Universidade Federal de Santa Catarina
Centro de Ciências Físicas e Matemáticas
Departamento de Matemática

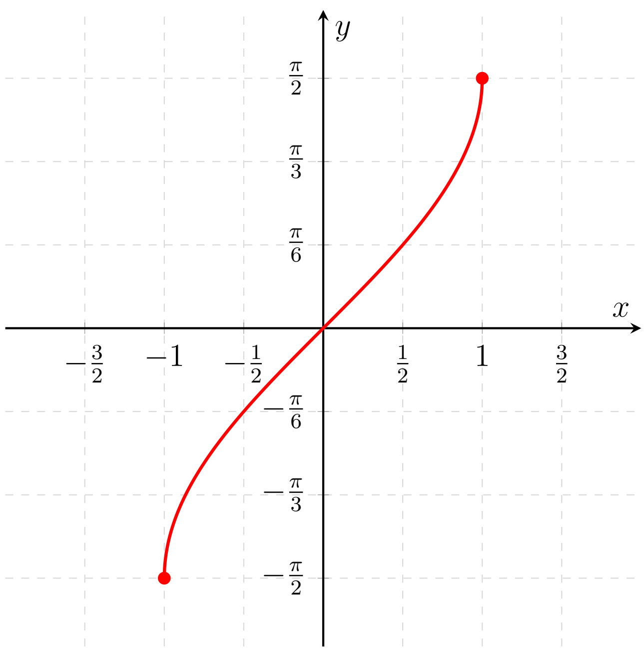
MTM3100 - Pré-cálculo

Gabarito parcial da 14ª lista de exercícios

1.
- **(a)** $S = \{k\pi \mid k \in \mathbb{Z}\}$.
- **(b)** $S = \{\pi/2 + 2k\pi \mid k \in \mathbb{Z}\}$.
- **(c)** $S = \{\pi/6 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{5\pi/6 + 2k\pi \mid k \in \mathbb{Z}\}$.
- **(d)** $x = 0$.
- **(e)** $x = \dfrac{\pi}{2}$.
- **(f)** $x = \dfrac{\pi}{6}$.

2.
- **(a)** $\operatorname{arcsen} 1 = \dfrac{\pi}{2}$.
- **(b)**
- **(c)** $\operatorname{arcsen}(\sqrt{2}/2) = \dfrac{\pi}{6}$.
- **(d)** $\operatorname{arcsen}(-1/2) = -\dfrac{\pi}{6}$.
- **(e)**
- **(f)** $\operatorname{arcsen}(-\sqrt{3}/2) = -\dfrac{\pi}{3}$.

3.



4.
- **(a)** $\operatorname{sen}(\operatorname{arcsen}(-1/2)) = -1/2$.
- **(b)** $\operatorname{sen}(\operatorname{arcsen}(1/2)) = 1/2$.
- **(c)**
- **(d)** $\operatorname{sen}(\operatorname{arcsen}(1/5)) = 1/5$.
- **(e)** $\operatorname{sen}(\operatorname{arcsen} x) = x$.
- **(f)** $\operatorname{arcsen}(\operatorname{sen}(\pi/6)) = \pi/6$.
- **(g)** $\operatorname{arcsen}(\operatorname{sen}(-\pi/2)) = -\pi/2$.
- **(h)** $\operatorname{arcsen}(\operatorname{sen}(4\pi/3)) = -\pi/3$.
- **(i)**
- **(j)** $\operatorname{arcsen}(\operatorname{sen}(20\pi/17)) = -3\pi/17$.
- **(k)** $\operatorname{arcsen}(\operatorname{sen} x) = y$, em que y é o único número pertencente a $[-\pi/2, \pi/2]$ tal que $\operatorname{sen} y = \operatorname{sen} x$.

5. **(a)** $S = \{\pi/2 + k\pi \mid k \in \mathbb{Z}\}$.

(b) $S = \{2k\pi \mid k \in \mathbb{Z}\}$.

(c) $S = \{3\pi/4 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{5\pi/4 + 2k\pi \mid k \in \mathbb{Z}\}$.

(d)

(e) $x = 0$.

(f) $x = \dfrac{3\pi}{4}$.

6. **(a)** $\arccos 1 = 0$.

(b) $\arccos(\sqrt{3}/2) = \dfrac{\pi}{6}$.

(c)

(d)

(e) $\arccos 0 = \dfrac{\pi}{2}$.

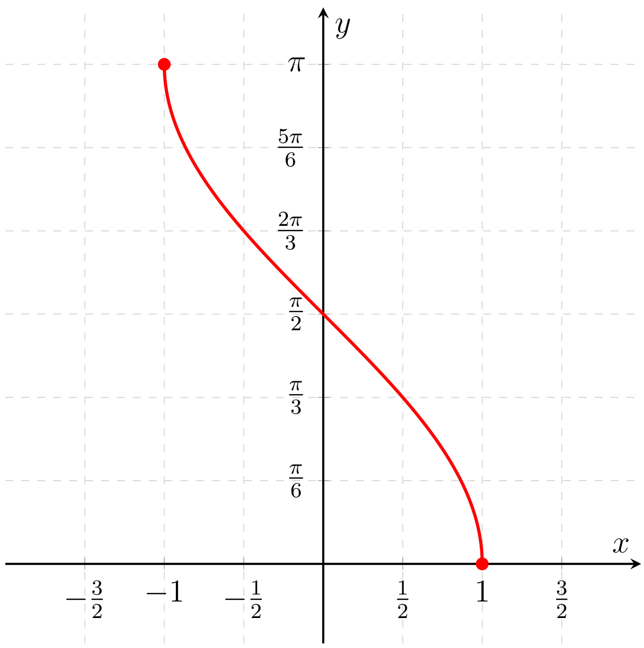
(f)

(g) $\arccos(-\sqrt{2}/2) = \dfrac{3\pi}{4}$.

(h) $\arccos(-\sqrt{3}/2) = \dfrac{5\pi}{6}$.

(i)

7.



8. **(a)** $\cos(\arccos(-1/2)) = -1/2$.

(b) $\cos(\arccos(\sqrt{3}/2)) = \sqrt{3}/2$.

(c) $\cos(\arccos(1/34)) = 1/34$.

(d) $\cos(\arccos x) = x$.

(e) $\arccos(\cos(\pi/6)) = \pi/6$.

(f) $\arccos(\cos(-\pi/2)) = \pi/2$.

(g) $\arccos(\cos(4\pi/3)) = 2\pi/3$.

(h) $\arccos(\cos(20\pi/17)) = 14\pi/17$.

(i) $\arccos(\cos x) = y$, em que y é o único número pertencente a $[0, \pi]$ tal que $\cos y = \cos x$.

9. **(a)** $S = \{k\pi \mid k \in \mathbb{Z}\}$.

(b) $S = \{\pi/4 + k\pi \mid k \in \mathbb{Z}\}$.

(c)

(d) $x = 0$.

(e)

(f) $x = \dfrac{\pi}{6}$.

10.

(a) $\operatorname{arctg}\sqrt{3} = \frac{\pi}{3}$.

(b) $\operatorname{arctg} 1 = \frac{\pi}{4}$.

(c) $\operatorname{arctg}(\sqrt{3}/3) = \frac{\pi}{6}$.

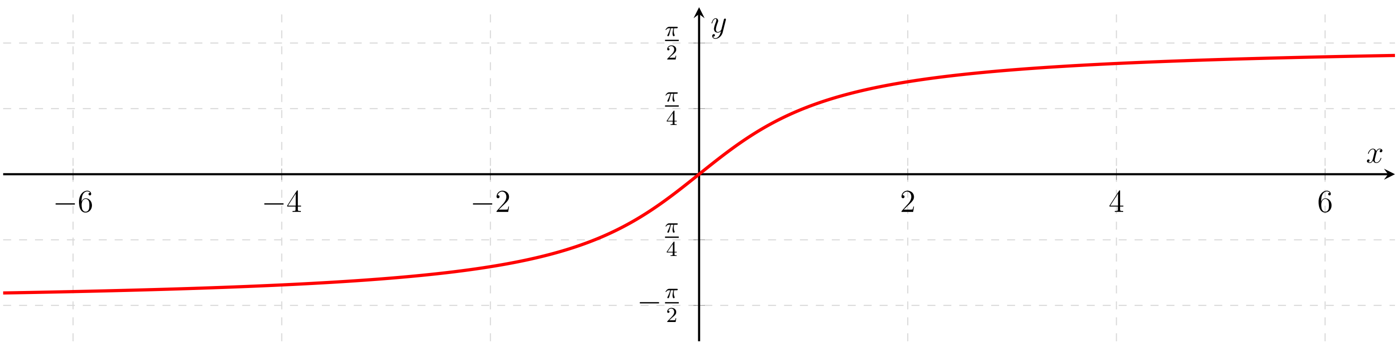
(d)

(e)

(f) $\operatorname{arctg}(-1) = -\frac{\pi}{4}$.

(g) $\operatorname{arctg}(-\sqrt{3}) = -\frac{\pi}{3}$.

11.



12. Aproximadamente $\frac{\pi}{2}$.

13.

(a) $\operatorname{tg}(\operatorname{arctg}\sqrt{3}) = \sqrt{3}$.

(b)

(c) $\operatorname{tg}(\operatorname{arctg}(-361)) = -361$.

(d) $\operatorname{tg}(\operatorname{arctg} x) = x$.

(e) $\operatorname{arctg}(\operatorname{tg}(\pi/6)) = \pi/6$.

(f) $\operatorname{arctg}(\operatorname{tg}(-2\pi/3)) = \pi/3$.

(g)

(h)

(i) $\operatorname{arctg}(\operatorname{tg} x) = y$, em que y é o único número pertencente a $(-\pi/2, \pi/2)$ tal que $\operatorname{tg} y = \operatorname{tg} x$.

14.

(a) $\cos(\operatorname{arcsen} x) = \sqrt{1-x^2}$;

(b) $\operatorname{sen}(\arccos x) = \sqrt{1-x^2}$.

15.

16.

(a) $\operatorname{sen}(-x) = -\operatorname{sen} x$, isto é, a função seno é ímpar.

(b) $\cos(-x) = \cos x$, isto é, a função cosseno é par.

(c) $\operatorname{tg}(-x) = -\operatorname{tg} x$, isto é, a função tangente é ímpar.

(d) $\operatorname{cotg}(-x) = -\operatorname{cotg} x$, isto é, a função cotangente é ímpar.

(e) $\sec(-x) = \sec x$, isto é, a função secante é par.

(f) $\operatorname{cossec}(-x) = -\operatorname{cossec} x$, isto é, a função cossecante é ímpar.

17.

(a) $\cos x + \operatorname{tg} x \operatorname{sen} x$.

Solução. $\cos x + \left(\frac{\operatorname{sen} x}{\cos x}\right) \operatorname{sen} x = \frac{\cos^2 x + \operatorname{sen}^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$.

(b) $\cos x \operatorname{tg} x = \operatorname{sen} x$.

(c) $\operatorname{sen} x \sec x = \operatorname{tg} x$.

(d)

(e) $\operatorname{sen} x + \operatorname{cotg} x \cos x = \operatorname{cossec} x$.

(f) $\frac{\sec x - \cos x}{\operatorname{sen} x} = \operatorname{tg} x$.

(g) $\frac{\operatorname{sen} x \sec x}{\operatorname{tg} x} = 1$.

(h) $\cos^3 x + \operatorname{sen}^2 x \cos x = \cos x$.

(i)

(j) $\frac{1 + \operatorname{sen} x}{\cos x} + \frac{\cos x}{1 + \operatorname{sen} x} = 2 \sec x$.

(k)

18.

19.

(a) $\operatorname{sen} 75° = \dfrac{\sqrt{2}(1+\sqrt{3})}{4}$;

(b) $\cos 105° = \dfrac{\sqrt{2}(1-\sqrt{3})}{4}$;

(c) $\operatorname{tg} 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$;

(d) $\operatorname{sen}(19\pi/12) = -\dfrac{\sqrt{2}(1+\sqrt{3})}{4}$;

(e) $\cos(17\pi/12) = \dfrac{\sqrt{2}(1-\sqrt{3})}{4}$;

(f) $\operatorname{tg}(-\pi/12) = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$.

20.

21.

(a) $\operatorname{sen}(2x) = \dfrac{120}{169}$, $\cos(2x) = \dfrac{119}{169}$ e $\operatorname{tg}(2x) = \dfrac{120}{119}$.

(b) $\operatorname{sen}(2x) = -\dfrac{24}{25}$, $\cos(2x) = -\dfrac{7}{25}$ e $\operatorname{tg}(2x) = \dfrac{24}{7}$.

(c)

(d) $\operatorname{sen}(2x) = -\dfrac{\sqrt{15}}{8}$, $\cos(2x) = \dfrac{7}{8}$ e $\operatorname{tg}(2x) = -\dfrac{\sqrt{15}}{7}$.

22.

(a) $\operatorname{sen} 15° = \dfrac{\sqrt{2}(\sqrt{3}-1)}{4}$;

(b) $\cos 15° = \dfrac{\sqrt{2}(1+\sqrt{3})}{4}$;

(c) $\operatorname{tg} 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$;

(d) $\cos 165° = -\dfrac{\sqrt{2}(1+\sqrt{3})}{4}$;

(e) $\operatorname{sen} 22{,}5° = \dfrac{\sqrt{2-\sqrt{2}}}{2}$;

(f) $\cos(\pi/8) = \dfrac{\sqrt{2+\sqrt{2}}}{2}$;

(g) $\operatorname{sen}(3\pi/8) = \dfrac{\sqrt{2+\sqrt{2}}}{2}$;

(h) $\operatorname{tg}(5\pi/12) = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$.

23.

(a) $\operatorname{sen}(x/2) = \dfrac{\sqrt{10}}{10}$, $\cos(x/2) = \dfrac{3\sqrt{10}}{10}$ e $\operatorname{tg}(x/2) = \dfrac{1}{3}$.

(b) $\operatorname{sen}(x/2) = \dfrac{3\sqrt{10}}{10}$, $\cos(x/2) = -\dfrac{\sqrt{10}}{10}$ e $\operatorname{tg}(x/2) = -3$.

24. **(a)** $\operatorname{sen}(2x)\cos(3x) = \frac{1}{2}[\operatorname{sen}(5x) - \operatorname{sen} x]$;

(b) $\operatorname{sen} x \operatorname{sen}(5x) = \frac{1}{2}[\cos(4x) - \cos(6x)]$;

(c)

(d) $\operatorname{sen}(4x)\cos x = \frac{1}{2}[\operatorname{sen}(5x) + \operatorname{sen}(3x)]$.

25. **(a)** $\operatorname{sen}(5x) + \operatorname{sen}(3x) = 2\operatorname{sen}(4x)\cos x$;

(b) $\operatorname{sen}(x) - \operatorname{sen}(4x) = -2\cos(5x/2)\operatorname{sen}(3x/2)$;

(c) $\cos(4x) - \cos(6x) = 2\operatorname{sen}(5x)\operatorname{sen} x$;

(d)

26. **(a)** $S = \{5\pi/4 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\pi/4 + 2k\pi \mid k \in \mathbb{Z}\}$;

(b) $S = \{\arccos(-1/4) + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\arccos(-1/4) + 2k\pi \mid k \in \mathbb{Z}\}$
$= \{\arccos(1/4) + (2k+1)\pi \mid k \in \mathbb{Z}\} \cup \{-\arccos(1/4) + (2k+1)\pi \mid k \in \mathbb{Z}\}$;

(c) $S = \{\pi/6 + k\pi \mid k \in \mathbb{Z}\} \cup \{-\pi/6 + k\pi \mid k \in \mathbb{Z}\}$;

(d) $S = \{\operatorname{arcsen}(1/3) + k\pi \mid k \in \mathbb{Z}\} \cup \{-\operatorname{arcsen}(1/3) + k\pi \mid k \in \mathbb{Z}\}$;

(e) $S = \{\pi/3 + k\pi \mid k \in \mathbb{Z}\} \cup \{-\pi/3 + k\pi \mid k \in \mathbb{Z}\}$;

(f) $S = \{\operatorname{arcsen}(1/3) + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\operatorname{arcsen}(1/3) + (2k+1)\pi \mid k \in \mathbb{Z}\}$;

(g) $S = \{-\pi/6 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{\pi/6 + (2k+1)\pi \mid k \in \mathbb{Z}\} \cup \{\pi/2 + 2k\pi \mid k \in \mathbb{Z}\}$.

(h) $S = \{\arccos(1/3) + 2k\pi \mid k \in \mathbb{Z}\} \cup \{-\arccos(1/3) + 2k\pi \mid k \in \mathbb{Z}\} \cup \{(2k+1)\pi \mid k \in \mathbb{Z}\}$.

(i) $S = \{\pi/9 + 2k\pi/3 \mid k \in \mathbb{Z}\} \cup \{-\pi/9 + 2k\pi/3 \mid k \in \mathbb{Z}\}$;

(j) $S = \{8\pi/3 + 4k\pi \mid k \in \mathbb{Z}\}$;

(k) $S = \{11\pi/18 + 2k\pi/3 \mid k \in \mathbb{Z}\} \cup \{7\pi/18 + 2k\pi/3 \mid k \in \mathbb{Z}\}$;

(l) $S = \{\pi/2 + k\pi \mid k \in \mathbb{Z}\} \cup \{-\pi/6 + 2k\pi \mid k \in \mathbb{Z}\} \cup \{\pi/6 + (2k+1)\pi \mid k \in \mathbb{Z}\}$;

(m) $S = \{k\pi/2 \mid k \in \mathbb{Z}\}$.

27. $E(t) = 220\sqrt{2}\cos(120\pi t) \cong 311\cos(120\pi t)$.

28. **(a)** $x(t) = 0{,}05\cos(10\sqrt{3}t)$.

(b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(c) A frequência diminui com o aumento da massa. Mais precisamente, a frequência é inversamente proporcional à raiz quadrada da massa.

(d) A frequência aumenta com o aumento da rigidez da mola. Mais precisamente, a frequência é diretamente proporcional à raiz quadrada da constante elástica.

29. $\theta = \arccos(1/3) \cong 70{,}5°$.

30. **(a)** $f(t) = C\operatorname{sen}(\omega t) + C\operatorname{sen}(\omega t + \alpha) = \underbrace{C(1+\cos\alpha)}_{A}\operatorname{sen}(\omega t) + \underbrace{C\operatorname{sen}\alpha}_{B}\cos(\omega t)$.

(b) $k = 10\sqrt{3}$, $\phi = \pi/6$ e, portanto, $f(t) = 10\sqrt{3}\operatorname{sen}(\omega t + \pi/6)$.

31.

32. $\theta = \frac{\operatorname{arcsen}(Rg/v_0^2)}{2} \cong 30°$.