)(x) = \frac{x}{4}$, $\text{Dom}(g \circ g) = \mathbb{R}$.

(b) $(f \circ g)(x) = x + 2$, $\text{Dom}(f \circ g) = \mathbb{R}$;
 $(g \circ f)(x) = \sqrt[3]{x^3 + 2}$, $\text{Dom}(g \circ f) = \mathbb{R}$;
 $(f \circ f)(x) = x + 4$, $\text{Dom}(f \circ f) = \mathbb{R}$;
 $(g \circ g)(x) = \sqrt[9]{x}$, $\text{Dom}(g \circ g) = \mathbb{R}$.

(c)

(d) $(f \circ g)(x) = \frac{x^2 - 4x + 3}{\sqrt{x^2 - 4x + 3} + 1}$, $\text{Dom}(f \circ g) = (-\infty, 1] \cup [3, \infty)$;

$(g \circ f)(x) = \frac{x^2}{(\sqrt{x} + 1)^2} - \frac{4x}{\sqrt{x} + 1} + 3$, $\text{Dom}(g \circ f) = [0, \infty)$;

$(f \circ f)(x) = \frac{x}{(\sqrt{x} + 1) \left(\sqrt{\frac{x}{\sqrt{x} + 1}} + 1 \right)}$, $\text{Dom}(f \circ f) = [0, \infty)$;

$(g \circ g)(x) = x^4 - 8x^3 + 18x^2 - 8x$, $\text{Dom}(g \circ g) = \mathbb{R}$.

31. (a) $(f \circ g \circ h)(x) = \sqrt{x - 1} - 1$.

(b) $(f \circ g \circ h)(x) = \frac{1}{(x^2 + 2)^3}$.

(c)

(d) $(f \circ g \circ h)(x) = \sqrt{\frac{\sqrt[3]{x}}{\sqrt[3]{x} - 1}}$.

32. (a) $f(x) = \sqrt[3]{x}$, $g(x) = x^4 + 3$ e $h(x) = x - 4$.

(b)

(c) $f(x) = \frac{2}{x}$, $g(x) = x^6$ e $h(x) = x^3 - x + 3$.

33. $I(x) = x$.

34. $I(x) = x$.