Partial Galois cohomology, Picard semigroups and the relative Brauer group

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Remark: A flow is called total if this par. action is global.

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Group V is defined by **partial actions** on **finite binary words** by J. C. Birget (2004) (following E. A. Scott (1984)) to study complexity (word problem etc.).

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Say θ is unital if $\forall\ \mathcal{A}_g = 1_g\mathcal{A}$, 1_g central idemp. ($1_g^2 = 1_g$).

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M. D. + M. Khrypchenko 2015.

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***n*-cochains:** $f : G^n \to A$, s. that $f(x_1, \dots, x_n) \in \mathcal{U}(A_{(x_1,\dots,x_n)})$.

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$C^n(G, A)$ is abel. grp with pointwise mult-n:

identity : $$e_n(x_1, \dots, x_n) = 1_{x_1} 1_{x_1 x_2} \dots 1_{x_1 \dots x_n},$$

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Partial gr. cohomolgy

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Galois Theory of commutative rings in:

M. Auslander, O. Goldman, The Brauer group of a commutative ring, *Trans. Amer. Math. Soc.* **52** (1960), 367-409.

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Recall: Let R be com. ring and assume a finite grp G acts (globally) on S. **Say** $R^G \subseteq R$ is Galois ext. if $\exists\, x_i, y_i \in R$, $1 \leq i \leq n$, s. that

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M. Dokuchaev, M. Ferrero, A. Paques, Partial Actions and Galois Theory, *J. Pure Appl. Algebra*, **208** (2007), (1), 77–87.

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Given k-algebra A write $A^e = A \otimes_k A^{\mathrm{op}}$, where A^{op} is the opposite alg. Then A is a left A^e-module via $(a \otimes b)a' = aa'b$.

Definition

Let A be an algebra over comm. ring k. Say that A is separable over k if A is projective as a left A^e-module.

Recall that a module over a ring A is called projective if it is a direct summand of a free A-module.

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Theorem

Let $K \subseteq F$ finite Galois ext. fields with Galois gr. G. Then $H^2(G, F^*) \ni \mathrm{cls}(f) \mapsto [R *_f G] \in B(F/K)$ is gr. iso.

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Then $\mathrm{PicS}(R)$ with respect to $\otimes_R$ is com. inv. monoid with 0 and

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Partial action version?

Thank you!