Parameter identification – tools and methods

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Inverse Methodology

J.B. Keller, 1976

One calls two problems inverse to each other if the formulation of one problem involves the other one

Historically, it is clear which problem is called the *direct/forward problem* and which is called the *inverse problem*.

The circumpherence of the earth

Erathosthenes was the first who used an inverse method to compute the circumpherence/diameter of the earth (about 240 b.C.). The assumption he used is that the rays of the sun hit the earth on each place in parallel directions.

He considered the shadow of a gnomon in Syrene and Alexandria (Egypt, lying both on the same meridian) on noon. In Syrene he observed that there is no shadow since the rays of the sun are shining parallel to a gnomon. In Alexandria, the angle of the shadow of the gnomon is determined to be about 7.2 degrees, the fiftieth of a circle. The distance from Syrene to Alexandria was about 5000 stadia. This gives for the circumference of the earth about 77.000 stadia or 37.000 km. The actual measurement is 40.075 km.

The discovery of the x-rays

C. Röntgen (Würzburg) discovered in 1985 per random the effect of a new radiation; he named them X-rays to underline the fact that their nature was unknown. He found out that X-rays generate on a special sheet of paper the image of his hand in a new fashion: one could distinguish bones from the soft tissue. This was the beginning of the investigation of inhomogeneous material by X-rays and ended about a century later in the development of computer tomography.

C. Röntgen received the nobel prize for physics in 1901.

In order to understand the emission of X-rays, H. Becquerel (Paris) investigated the role of the phosphorescence of the glass of the tube and while doing so discovered radioactivity in March 1896. X-rays and radioactivity were at the origin of the scientific revolution at the end of the 19th and the beginning of the 20th centuries

Inverse problems

- Computer tomography
- MR-spectroscopy
- Inverse heat conduction
- The inverse problem of geomagnetic induction
- Impedance tomography
- Acoustic scattering, scattering in quantum mechanics
- Radio-astronomical imaging, image analysis
- Locating cracks or mines by electrical prospecting
- Seismic exploration, seismic tomography
- Electrocardiography and magnetocardiography
- Evolution backwards in time
- „Can you hear the shape of a drum/manifold?“
- Inverse problems in potential theory
- Deconvolution, inversion of autoconvolution
- Reconstruction of signals
- Compartmental analysis, parameter identification
- Determing the volatility in models for financial markets
- Shape from probing

Well-posedness

Well-posedness/Hadamard 1902
In a **well-posed problem** *three conditions have to be satisfied:*

1. *Existence of a solution*
2. *Uniqness of a solution*
3. *Continuous dependency of a solution on the data*

If for a problem one of the conditions above is not satisfied then the problem is called **ill-posed**.

Notice
Continous dependency depends on the topologies chosen. In general, these topologies are predetermined by the applied situation

Ill-posedness

As a rule, inverse problems are ill-posed*.*

Reasons for this statement:

- Irreversibility in physics
- Overdeterminded objects
- Smoothing of the forward operator
- Small errors in the data are amplified dramatically
- Noise cannot be avoided in applications!

As a rule,
identification and estimation of parameters is an inverse problem and therefore ill-posed in general

Classification of problems

Description of a system (differential equation, . . .) in mathematical terms:

X	space of input quantities;
Y	space of output quantities;
P	space of system parameters;
$F(p)$	system operator from X into Y associated to the admissible parameter $p \in P_{\text{ad}} \subset P$.

(A) *Given* $x \in X$ *and* $p \in P_{ad}$, *find* $y := F(p)x$.

(B) *Given* $y \in Y$ *and* $p \in P_{ad}$, *find* $x \in X$ *such that* $F(p)x = y$.

(C) *Given* $x \in X$ *and* $y \in Y$, *find* $p \in P_{ad}$ *such that* $F(p)x = y$.

For the description of $X, Y, P, F(p)$ and P_{ad} we use methods and results from **analysis, linear algebra, linear and nonlinear functional analysis**.

Outline of the course

Here are the issues which we want to discuss.

- Systems, observability, identifiability – finite dimensional case
- Identification in partial differential equations/Tutorial coniderations
- Functionalanalytic tools and linear equations
- Identification in elliptic equations
- Functionalanalytic tools and nonlinear mappings
- Model reference adaptive systems – complete observation
- Model reference adaptive systems – incomplete observation