EXERCISES 1 (C*-DYNAMICAL SYSTEMS)

ALCIDES BUSS

(1) Given C^*-algebras A, B, show that there is an isomorphism $A \otimes_\mu B \cong B \otimes_\mu A$ sending $a \otimes b$ to $b \otimes a$ for $\|\cdot\|_\mu = \|\cdot\|_{\min}$ and $\mu = \|\cdot\|_{\max}$.

(2) Given C^*-algebras A, B, C, show that there are canonical isomorphisms

$$(A \otimes_\mu B) \otimes_\mu C \cong A \otimes_\mu (B \otimes_\mu C)$$

for $\|\cdot\|_\mu = \|\cdot\|_{\min}$ and $\mu = \|\cdot\|_{\max}$.

(3) If $A = \mathcal{C}_0(X)$ and $B = \mathcal{C}_0(Y)$ are commutative C^*-algebras, show that $A \otimes B \cong \mathcal{C}_0(X \times Y)$.

(4) Given C^*-algebras A, B and $x \in A \otimes_{\min} B$, show that if $(\tau \otimes_{\min} \rho)(x) = 0$ for all $\tau \in A^*_+$ and $\rho \in B^*_+$, then $x = 0$.

(5) If A is a C^*-subalgebra of a C^*-algebra B and C any other C^*-algebra, show that the canonical inclusion $A \otimes C \hookrightarrow B \otimes C$ extends to an isometric embedding $A \otimes_{\min} C \hookrightarrow B \otimes_{\min} C$.

Warning: The above property of minimal tensor products does not have an analogue for maximal tensor products, that is, in general $A \otimes_{\max} C$ does not embed into $B \otimes_{\max} C$. The problem is that there are non-nuclear C^*-algebras that embed into non-nuclear ones (for example, a theorem by Kirchberg asserts that every separable nuclear C^*-algebra embeds into $\mathcal{O}_2$, a nuclear C^*-algebra). If A is a non-nuclear C^*-subalgebra of a nuclear C^*-algebra B, let C be a C^*-algebra for which $A \otimes_{\max} C \to A \otimes_{\min} C$ is not injective. Then $A \otimes_{\max} C \to B \otimes_{\max} C = B \otimes_{\min} C$ is not injective (why?).

E-mail address: `alcides.buss@ufsc.br`

Departamento de Matemática, Universidade Federal de Santa Catarina, 88.040-900 Florianópolis-SC, Brazil