

# mathematics & finance:

# Rio

Research in Options

UFSC (online), Brazil - November 21 to 24, 2021

## *Abstracts*



**LA CA**  
laboratório de análise e modelagem  
matemática em ciências aplicadas

# **Research in Options**

November, 2021

## **Foreword**

Dear Friends and Colleagues,

Welcome to the 16th edition of RiO!

We've come a long way from the initial meetings in Angra dos Reis and Buzios! As the prospect of a new pos-pandemics era ushers in, we hope to see you again next year.

In the meantime, we are having our second year online, with the usual vibrant group of speakers and a diversified program in a broad range of topics in Mathematical Finance.

Last, but not least, we would like to thank all those who helped make this event unique. In particular, Ms. Raphaela Pelosi.

On behalf of the organizing committee,

Jorge P. Zubelli

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# **Abstracts**

# COST-EFFICIENT PAYOFFS UNDER MODEL AMBIGUITY

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## Abstract

A payoff that is the cheapest possible in reaching a given target distribution is called cost-efficient. In the presence of ambiguity the distribution of a payoff is no longer known.

A payoff is called robust cost-efficient if its worst-case distribution stochastically dominates a target distribution and is the cheapest possible in doing so. We study the link between this notion of "robust cost-efficiency" and the maxmin expected utility setting of Gilboa and Schmeidler, as well as more generally with robust preferences in a possibly non-expected utility setting. We illustrate our study with examples involving uncertainty both on the drift and on the volatility of the risky asset.

\*joint work with Gero Junike, Thibaut Lux and Steven Vanduffel

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# OPTIMAL OPTION PORTFOLIOS WITH VOLATILITY ASSET CLASS IN A DISCRETE MARKET

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## Abstract

Derivatives-based nonlinear portfolios in a discrete multistage stochastic programming (MSP) framework can be potentially very beneficial to shape dynamically portfolios' return distributions and attain superior performance. In this work we extend our dynamic discrete portfolio optimization model with equity options (RiO 2018) to include a volatility-based exchange-traded-fund (ETF on the VIX) and a more extended set of option strategies and assess the effectiveness of this more comprehensive model instance during the January 2020-June 2021 period. The asset universe includes equity, bonds, money market, volatility and option contracts on the equity (ATM, OTM, ITM options with 1, 3, 6 month maturity). Relying on this market structure we formulate and analyse a comprehensive set of optimal option strategies in a discrete framework, including canonical protective puts, covered calls and straddles, as well as more advanced combined strategies based on equity options and the volatility index. By introducing an optimal trade-off problem based on expected wealth and Conditional Value-at-Risk (CVaR), we formulate the problem as a stochastic linear program and present an extended set of in-sample and out-of-sample numerical results across different market phases, to discuss the interplay among asset classes and options, relevant to financial engineers and fund managers.

# DARWINIAN MODEL RISK AND REVERSE STRESS TESTING

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## Abstract

We consider the risk of adverse model selection, whereby a trader selects a model leading to superiorly competitive prices, the ensuing valuation loss for the bank being more than offset by gains on the hedging side of the position. But this is only the case until a stress event reveals the erroneous nature of the model used, forcing the bank to liquidate the position and its hedge at the cost of heavy losses. This "Darwinian model risk" is directional (related to a long-term moment of order one of the PnL) and likely to stay unnoticed from traditional risk systems, which are focused on shorter-term moments of order two and beyond. One possible approach to detect it consists of long-term, large-scale simulations, revealing the consequences of using various models in extreme scenarios.

BASED ON: Claudio Albanese, Stéphane Crépey, and Stefano Iabichino (2021). A Darwinian theory of model risk. Risk Magazine July pages 72-77. And Claudio Albanese, Stéphane Crépey, and Stefano Iabichino (2021). Reverse Stress Testing. [ssrn.3544866](https://ssrn.com/abstract=3544866)

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## SIGNATURES: A BASIS FOR PATH DEPENDENT OPTIONS

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## Abstract

European option payoffs can be generated by combinations of hockeystick payoffs or of monomials. Interestingly, path dependent options can be generated by combinations of signatures, which are the building blocks of path dependence.

We focus on the case of 1 asset together with time, typically the evolution of the price  $x$  as a function of the time  $t$ .

The signature of a path for a given word with letter in the alphabet  $\{t,x\}$  is an iterated integral with respect to the letters of the word and it plays the role of a monomial in a Taylor expansion.

We first review and establish properties of the signature in that context and then study the Taylor expansion of a functional. It is a sum over the words of the functional derivative with respect to this word at the origin times the signature associated to the word. We explore the implications of this expansion in terms of pricing and hedging of exotic options.

# FOURIER BASED METHODS FOR THE MANAGEMENT OF COMPLEX LIFE INSURANCE PRODUCTS

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joint project with Laura Ballotta, Thorsten Schmidt, and Raghid Zeineddine

## **Abstract**

This paper proposes a framework for the valuation and the management of complex life insurance contracts, whose design can be described by a portfolio of embedded options, which are activated according to one or more triggering events. These events are in general monitored discretely over the life of the policy, due to the contract terms. Similar designs can also be found in other contexts, such as counterparty credit risk for example. The framework is based on Fourier transform methods as they allow to derive convenient closed analytical formulas for a broad spectrum of underlying dynamics. Multidimensionality issues generated by the discrete monitoring of the triggering events are dealt with efficiently designed Monte Carlo integration strategies. We illustrate the tractability of the proposed approach by means of a detailed study of ratchet variable annuities, which can be considered a prototypical example of these complex structured products.



# PRICING IN AFFINE FORWARD VARIANCE MODELS

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## Abstract

The class of affine forward variance (AFV) models was defined in Gatheral and Keller-Ressel (2019); this class includes both the conventional Heston model and its celebrated extension, the rough Heston model of El Euch and Rosenbaum. The AFV characteristic function may be expressed in terms of the solution of a Volterra integral equation, which can be solved numerically using the Adams scheme. I will present a rational approximation to the solution of this integral equation in the special case of the rough Heston model. Until now, simulation of AFV models using the Markovian approximation of Abi Jaber and El Euch has proved relatively complicated and time-consuming, I will present a new efficient and easy-to-implement method for simulating AFV models for general kernels. Numerical results will be benchmarked against the rational approximation and the Adams scheme.

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# OPTIMAL ECOLOGICAL TRANSITION PATH OF A CREDIT PORTFOLIO DISTRIBUTION, BASED ON MULTIDATE MONGE- KANTOROVICH FORMULATION

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## Abstract

Accounting for climate transition risks is one of the most important challenges in the transition to a low-carbon economy. Banks are encouraged to align their investment portfolios to CO2 trajectories fixed by international agreements, showing the necessity of a quantitative methodology to implement it. We propose a mathematical formulation for this problem and a multistage optimization criterion for a transition between the current bank portfolio and a target one. The optimization Problem combines the Monge-Kantorovich formulation of optimal transport, for which the cost is defined according to the financial context, and a credit risk measure. We show that the problem is well-posed, and can be embedded into a saddle-point problem for which Primal-Dual algorithms can be used. We design a numerical scheme that is able to solve the problem in available time, with nice scalability properties according to the number of decision times; its numerical convergence is analysed. Last we test the model using real financial data, illustrating that the optimal portfolio alignment may differ from the naive interpolation between the initial portfolio and the target.

\*Joint work with Clara Lage.

# LONG VS SHORT TIME SCALES: THE ROUGH DILEMMA AND BEYOND

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## Abstract

Using a large dataset on major FX rates, we test the robustness of the rough fractional volatility model over different time scales, by including smoothing and measurement errors into the analysis.

Our findings lead to new stylized facts in the log-log plots of the second moments of realized variance increments against lag which exhibit some convexity in addition to the roughness and stationarity of the volatility. The very low perceived Hurst exponents at small scales is consistent with the rough framework, while the higher perceived Hurst exponents for larger scales leads to a nonlinear behavior of the log-log plot that has not been described by models introduced so far. In the last part of the talk we introduce some models that are able to explain the new stylized facts.

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## TESTING THE KEEN MODEL FOR THE UNITED STATES

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## Abstract

In this talk I describe recent work with Alex Shkolnik (UCSB) and Steve Li (University of Chicago), where we use 60 years of macroeconomic data for the United States to test the Keen model for the dynamics of wages, employment, and private debt. We begin with a careful construction of the time series for net debt of the firms sector from balance sheet information obtained from the US national accounts and show how this differs from a commonly used metric for private debt published by the Bank of International Settlements. This is followed by a detailed examination of each assumption of the model in light of the statistical properties of the corresponding time series, including the use of a Leontief production function with a constant capital-to-output ratio. A comparison between the observed change in debt and the difference between investment and profits (which are assumed to be the same in the model, but are far from it in the data), leads to introduction of successive extensions, such as the inclusion of taxes, dividends, and ultimately a residual term identified with financial speculation. We conclude with an extensive sensitivity analysis of the final version of the model with respect to the underlying parameters and compare its predictive performance with that of a vector autoregressive (VAR) model for the same

three state variables. If time permits, I'll explore the consequences of these estimates when the Keen model is used as the economic core of an integrated assessment model (IAM) for climate change.

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## **A NEW LOOK AT BERGOMI MODELS: CLOSED-FORM VIX FUTURES EXPANSIONS, JOINT S&P 500/VIX CALIBRATION, AND THE APPEAL OF PATH-DEPENDENT VOLATILITY**

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### **Abstract**

We derive the formal expansion of the price of a VIX future (and more generally a VIX power payoff) in various Bergomi models at any order in powers of the volatility-of-volatility. We introduce the notion of volatility of the VIX squared implied by the VIX future, which we call "VIX2 implied volatility", and also expand this quantity at any order. We cover the one-factor, two-factor, and skewed two-factor Bergomi models and allow for maturity-dependent and/or time-dependent parameters. When the initial term-structure of variance swaps is flat, the expansions are in closed form; otherwise, they involve one-dimensional integrals which are extremely fast to compute. Our extensive numerical experiments show that both the price and implied volatility expansions provide very accurate approximations for a wide range of model parameters which covers the values typically used in the equity and FX derivatives markets and even well beyond---in particular, the expansions are accurate even for very large values of the volatility-of-volatility. They also show that the implied volatility expansion converges much faster than the price expansion. Its truncation at order 1 is already so accurate that it leads to a simple formula for the price of the VIX future that can virtually be considered exact.

The derivation of the expansions naturally involves the (classical or dual bivariate) Hermite polynomials and exploits their orthogonality properties; it is interesting in itself. The expansions allow us to precisely pinpoint the roles of all the model parameters (volatility-of-volatility, mean reversions, correlations, mixing fraction) in the formation of the prices of VIX futures and VIX power payoffs in Bergomi models. We also use them together with the Bergomi-Guyon expansion of the S&P 500 (SPX) smile to (instantaneously) calibrate the two-factor Bergomi model jointly to the term-structures of SPX at-the-money skew and VIX2 implied volatility. Our tests and the new expansions shed more light on the inability of traditional stochastic volatility models to jointly fit SPX and VIX market data. Most importantly, the (imperfect but decent) joint fit requires much larger values of volatility-of-volatility and fast mean reversion than the ones previously reported in the literature (Bergomi 2005, Bergomi 2016), as well as perfectly correlated Brownian motions. Very interestingly, the joint SPX/VIX calibration thus naturally leads to path-dependent volatility, via "zero-factor stochastic volatility models".

# AXES THAT MATTER: PCA WITH A DIFFERENCE

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## Abstract

Differential PCA is introduced to reduce the dimensionality in derivative pricing problems.

Brian Huge and Antoine Savine extend differential machine learning and introduce a new breed of supervised principal component analysis to reduce the dimensionality of derivatives problems, with applications including the specification and calibration of pricing models, identification of regression features in least-squares Monte Carlo and preprocessing simulated datasets for (differential) machine learning.

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## INFORMATION-BASED TRADING

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## Abstract

We consider a pair of traders in a market where the information available to the second trader is a strict subset of the information available to the first trader. The traders make prices based on the information available concerning a security paying a random cash flow at a fixed time  $T$  in the future. Market information is modelled in line with the scheme of Brody, Hughston & Macrina (2007, 2008) and Brody, Davis, Friedman & Hughston (2009). The risk-neutral distribution of the cash flow is known to the traders, who make prices with a fixed multiplicative bid-offer spread and report their prices to a game master who declares that a trade has been made when the bid price of one of the traders first reaches the offer price of the other. We prove that the value of the first trader's position is strictly greater than that of the second. The results are analyzed by use of simulation studies and generalized to the situation where there is a hierarchy of traders.

\*Based on work carried out in collaboration with G. Bouzianis (Goldsmiths University of London) and L. Sánchez-Betancourt (Imperial College London). References: D C Brody, L P Hughston & A Macrina (2007) Beyond Hazard Rates: a New Framework for Credit-Risk Modelling, in: Advances in Mathematical Finance (M Fu, R Jarrow, Ju-Yi Yen & R Elliott, eds) Birkhauser, 231-257. D C Brody, L P Hughston & A Macrina (2008) Information-Based Asset Pricing, Int. J. Theor. Appl. Finance 11 (1), 107-142. D C Brody, M H A Davis, R L Friedman & L P Hughston (2009) Informed Traders, Proc. Roy. Soc. Lond. A 465, 1103-1122.

# DEEP LEARNING FOR PRINCIPAL-AGENT MEAN FIELD GAMES

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## Abstract

Here, we develop a deep learning algorithm for solving Principal-Agent (PA) mean field games with market-clearing conditions -- a class of problems that have thus far not been studied and one that poses difficulties for standard numerical methods. We use an actor-critic approach to optimization, where the agents form a Nash equilibria according to the principal's penalty function, and the principal evaluates the resulting equilibria. The inner problem's Nash equilibria is obtained using a variant of the deep backward stochastic differential equation (BSDE) method modified for McKean-Vlasov forward-backward SDEs that includes dependence on the distribution over both the forward and backward processes. The outer problem's loss is further approximated by a neural net by sampling over the space of penalty functions. We apply our approach to a stylized PA problem arising in Renewable Energy Certificate (REC) markets, where agents may rent clean energy production capacity, trade RECs, and expand their long-term capacity to navigate the market at maximum profit. Our numerical results illustrate the efficacy of the algorithm and lead to interesting insights into the nature of optimal PA interactions in the mean-field limit of these markets.

\*joint work with Steven Campbell, Yichao Chen, and Arvind Shrivats

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# DUALITY IN CONVEX STOCHASTIC OPTIMIZATION

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## Abstract

We present a general duality theory for convex stochastic optimization in discrete time. Our framework unifies and extends many well-known problem classes from operations research, control, and mathematical finance. We find dual representations and optimality conditions by combining general functional analytic duality arguments with some stochastic analysis. The theory is illustrated by applications to optimal stopping, stochastic control, and semi-static hedging. Besides simpler proofs, our approach yields various generalizations to earlier results. In semi-static hedging, in particular, our approach allows for nonlinear illiquidity effects and portfolio constraints.

\* based on joint work with Ari-Pekka Perkkiö.

## **BARRETT'S PARADOX**

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### **Abstract**

Baliga and Maskin (2003) considered a model where clean air is the public good and its contamination by diverse communities is a negative externality. For this model, we prove that the agents form spontaneously a stable cooperative coalition in the bargaining stochastic game. Furthermore, there is a unique stable high coalition. The utility of the stable high coalition is significantly larger than the utility of the competitive Nash coalition. However, the Barrett's paradox holds unless the intensity of the preservation parameter is high enough to stabilize the grand coalition.

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## **ESCAPING THE SISYPHEAN TRAP: HOW QUANTS CAN ACHIEVE THEIR FULL POTENTIAL**

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### **Abstract**

Investing can be characterized as a data science problem. While investment firms have attracted scientific talent, they have done a poor job at developing it. Firms hire specialists, but entice them to become generalists (e.g., portfolio managers). Under the ubiquitous silo/platform structure, quants succumb to the Sisyphian trap, and do not achieve their full potential. A research lab structure offers a unique environment for developing scientists, by means of: (a) co-specialization, working in a highly cooperative lab environment; (b) tackling well-defined open investment problems; and (c) applying the scientific method.

# MODEL FREE DEEP HEDGING

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## Abstract

Deep Hedging is a simple machine learning inspired algorithm, which produces risk management strategies and which is built upon a given market environment and a given preference structure. In this talk we introduce a novel algorithm inspired by Moritz Duembgen's and Chris Rogers' Bayesian approach, which can additionally deal with model uncertainty. We discuss cases where the approach converges and why it might be successful beyond.

\*joint work with Thorsten Schmidt.