Mini-course 01: Identification of parameters in time dependent differential equtions

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September 20, 2017

Abstract: Mathematical models for a physical processes contain parameters which cannot be measured directly, in general. They must be determined by using recorded consequences they produce. Such problems are called *inverse problems*. Identification problems are inverse problems since one wants to reconstruct causes (parameter in a model) from consequences (output variables). *Ill-posedness* is a characteristic feature of inverse problems.

Ill-posedness means that at least one of the following properties is violated:

existence, uniqueness, stability

Mostly, we are concerned with the case that the property of stability is violated, i.e. that the solution does not depend continously on the data. This is connected to the fact that in inverse problems the consequences are determined by some measure process and therefore corrupted by noise. It is a challence to solve such problems without that a priori the stability property holds. The approach which makes it possible to handle such situations is *regularization*, i.e. *restoration of stability* by an appropriate approach. Such regularization approaches may be realizaed in many different ways:

- Change of the solution concept
- Truncation of small singular values in systems of linear equations
- Discretization with an appropriate discretization level
- Stopping rules in iterative computational schemes
- ......

As a rule, the degree of efficiency of restoration of stability is deeply connected to the quality and quantity of *a priori knowledge* that we put in in the approach. Usually, a priori knowledge is introduced by adding qualitative and quantitative constraints to the solution we want to find out.

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The aim of this mini-course is to present some topics which are relevant for studying parameter identification problems for ordinary and partial differential equations. The following outline for the lectures in the course is meant to describe a tentative rather than a final plan. Here are issues which we want to present:

(1) Systems theory, modelling, identification

(2) Parametric distributed systems – problems and examples

(3) Functionalanalytic tools

(4) Special concepts for regularizing (nonlinear) problems

(5) Solving parameter identification problems by the *model reference adaptive systems*–appoach

(6) Identification of parameters by the model refernce adaptive systems–approach from a projected observation

Ad (1)
Systems theory is the (interdisciplinary) study of interrelated and interdependent components under a common point of view. The common view is the dynamic behavior of a system, i.e. the time dependent behavior of the components of the system and their interaction. In this chapter we present some basic remarks on *modelling, controlled systems, classification of systems, identification/estimation of systems, model order reduction, compartmental sytems*. See [2].

Ad (2)
Parameter identification in partial differential equations arise in a variety of applications. Classical applications are heat conduction problems, population models, seismic explorations, groundwater flow, impedance–tomography, finance instruments. We present some of these problems, have a look on the identifiabilty for these problems and present some counterexamples concerning stability. See [1, 7]

Ad (3)
Here we sketch material for studying distributed systems. Our main focus is on parametric parabolic and elliptic partial differential equations. These considerations prepare the discussion of identification methods for time dependent partial differential equations. Tentative subjects are: Gelfand triples, Lax-Milgram Lemma, function spaces, solution of evolution equations. See [6].

Ad (4)
This issue is devoted to a short introduction to ill-posed equations. The considerations should introduce the main related problems and notations in order to motivate the Hilbert uniqueness method (HUM) in the linear and nonlinear context. Moreover, we discuss the so called *tangential cone condition* which is an important tool in solving nonlinear ill-posed problems and sketch the Landweber methods and the so called iterative Tihonov methods. See [2, 3, 8, 9].

Ad (5)
Online or real time parameter identification is the task of inferring model parameters

simultaneously to the process of data sensing of a dynamical system in operation. Such techniques have been developed by engineers for control problems governed by ordinary differential equations. These techniques can also be used in the case of partial differential equations. We present the realization of the so called *model reference adaptive system*-approach and discuss their efficiency in the case of complete observation. See [4].

Ad (6)
Again, we consider the *model reference adaptive system*-approach but without complete observation of the state. Instead we try to identify the parameter by using the observable part only. This study is inspired by a recent paper of Boiger and Kaltenbacher; see [5].

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