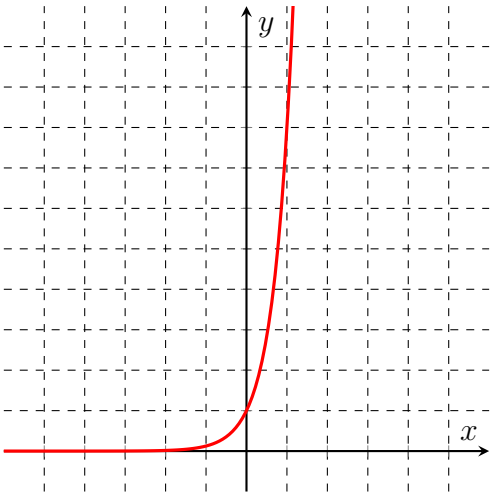
Universidade Federal de Santa Catarina
Centro de Ciências Físicas e Matemáticas
Departamento de Matemática

MTM3100 - Pré-cálculo

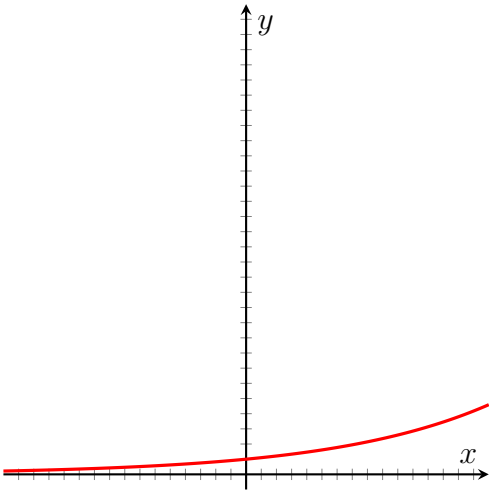
Gabarito da 12ª lista complementar de exercícios

1. $f(0{,}7) = 0{,}850$, $f(\sqrt{7}/2) = 0{,}176$, $f(2/3) = 0{,}874$ e $f(-1/2) = \dfrac{9}{4} = 2{,}250$.

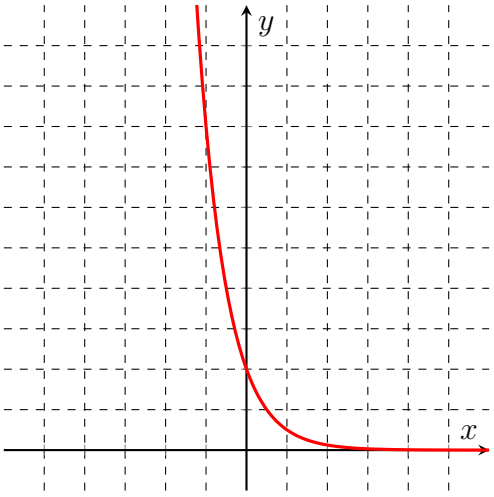
2. **(a)**



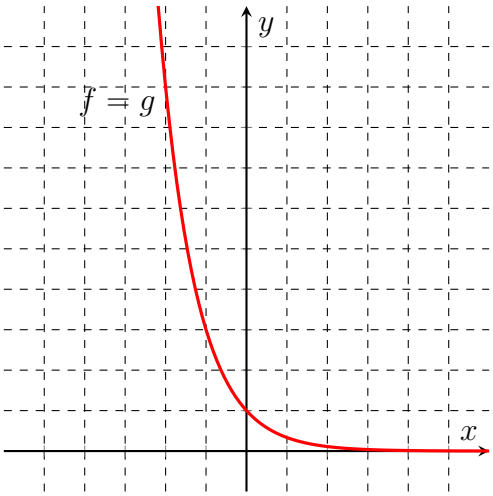
(b)



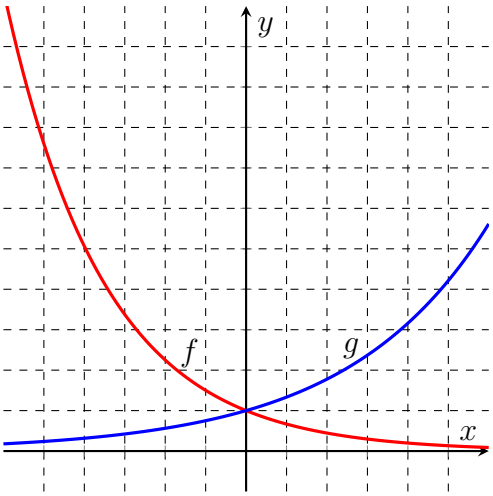
(c)



3. **(a)**



(b)

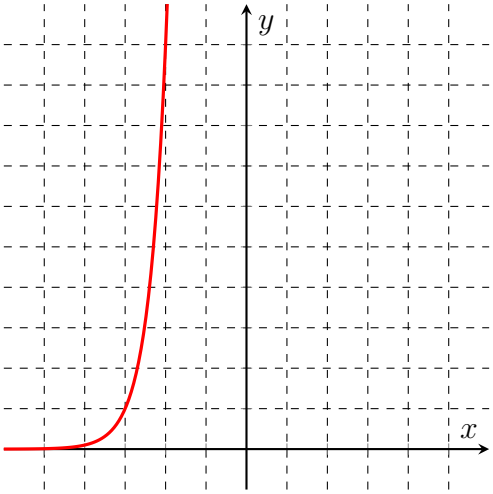


4.

(a) $f(x) = 5^x$.

(b) $f(x) = \left(\frac{1}{2}\right)^x$.

5. **(a)**



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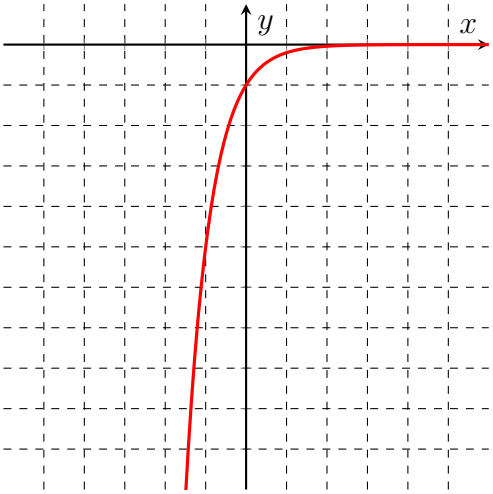
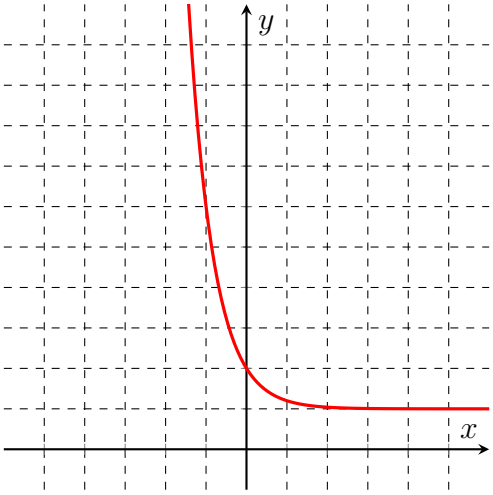
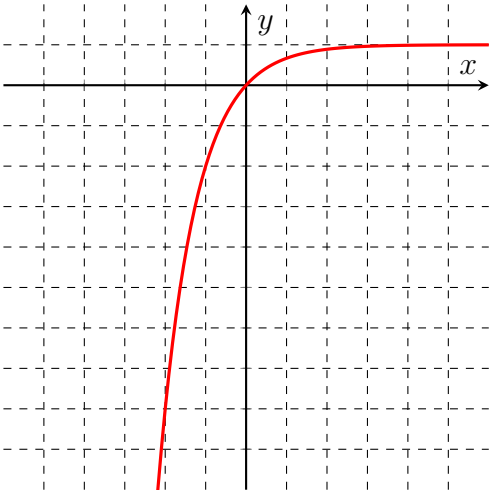


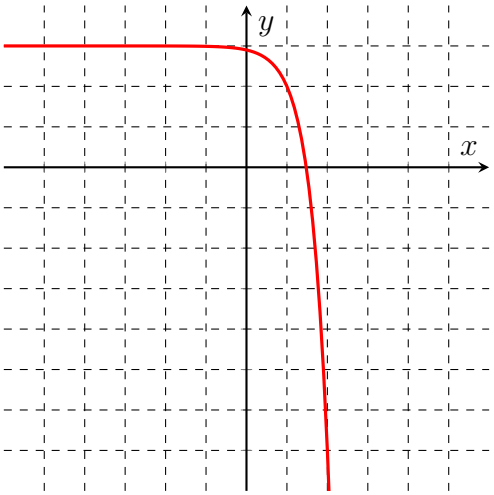
(c)



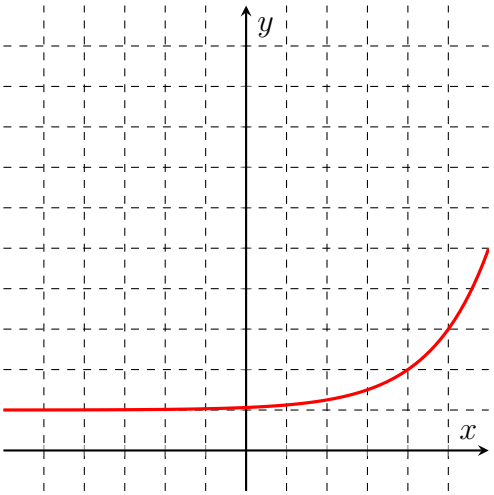
(d)



(e)



(f)



6.

7.

8. **(a)** $N(t) = 320 \cdot 2^t$.

(b) $N(8) = 320 \cdot 2^8 = 81.920$.

9. R$ 158.687,43.

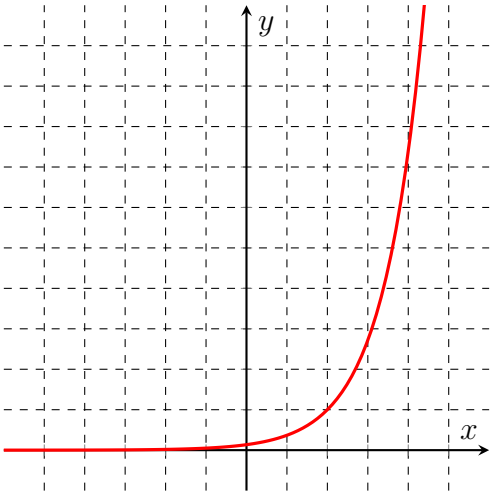
10. $M = C(1+i)^n$.

11. $i_a = (1+0{,}007)^{12} - 1 \cong 0{,}0873 = 8{,}73\%$.

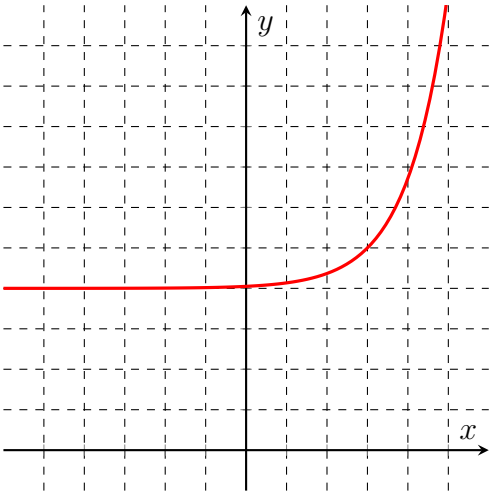
12. $i_m = \sqrt[12]{1+0{,}1} - 1 \cong 0{,}00797 = 0{,}797\%$.

13. $e \cong 2{,}71828$.

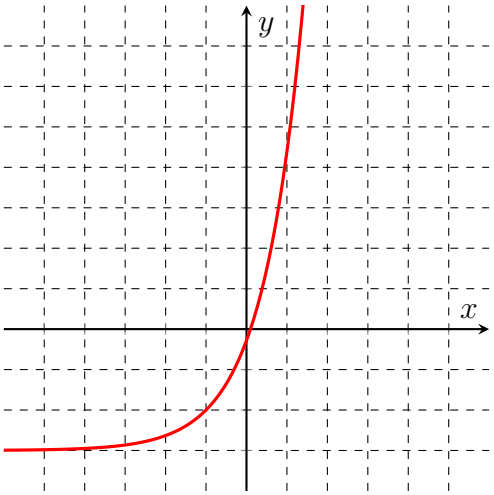
14. **(a)**



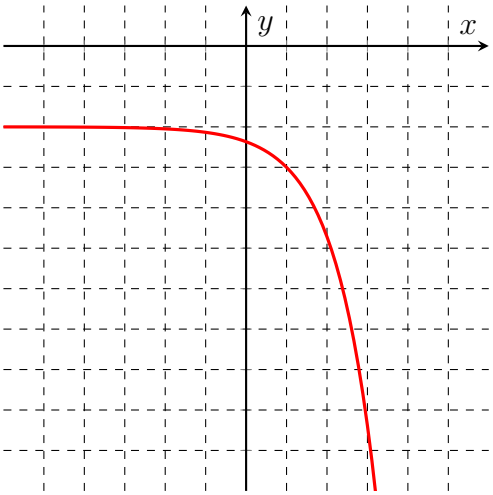
(b)



(c)



(d)



15. **(a)** $m(0) = 13\,kg$.

(b) $m(45) = 13e^{-0{,}015 \cdot 45} \cong 6{,}619\,kg$.

(c) $6{,}381\,kg$.

16. Sabemos que um objeto em queda livre tem sua velocidade dada por $v(t) = v_0 + gt$, em que v_0 é a velocidade inicial e g é a aceleração da gravidade. Porém, esse resultado é válido quando desprezamos a resistência do ar. Em uma situação mais fiel, onde a resistência do ar é considerada, a modelagem é bem diferente. Você verá em Cálculo 1 que quando a resistência do ar é modelada proporcional à velocidade do objeto, então uma expressão típica da formulação da velocidade de queda é dado por

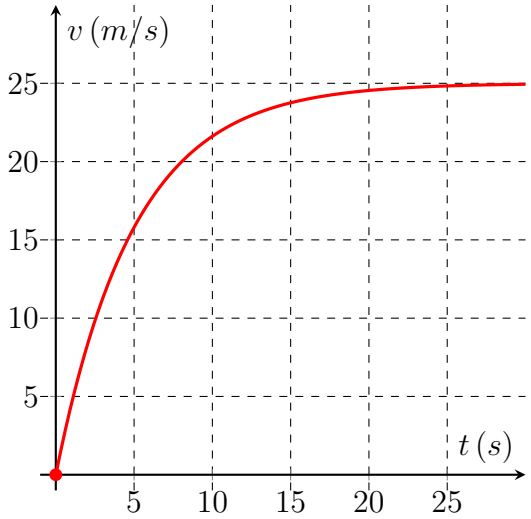
$$v(t) = 25(1 - e^{-0{,}2t}),$$

em que t está medido em segundos e $v(t)$ em metros por segundo. Considerando esta equação, determine o que se pede.

(a) $v(0) = 0\,m/s$.

(b) $v(5) = 25(1 - e^{-0{,}2 \cdot 5}) \cong 15{,}80\,m/s$ e $v(5) = 25(1 - e^{-0{,}2 \cdot 10}) \cong 21{,}62\,m/s$.

(c)



(d) $v_t = 25\,m/s$.

17.

18.

19.
- Tangente hiperbólica. $\mathbb{R}$;
- Cotangente hiperbólica. $\mathbb{R}^*$;
-
-

.

20.

21. **(a)** $\log_5 5^4 = 4$. **(b)** $\log_4 64 = 3$. **(c)** $\log_3 9 = 2$. **(d)**
(e) $\log_2 32 = 5$. **(f)** **(g)** $\log_6 1 = 0$. **(h)**
(i) $e^{\ln \pi} = \pi$. **(j)** $10^{\log 87} = 87$. **(k)** **(l)**
(m) $\log_4 \frac{1}{2} = -\frac{1}{2}$. **(n)** $\log_4 8 = \frac{3}{2}$.

22. **(a)** 3. **(b)** -2.
(c) **(d)** 99.
(e) 4. **(f)** 200.

23. **(a)** $\log 6^{10} = 10 \log 6$.

(b) $\ln \sqrt{z} = \frac{1}{2} \ln z$.

(c) $\log_6 \sqrt[4]{17} = \frac{1}{4} \log_6 17$.

(d)

(e) $\ln \sqrt{ab} = \frac{1}{2} \ln a + \frac{1}{2} \ln b$.

(f) $\log \left(\frac{x^3 y^4}{z^6} \right) = 3 \log x + 4 \log |y| - 6 \log |z|$.

(g)

(h) $\log_5 \sqrt{\frac{x-1}{x+1}} = \frac{1}{2} \log_5 (x-1) - \frac{1}{2} \log_5 (x+1)$.

(i)

(j) $\ln \frac{3x^2}{(x+1)^{10}} = \ln 3 + 2 \ln |x| - 10 \ln |x+1|$.

(k)

(l) $\log \left(\frac{x}{\sqrt[3]{1-x}} \right) = \log x - \frac{1}{3} \log (1-x)$.

(m) $\ln \sqrt{x \sqrt{y \sqrt{z}}} = \frac{1}{2} \ln x + \frac{1}{4} \ln y + \frac{1}{8} \ln z$.

(n) $\ln \left(\frac{x^3 \sqrt{x-1}}{3x+4} \right) = 3 \ln x + \frac{1}{2} \ln (x-1) - \ln (3x+4)$.

24. **(a)** $\ln 5 + 2\ln x + 3\ln(x^2+5) = \ln(5x^2(x^2+5)^3)$.

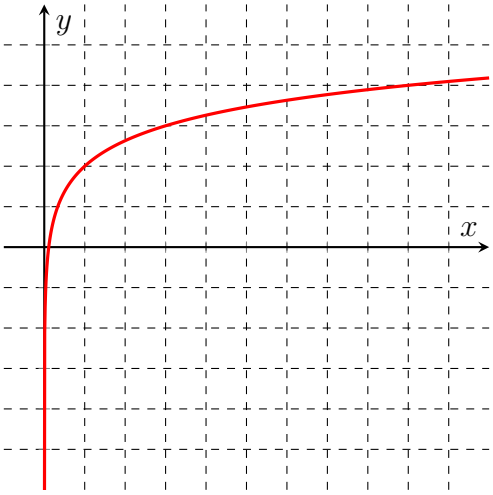
(b)

(c) $\dfrac{1}{3}\log(x+2)^3 + \dfrac{1}{2}[\log x^4 - \log(x^2-x-6)^2] = \log\left(\dfrac{(x+2)x^2}{|x^2-x-6|}\right)$.

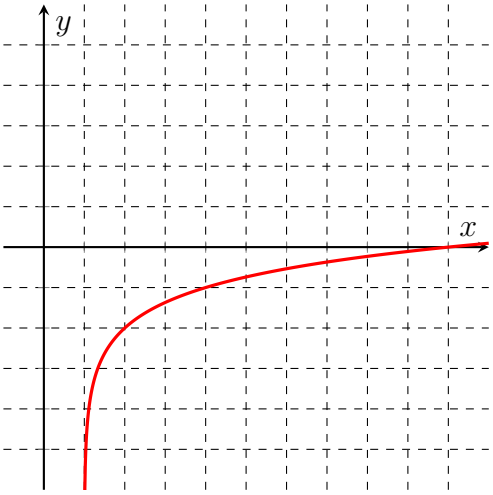
(d) $\log_a b + c\log_a d - r\log_a s = \log_a\left(\dfrac{bd^c}{s^r}\right)$.

25. $(\log_2 5)(\log_5 7) = \log_2 7$.

26. **(a)**



(b)



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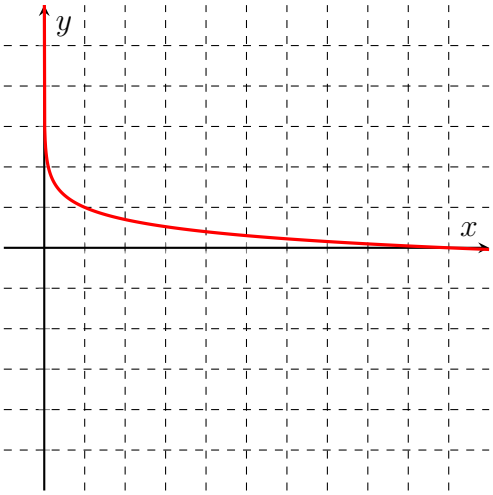

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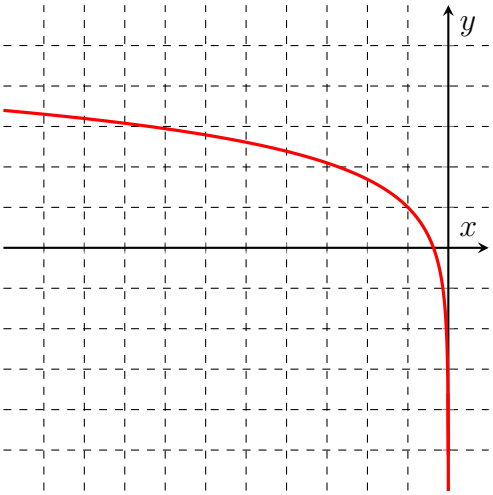
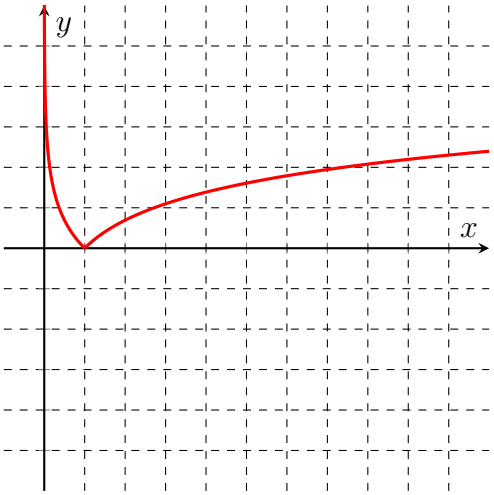
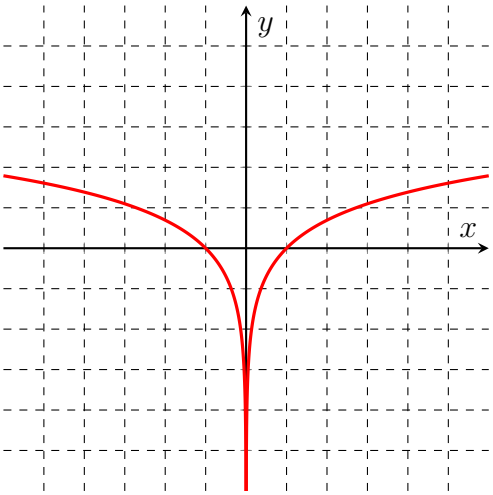


(e)



(f)



27. **(a)** $\mathrm{Dom}(f) = (0, 1)$.

(b) $\mathrm{Dom}(f) = [2, 10)$.

(c) $\mathrm{Dom}(f) = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$.

(d) $\mathrm{Dom}(f) = (4, 5) \cup (5, \infty)$.

(e) $\mathrm{Dom}(f) = (1, \infty)$.

(f) $\mathrm{Dom}(f) = (e, \infty)$.

28. $f^{-1}(x) = \log_2\left(\dfrac{x}{1-x}\right)$ e $\mathrm{Dom}(f^{-1}) = (0, 1)$.

29. **(a)** 2.

(b) 2.

30. $\log(2^{200}) = 200 \log 2 \cong 60{,}2$ e, portanto, 2^{200} possui 61 algarismos.