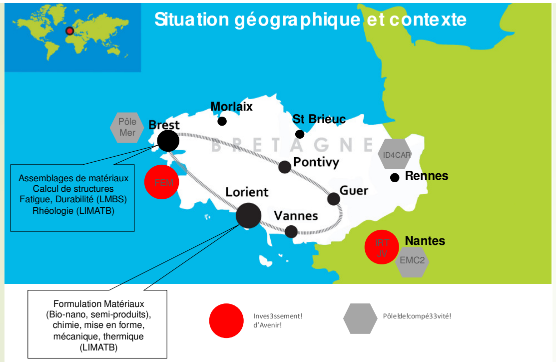
Radiative measurements and inverse problem

Philippe Le Masson / Thomas Pierre – IRDL – UBS.



My Team in our laboratory IRDL (**21 permanent research professors – 2 engineers – 15 phd students and 2 post doc)**

The assemblies

in Lorient : Welding, additive manufacturing, sintering…
in Brest: Welding, Bonding…

3 axis: Modelisation, characterisation and instrumentation.
- Multiphysic model (knowledge model for the heat input) and reduced model (for the calculation of mechanical effects: residual stresses and distortions)
- characterisation: definition of the parameters for the simulation
- instrumentation of the experimental characterisation and in situ experiments : thermocouples, infrared camera, speed camera, multispectral pyrometer…

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Context:

Estimation of the thermophysical characteristics at high temperature

Thermal measurements between 1000°C and 3000°C…

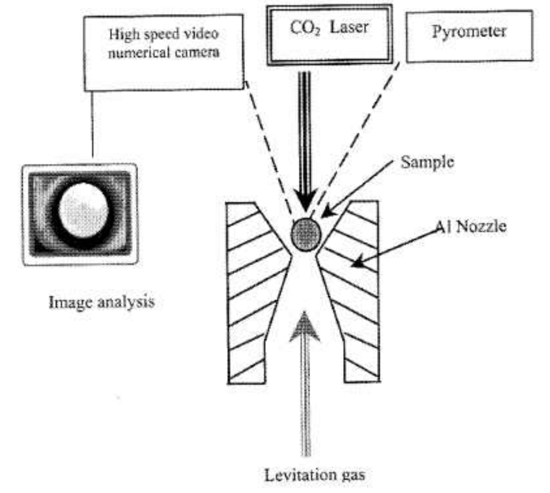
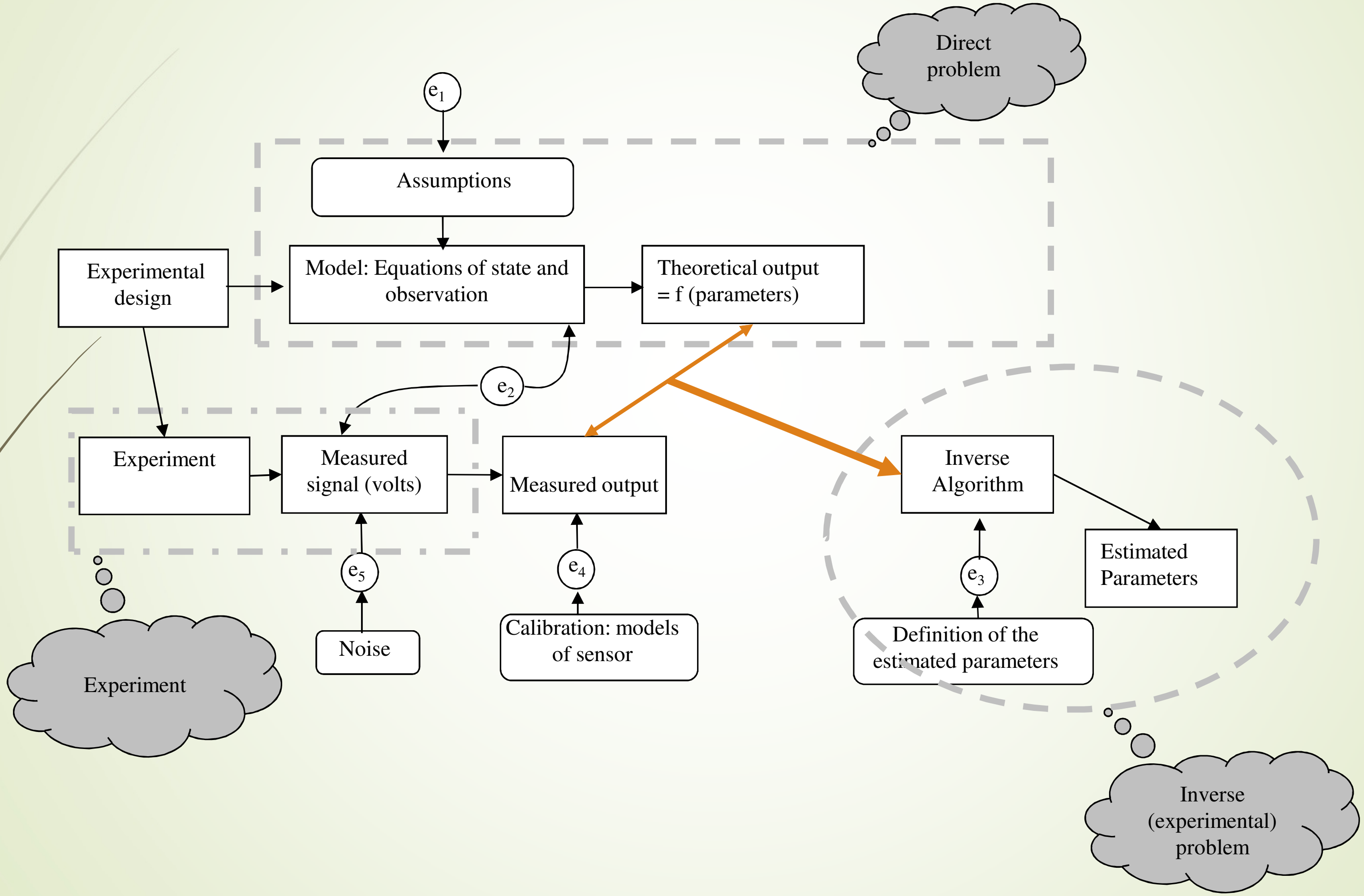


FIGURE 1. Aerodynamic Levitation Set-Up with Aluminum Nozzle.



Direct problem

e_1

Assumptions

Experimental design

Model: Equations of state and observation

Theoretical output = f (parameters)

e_2

Experiment

Measured signal (volts)

Measured output

Inverse Algorithm

Estimated Parameters

e_5

e_4

e_3

Noise

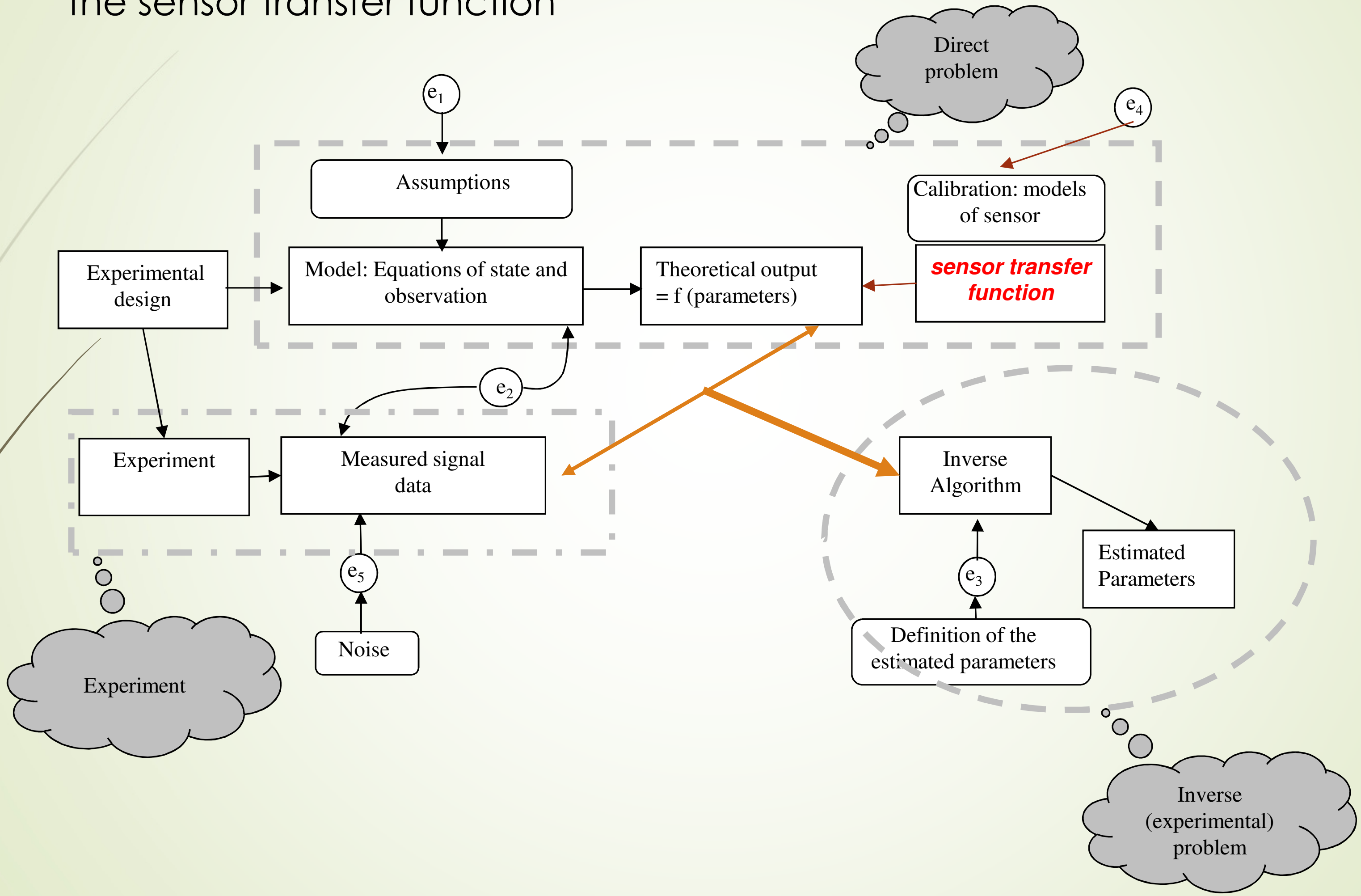
Calibration: models of sensor

Definition of the estimated parameters

Experiment

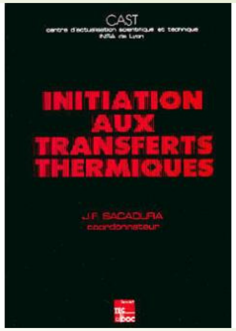
Inverse (experimental) problem

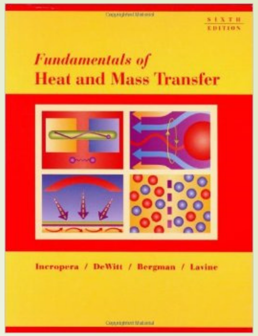
New approach for the bayesian method : the direct problem with the sensor transfer function

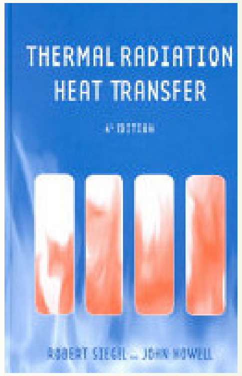


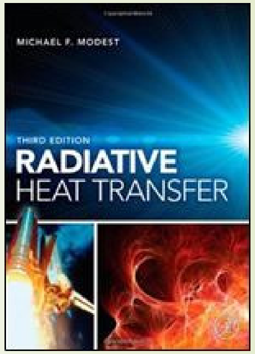
Thermal Radiation

- J.-F. Sacadura, *Initiation aux transferts thermiques*, Ed. Tec&Doc, Paris, 2000.
- F. P. Incropera, D. P. DeWitt, *Fundamentals of heat and mass transfer*, Fifth Ed., Wiley, New-York, 2002.
- R. Siegel, J.R. Howell, *Thermal radiation heat transfer*, Éditions Taylor and Francis, New York (1992).
- Y.S. Touloukian, *Thermal radiative properties*, Plenum, New York (1970).
- N. Ozisik, *Radiative transfer*, Wiley and sons, New-York, 1973.
- M.F. Modest, *Radiative heat transfer*, Academic Press, Ney-York, 2003.

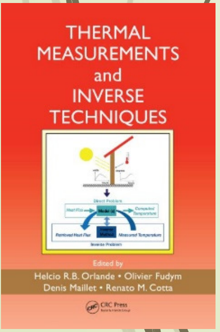


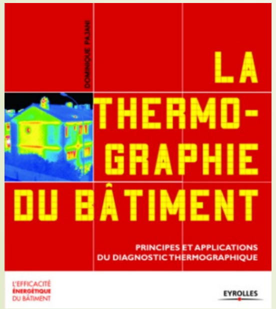


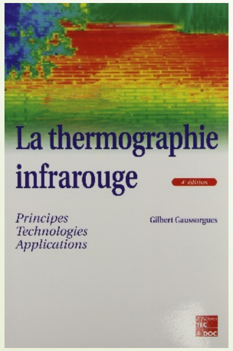




Thermal Measurements







- H. R. B. Orlande, O. Fudym, D. Maillet, R. M. Cotta, *Thermal measurements and inverse techniques*, CRC Press, Taylor & Francis Group, Boca Raton, 2011.
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- G. Gaussorgues, *La thermographie infrarouge*, Tec&Doc, Paris, 2000.

… et les techniques de l’ingénieur.

- ***1. Thermal Radiation***
- 2. Radiatives Properties
- 3. Radiative measurements
- 4. Calibration
- 5. Other techniques
- 6. Applications
- 7. Inverse method

8. Prospect…

1. Thermal Radiation

Principle : heat flux measurement for the evaluation of the temperature.

- Propagation of the electromagnetic waves.
- no contact solid-solid between sensor and the surface.
- In first approximation, radiative heat transfer only.
- the intermediate medium may be transparent or participatory (semi-transparent).

Maxwell equations:

$$\left\{\begin{aligned} &\nabla\cdot\mathbf{E}=0\\ &\nabla\cdot\mathbf{H}=0\\ &\nabla\times\mathbf{H}=\varepsilon\frac{\partial\mathbf{E}}{\partial t}+\sigma\mathbf{E}\\ &\nabla\times\mathbf{E}=-\mu\frac{\partial\mathbf{H}}{\partial t}\end{aligned}\right.$$

E and **H** electric and magnetic field vectors

ε : electrical permittivity

μ : magnetic permeability

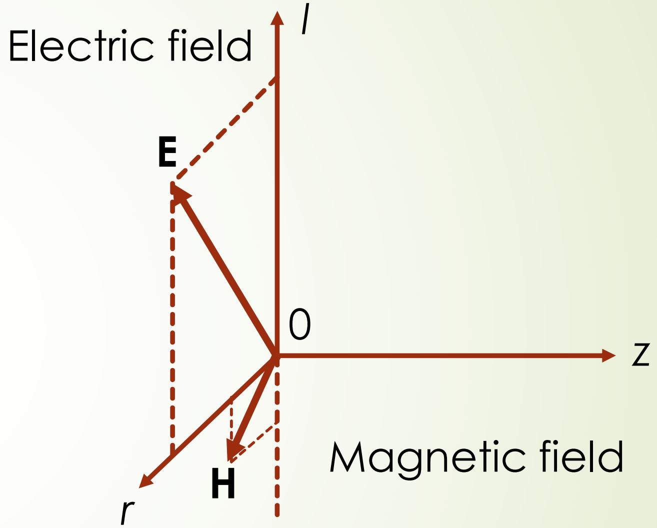
σ : specific conductivity

The propagation of radiation as electromagnetic waves is analyzed from the study of plane waves.

1. Thermal Radiation

Propagation in a dielectric medium:

- isotropic medium, homogeneous.
- plane wave: the electric and magnetic fields are perpendicular to the direction of propagation 0z.
- constant parameters in the plane r01.



$$\left\{\begin{array}{ll} \partial / \partial l = 0 & \partial / \partial r = 0 \\ E_z = 0 & H_z = 0 \\ \sigma = 0 & \end{array}\right.$$

$$\frac{\partial^2 E_l}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_l}{\partial t^2} \quad \text{et} \quad \frac{\partial^2 E_r}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_r}{\partial t^2}$$

with $c^2 = \frac{1}{\mu\varepsilon} = \frac{c_0^2}{n^2}$ c_0 : celerity in the vaccum.

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c_0} = \omega\sqrt{\varepsilon_0\mu_0}$$

solutions : $\left\{\begin{array}{l} E_l = a_l \exp\left\{i\left[\left(\omega t - k_0 n z\right) + \gamma_l\right]\right\} \\ E_r = a_r \exp\left\{i\left[\left(\omega t - k_0 n z\right) + \gamma_r\right]\right\} \end{array}\right.$

n : refractive index
ω : pulsation
λ : wavelenght
k : wave number
γ : phase shift

1. Thermal Radiation

Propagation in a conductive medium :

- same items than before.

$$\left\{\begin{array}{ll} \partial / \partial l = 0 & \partial / \partial r = 0 \\ E_z = 0 & H_z = 0 \end{array}\right.$$

$$\rightarrow \frac{\partial^2 E_l}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_l}{\partial t^2} + \mu\sigma \frac{\partial E_l}{\partial t} \quad \text{et} \quad \frac{\partial^2 E_r}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_r}{\partial t^2} + \mu\sigma \frac{\partial E_r}{\partial t}$$

solutions :

$$\left\{\begin{array}{l} E_l = a_l \exp\left\{\left[i\left(\omega t - k_0 m z\right)\right] + \gamma_l\right\} \\ E_r = a_r \exp\left\{\left[i\left(\omega t - k_0 m z\right)\right] + \gamma_r\right\} \end{array}\right.$$

with $m = n - in'$ *m* : complex refractory index for a conductive medium.

et $c = \frac{c_0}{|m|} = \frac{c_0}{\sqrt{n^2 + n'^2}}$

n and *n*’ are not constants for a same material, they depend on the wavelenght, on the temperature.

1. Thermal Radiation

Energy transmitted by electromagnetic waves :
- if no energy, no detection possible!
  given by the vector of Poynting S.

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

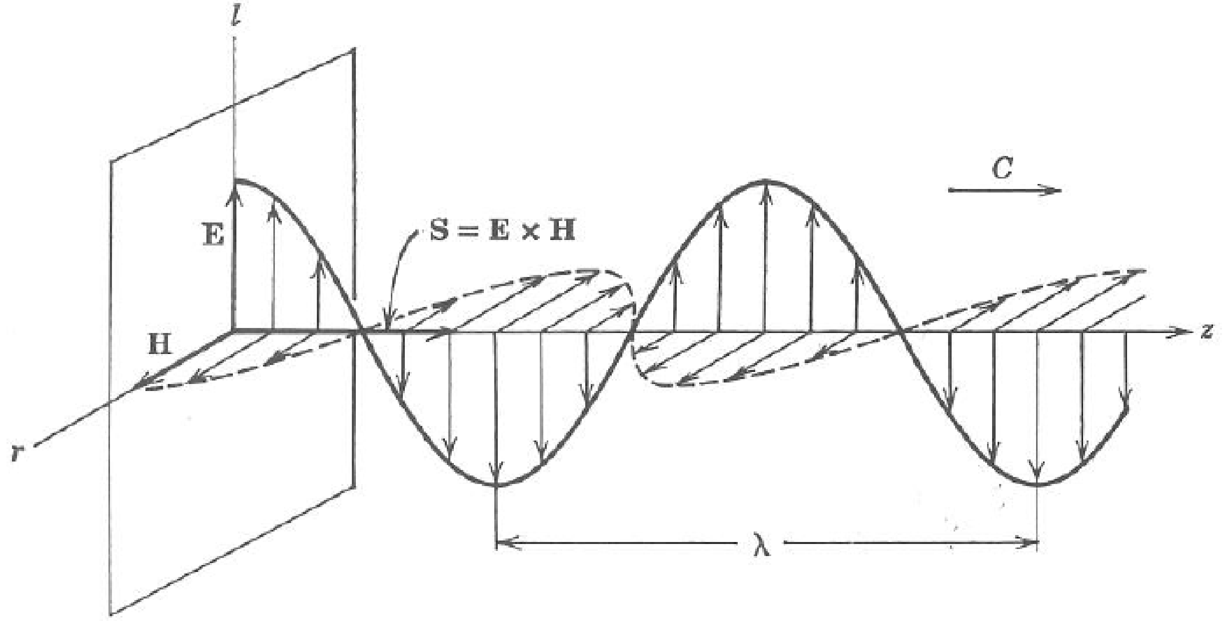


Fig. 1-3. The electric and magnetic field vectors and the direction of the corresponding Poynting vector.

N. Ozisik, *Radiative transfer and interactions with conduction and convection*, Wiley, New-York, 1973.

1. Thermal Radiation

For a plane wave, E_z and H_z being zero, the Poynting vector becomes :

$$\mathbf{S} = \hat{\mathbf{k}}\left[E_l H_r - E_r H_l\right]$$

Unit vector in 0z *direction*.

Since the magnetic fields H_l and H_r are proportional to the electric fields E_r and E_l respectively, it happens that:

In the dielectric medium

$$\mathbf{S} = \hat{\mathbf{k}}\frac{n}{c_0\mu}\left[E_l^2 + E_r^2\right]$$

In the conductive medium

$$\mathbf{S} = \hat{\mathbf{k}}\frac{m}{c_0\mu}\left[E_l^2 + E_r^2\right]$$

The instantaneous value of the energy associated with a radiation traveling through a dielectric medium is :

$$\left|\mathbf{S}\right| = \hat{\mathbf{k}}\frac{n}{c_0\mu}\left[E_l E_l^* + E_r E_r^*\right]$$

* : conjuguated complex

1. Thermal Radiation

Radiation consists of a rapid succession of a multitude of waves of amplitudes and phases subject to continuous variations (several millions of times per second).
Consequently, during a sufficient measurement period, it is finally an average energy of the radiation that is perceived :

$$\overline{|\mathbf{S}|} = \frac{n}{c_0 \mu}\left[\overline{E_l E_l^*} + \overline{E_r E_r^*}\right] \equiv \frac{n}{c_0 \mu}\left(I_l + I_r\right)$$

where I_l and I_r are the intensity components relative to the vibrations of the electric field along the directions 0*l* and 0*r*, respectively.

Radiation, in the sense of Planck's law, is a natural or depolarized radiation, that is to say, random and chaotic in nature. It is therefore not technically possible to distinguish independently the two components I_l and I_r (amplitudes and phase shifts).

In this case, only $I = I_l + I_r$ is sufficient to characterize the depolarized radiation.

1. Thermal Radiation

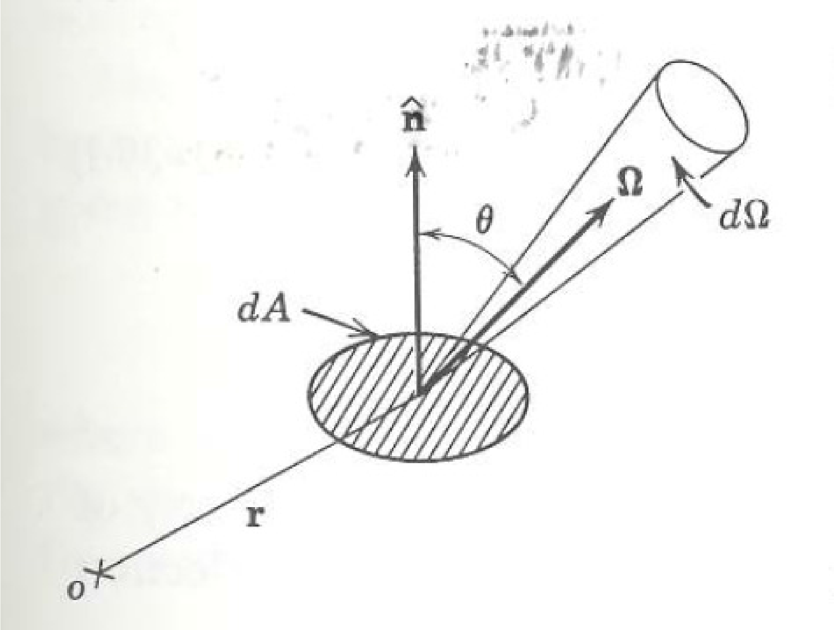
It is I which is measurable, intensity or monochromatic luminance (L):

$$I_\nu\left(\mathbf{r},\hat{\boldsymbol{\Omega}},t\right)=\left[\frac{dE_\nu}{dA\cos\theta d\hat{\Omega}d\nu dt}\right]_{\lim dA,d\omega,d\nu,dt\to 0}$$

Beware, here E is an energy!

It is the energy quantity between ν and $\nu + d\nu$, confined in a solid angle $d\Omega$ around a propagation direction $\mathbf{\Omega}$ coming from a surface element dA during a time interval between t and $t + dt$.

θ Being the angle between the normal **n** and the direction $\mathbf{\Omega}$.



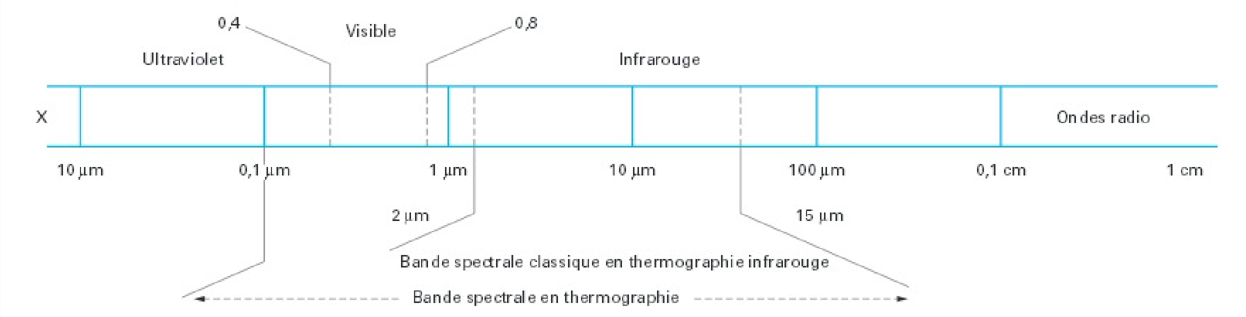
Or more familiarly: $L_\lambda\left(\mathbf{r},\hat{\boldsymbol{\Omega}},t\right)=\left[\frac{dQ_\lambda}{dA\cos\theta d\hat{\Omega}d\lambda dt}\right]_{\lim dA,d\omega,d\lambda,dt\to 0}$

The frequency is substituted for the wavelength.

1. Thermal Radiation

For a température *T*, a surface emits an infinity of waves of different wavelengths λ in all directions of space. These wavelengths are distributed over a spectral range as a function of temperature.

electromagnetic spectrum



Radiative transfers concern only the [0.1 µm - 100 µm] range of thermal radiation covering the ultraviolet (UV), visible and infrared (IR) ranges.

The International Commission on Illumination says (1):

UV

- [0,250 µm – 0,280 µm] : UV - type C
- [0,280 µm – 0,315 µm] : UV - type B
- [0,315 µm – 0,380 µm] : UV - type A

IR

- [0,760µm – 25µm] : Infrared

Visible

- [0,380 µm – 0,439 µm] : purple (maximum 0,412 µm)
- [0,439 µm – 0,498 µm] : blue (maximum 0,470 µm)
- [0,498 µm – 0,568 µm] : green (maximum 0,515 µm)
- [0,568 µm – 0,592 µm] : yellow (maximum 0,577 µm)
- [0,592 µm – 0,631 µm] : orange (maximum 0,600 µm)
- [0,631 µm – 0,760 µm] : red (maximum 0,673 µm)

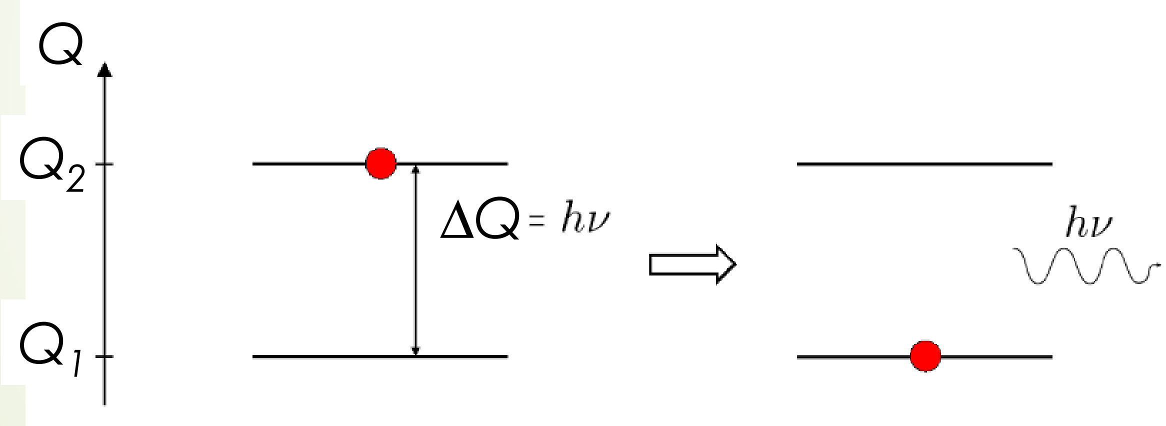
1. A. Chiron de la Casinière, Le rayonnement solaire dans l'atmosphère terrestre, Publibook, 2003.

1. Thermal Radiation

The radiative emission depends on the temperature, which is the control of the level of excitation of the material: the more the particles are stirred, excited, the higher the temperature.

But how are these movements connected with the emission of waves?

The particles can be represented on several energy levels as a function of their excitation state Qn; the ground state is the quiescent state Q0.



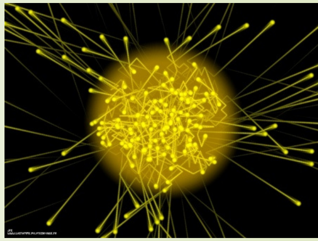
In an excited state Q_2, the particle will rejoin a lower energy state Q1 by releasing an energy quantity **$\Delta Q = h\nu$ (photon)**.

ν Is the frequence and h = 6,62 × 10-34 J·s is the universal constant of Planck. $\nu = \frac{c}{\lambda}$

1. Thermal Radiation

A photon is therefore a quantum of energy $h\nu$. More the wavelenght is low, more the photon is energetic.
This emission is a random phenomenon very well explained by statistical physics[1]. But the temperature T does not appear explicitly.

Nevertheless this quantum of energy can be approached as a packet of waves moving undulating.



Max Planck in 1905 succeeded in linking energy and temperature by means of statistical physics[1].

$$\boxed{L_\lambda^0(T) = \frac{2hc^2\lambda^{-5}}{e^{\frac{hc}{\lambda kT}} - 1}}$$

L : spectral luminance (W·m^{-2}·µm^{-1}·sr^{-1}). The index « 0 » of *L* represents the black body. It shows the temperature, but also the wavelength. k is the universal Boltzmann constant.

---

[1] G. Bruhat, Cours de physique générale, thermodynamique, Masson, Paris, 1926.

1. Thermal Radiation

Three photometric quantities:

Luminance
Directional quantity

$$d^5\Phi_{émis}(\theta,\varphi,\lambda)=L(\theta,\varphi)\cos\theta d^2\Sigma d^2\Omega d\lambda$$

Emittance (isotropic emission)
Hemisperical quantity.

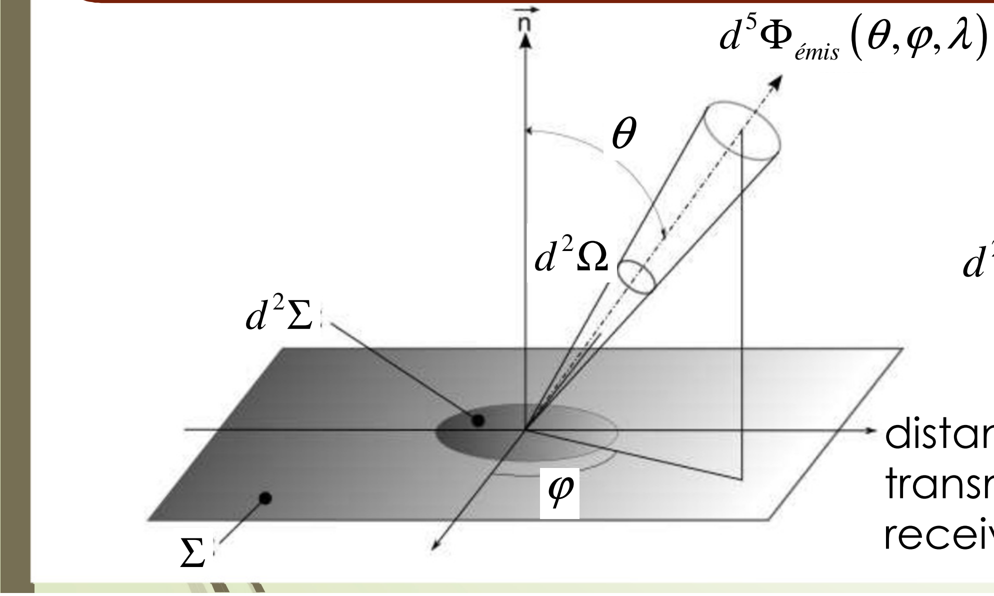
$$d^3\Phi_{émis}(\lambda)=\int_{\cap} L\cos\theta d^2\Sigma d\lambda d^2\Omega$$

$$d^3\Phi_{émis}(\lambda)=\pi L d^2\Sigma d\lambda$$

$$d^3\Phi_{émis}(\lambda)=M(\lambda,T)d^2\Sigma d\lambda$$

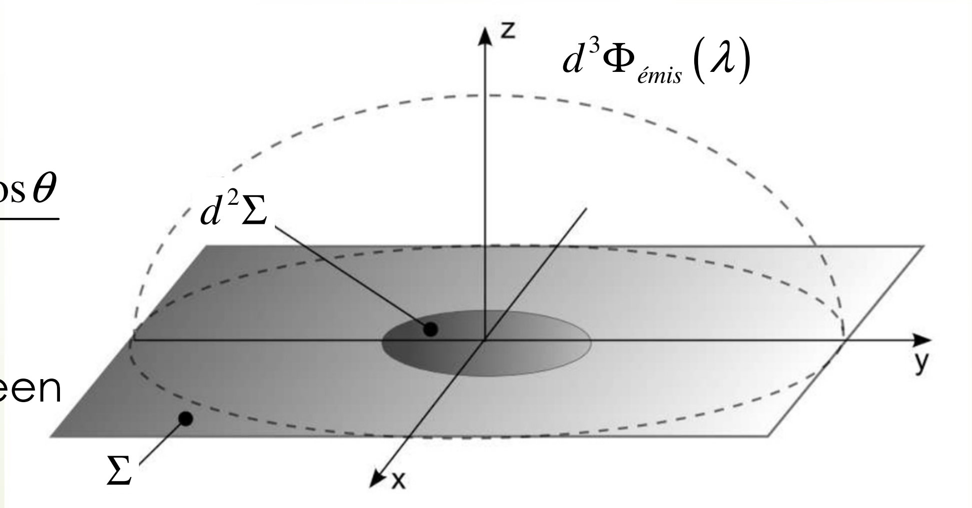
Irradiance (isotropic emission)
Hemisperical quantity.

$$d^3\Phi_{reçu}(\lambda)=E(\lambda,T)d^2\Sigma d\lambda$$



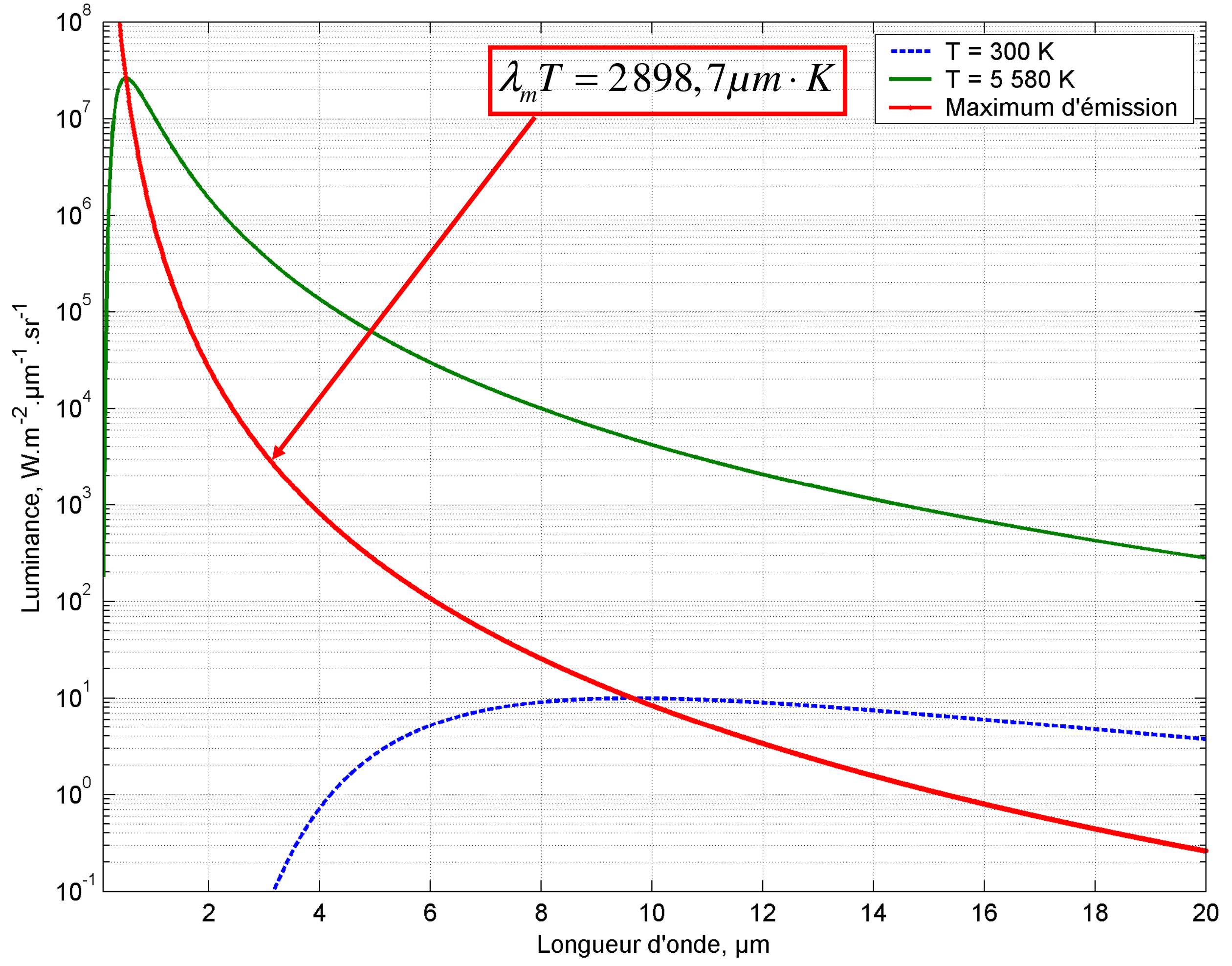
$$d^2\Omega=\frac{d^2\Sigma\cos\theta}{D^2}$$

distance between transmitter and receiver



1. Thermal Radiation

Two traces for two temperatures: human body and surface of the sun that pass through a maximum.



- For the sun, $\lambda_m \approx 0.5$ µm.
- For the human body $\lambda_m \approx 10$ µm.

Useful spectrum between $0.5\lambda_m$ µm and $5\lambda_m$ µm : 95% of the energy emission.

For the sun: UV-IR.

For the human body: IR.

1. Thermal Radiation

2. Radiatives Properties

3. Radiative measurements

4. Calibration

5. Other techniques

6. Applications

7. Inverse method

8. Prospect…

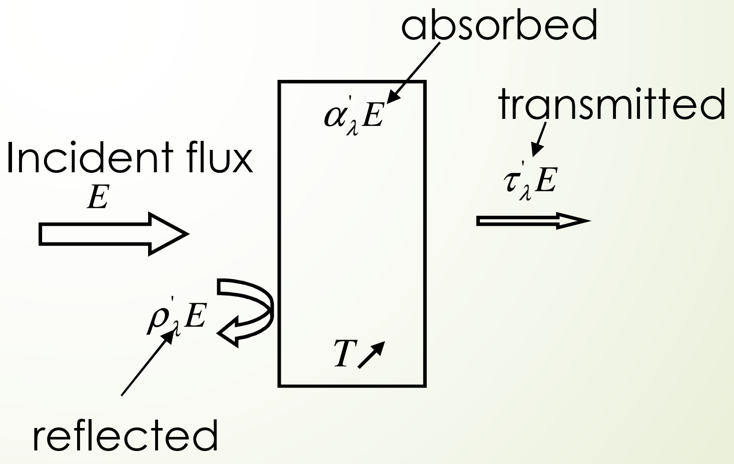
2. Radiative properties

The amount of energy emitted by a surface is a function of these **radiative properties**

In general, for a certain wavelength (λ) and from a certain direction ('), the conservation of energy predicts that on 100% of a heat flux reaching a surface, there are:

- an **absorbed** portion,
- a **reflected** portion and
- a portion **transmitted**.

$$\boxed{\alpha'_\lambda + \rho'_\lambda + \tau'_\lambda = 1}$$



2. Radiative properties

The radiative properties can be independent of the direction, the surfaces are lambertian:

$$\boxed{\alpha_\lambda = \varepsilon_\lambda} \quad \boxed{\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1}$$

They can also be independent of the wavelength, the surfaces are gray:

$$\boxed{\alpha\left(\mathbf{r},\hat{\Omega}\right) = \varepsilon\left(\mathbf{r},\hat{\Omega}\right)} \quad \boxed{\alpha\left(\mathbf{r},\hat{\Omega}\right) + \rho\left(\mathbf{r},\hat{\Omega}\right) + \tau\left(\mathbf{r},\hat{\Omega}\right) = 1}$$

Kirchhoff has demonstrated that the emission properties of a surface are the same as those of absorption:

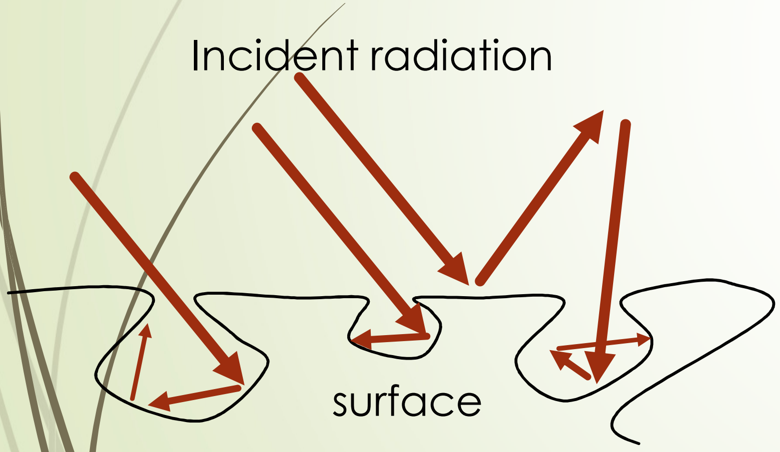
$$\boxed{\alpha_\lambda\left(\mathbf{r},\hat{\Omega}\right) = \varepsilon_\lambda\left(\mathbf{r},\hat{\Omega}\right)}$$

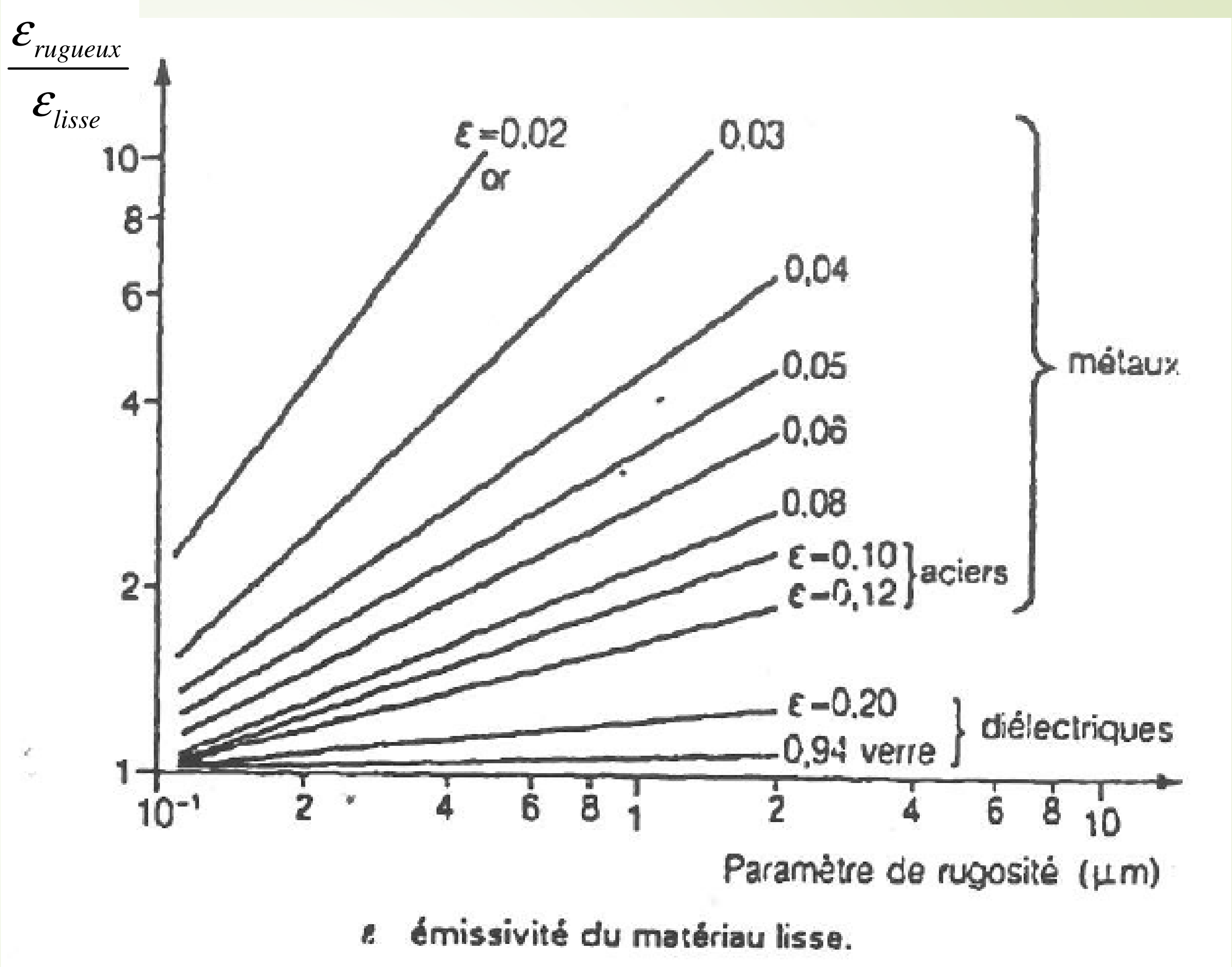
The particular case is that of the black body which absorbs the radiation received whatever the wavelength and the incident direction. But the black body does not exist, it only approaches ($\varepsilon = 0.999$).

$$\boxed{\alpha = \varepsilon = 1}$$

2. *Radiative properties:* Dependency on roughness

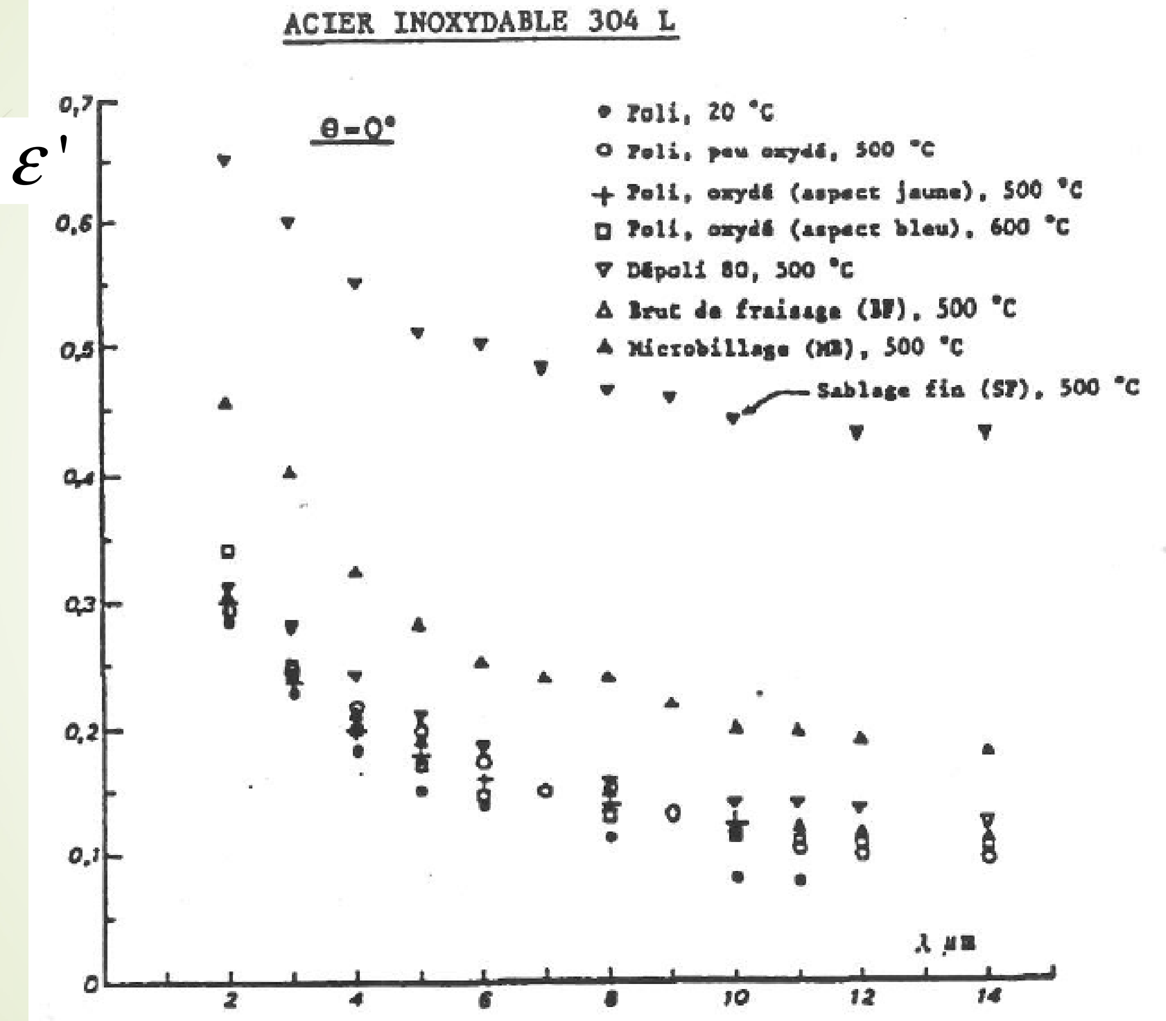
Roughness reveals cavities that serve as small "black bodies" that trap radiation, which is partially absorbed by matter.





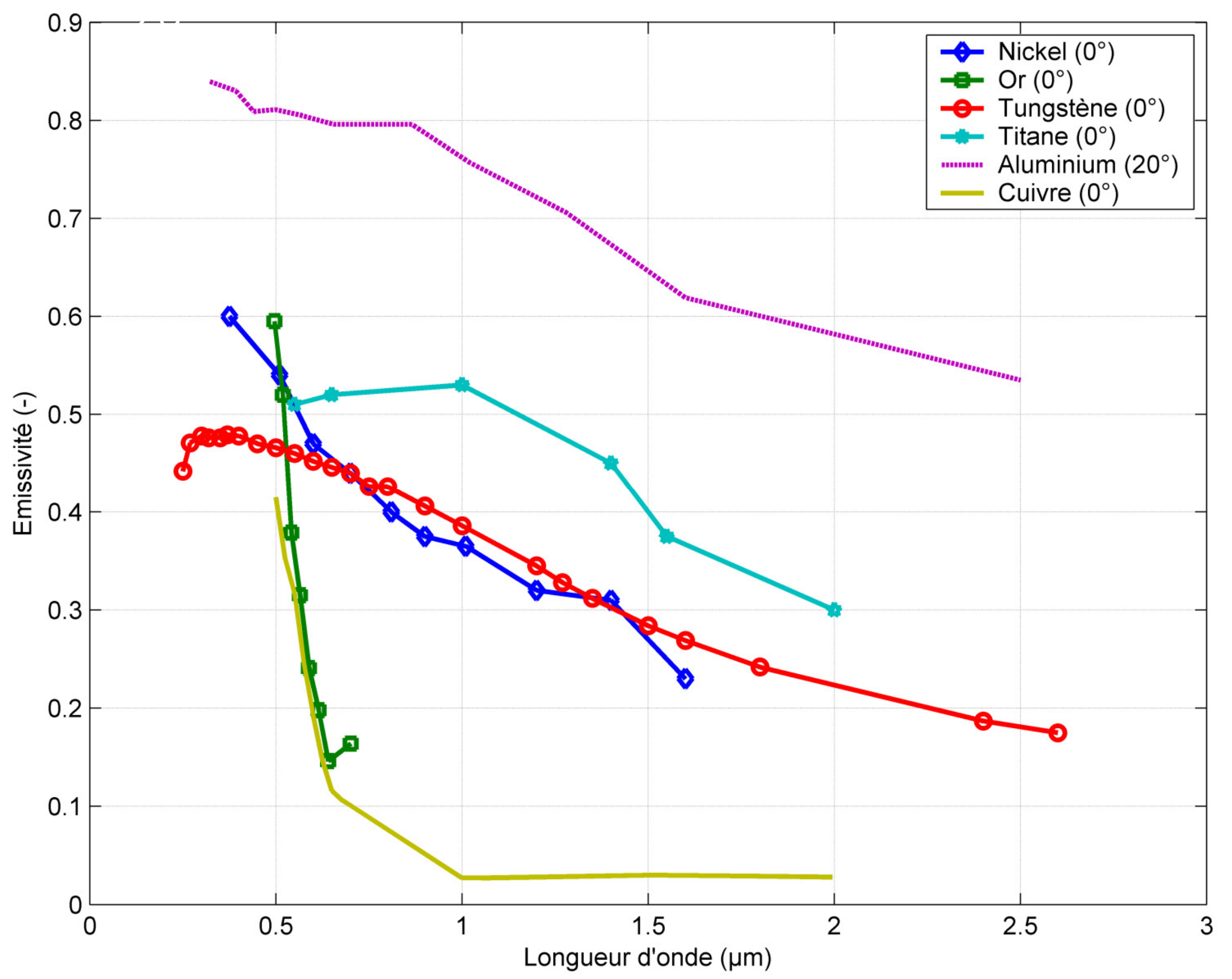
P. Hervé, Influence de l'état de surface sur le rayonnement thermique des matériaux solides, thèse, 1977.

2. Radiative properties: Dependency on roughness



J.-F. Sacadura, Les méthodes de mesure des propriétés radiatives, colloque SFT, ISITEM Nantes, 1990.

2. Radiative properties: spectral dependency



1 Y.S. Touloukian, Thermal radiative properties, Plenum, New York,1970.

2. *Radiative properties:* spectral dependency

Spectral properties

Spectral integration

$$\int_0^\infty d\lambda$$

Total properties

$$\varepsilon_\lambda = \frac{d^3\Phi_{émis}(\lambda,T)}{d^3\Phi^0_{émis}(\lambda,T)}$$

Spectral emissivity

In front of the emission of a black body at the same temperature.

$$\alpha_\lambda = \frac{d^3\Phi_a(\lambda)}{d^3\Phi_{reçu}(\lambda)} \qquad \rho_\lambda = \frac{d^3\Phi_r(\lambda)}{d^3\Phi_{reçu}(\lambda)} \qquad \tau_\lambda = \frac{d^3\Phi_t(\lambda)}{d^3\Phi_{reçu}(\lambda)}$$

Spectral absorptivity

Spectral reflectivity

Spectral transmittivity

Ratio between the analysed flux (α, ρ or τ) and the recepted incident flux.

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda d^3\Phi_{émis}(\lambda)d\lambda}{\int_0^\infty d^3\Phi^0_{émis}(\lambda)d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda M^0(\lambda,T)d\lambda}{\int_0^\infty M^0(\lambda,T)d\lambda}$$

$$\boxed{\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda M^0(\lambda,T)d\lambda}{\sigma T^4}}$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda d^3\Phi_a(\lambda)d\lambda}{\int_0^\infty d^3\Phi_a(\lambda)d\lambda} = \frac{\int_0^\infty \alpha_\lambda E(\lambda,T)d\lambda}{\int_0^\infty E(\lambda,T)d\lambda}$$

If the illumination comes from the sun then :

$$E(\lambda,T) \approx M^0(\lambda,T_s)$$

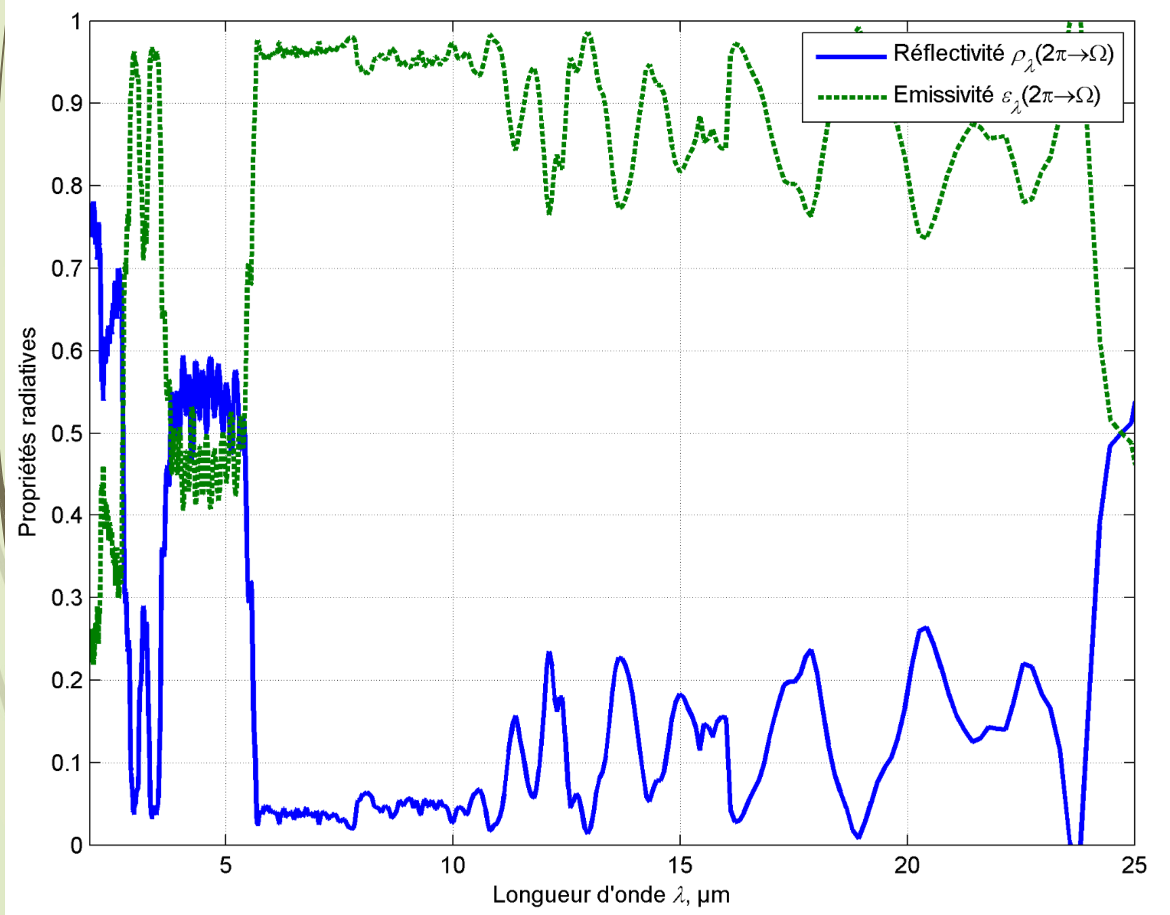
Same results for the total reflectivity and the total transmittivity

$$\boxed{\rho = \frac{\int_0^\infty \rho_\lambda E(\lambda,T)d\lambda}{\int_0^\infty E(\lambda,T)d\lambda}} \qquad \boxed{\tau = \frac{\int_0^\infty \tau_\lambda E(\lambda,T)d\lambda}{\int_0^\infty E(\lambda,T)d\lambda}}$$

2. ***Radiative properties:*** spectral dependency

Example of the total emissivity (camera LW [7 µm – 13 µm]) « apparent » :

Metallic adhesif tested with a spectroradiometer IR :



$$\varepsilon = \frac{\int_{\lambda_1}^{\lambda_2} \varepsilon_\lambda M^0(\lambda,T)\,d\lambda}{\int_{\lambda_1}^{\lambda_2} M^0(\lambda,T)\,d\lambda}$$

$$\varepsilon = \frac{\sum_{i=1}^{N} \varepsilon_\lambda M^0(\lambda,T)\Delta\lambda}{\sum_{i=1}^{N} M^0(\lambda,T)\Delta\lambda}$$

apparent emissivity calculated for the scale :

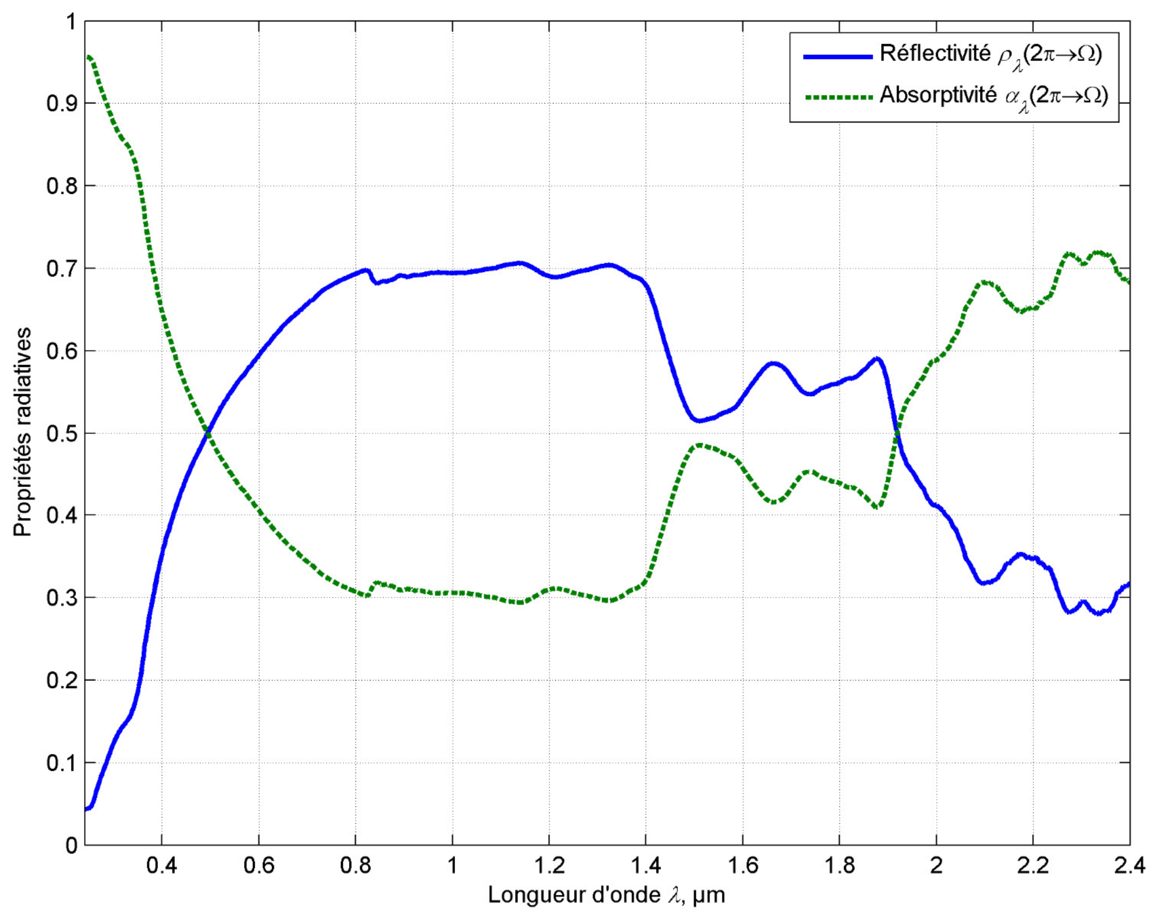
- [2 µm – 25 µm] : 0.41
- [7 µm – 13 µm] : **0.94**

For the temperature T = 300 K.

2. Radiative properties: spectral dependency

Example of the total absorptivity and refectivity « apparents » of a material under a solar radiation :

Linen tested with a spectroradiometer UV-visible (useful extent of the sun) :



$$\rho = \frac{\int_{\lambda_1}^{\lambda_2} \rho_\lambda E(\lambda,T)\,d\lambda}{\int_{\lambda_1}^{\lambda_2} E(\lambda,T)\,d\lambda}$$

$$\rho = \frac{\sum_{i=1}^{N} \rho_\lambda M^0(\lambda,T_s)\Delta\lambda}{\sum_{i=1}^{N} M^0(\lambda,T_s)\Delta\lambda}$$

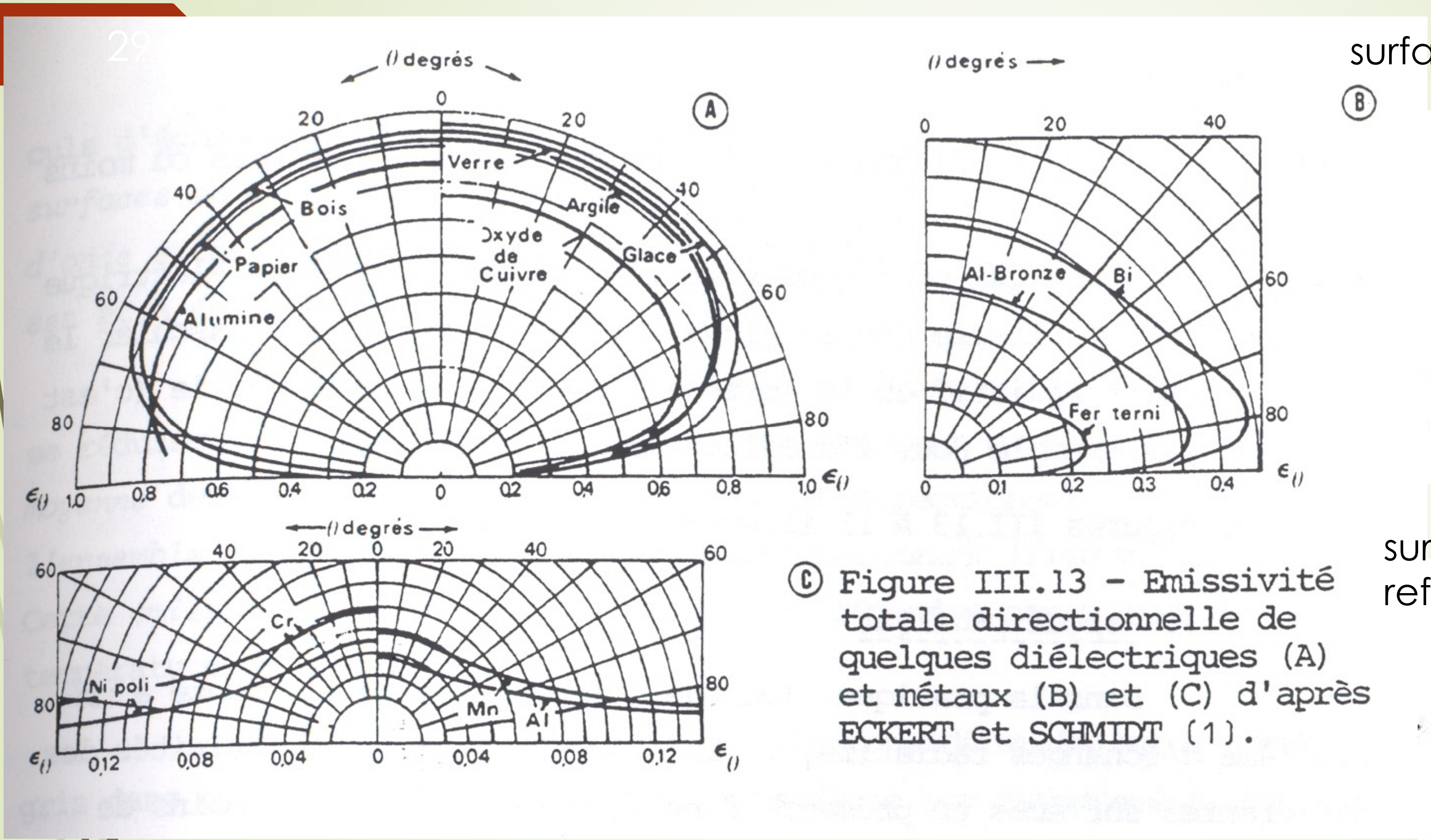
If the material is opaque : $\alpha = 1 - \rho$

apparent absorptivité calculated on the scale:

- [0.24 µm – 2.4 µm] : 0.44

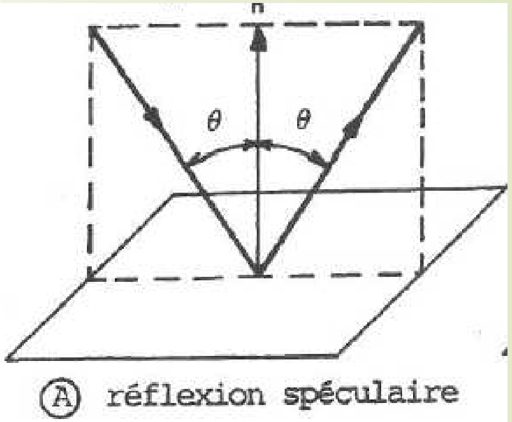
For a temperature T = 5 800 K.

2. *Radiative properties:* directional dependency



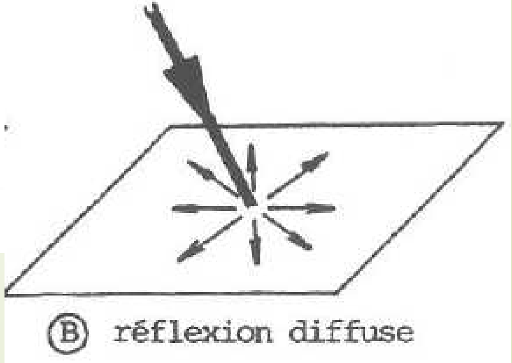
Ⓒ Figure III.13 - Emissivité totale directionnelle de quelques diélectriques (A) et métaux (B) et (C) d'après ECKERT et SCHMIDT (1).

surface with a specular reflection



Ⓐ réflexion spéculaire

surface with a isotropic reflection (diffus, lambertian)



Ⓑ réflexion diffuse

1 J.-F. Sacadura, *Initiation aux transferts thermiques*, Tec&Doc, Paris,1993.

2. ***Radiative properties:*** directional dependency

Reflection Properties of the surfaces : **function of the bireflection distribution**

portion of the reflected flux

$$f_\lambda\left(\mathbf{r},\hat{\Omega},\hat{\Omega}'\right)=\frac{dI_\lambda\left(\mathbf{r},\hat{\Omega}\right)}{I_\lambda\left(\mathbf{r},\hat{\Omega}'\right)\cos\theta' d\Omega'}$$

Incident flux

or

$$\boxed{f_\lambda\left(\mathbf{r},\hat{\Omega},\hat{\Omega}'\right)=\frac{dL_\lambda\left(\mathbf{r},\hat{\Omega}\right)}{L_\lambda\left(\mathbf{r},\hat{\Omega}'\right)\cos\theta' d\Omega'}}$$

Reciprocity relation :

$$f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}',\hat{\mathbf{\Omega}}\right)=f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}'\right)$$

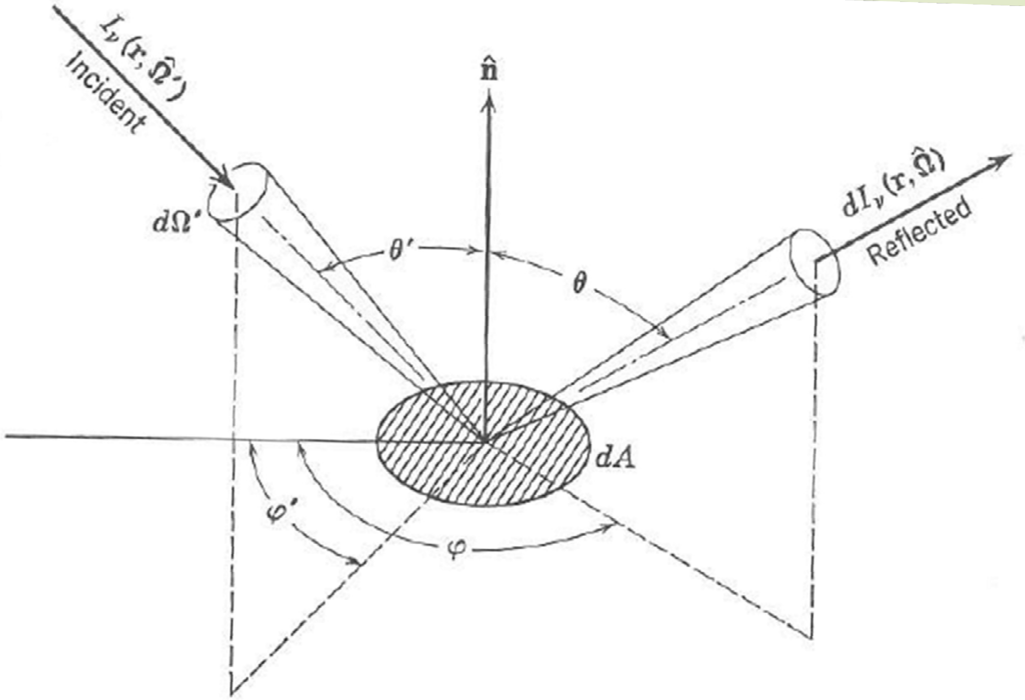


Fig. 1-11. Symbols for the definition of reflection distribution function, $f_\nu(\mathbf{r}, \hat{\Omega}', \hat{\Omega})$.

Very precise quantity, but difficult to quantify and impractical.

2. Radiative properties: directional dependency

Reflection properties of the surfaces : **directional-hemespherical reflectivity**

$$\rho_\lambda\left(\mathbf{r},\hat{\boldsymbol{\Omega}}'\to 2\pi\right)=\frac{\int_{2\pi} dL_\lambda\left(\mathbf{r},\hat{\boldsymbol{\Omega}}\right)\cos\theta d\Omega}{L_\lambda\left(\mathbf{r},\hat{\boldsymbol{\Omega}}'\right)\cos\theta' d\Omega'}$$

$$\boxed{\rho_\lambda\left(\mathbf{r},\hat{\boldsymbol{\Omega}}'\to 2\pi\right)=\int_{2\pi} f_\lambda\left(\mathbf{r},\hat{\boldsymbol{\Omega}},\hat{\boldsymbol{\Omega}}'\right)\cos\theta d\Omega}$$

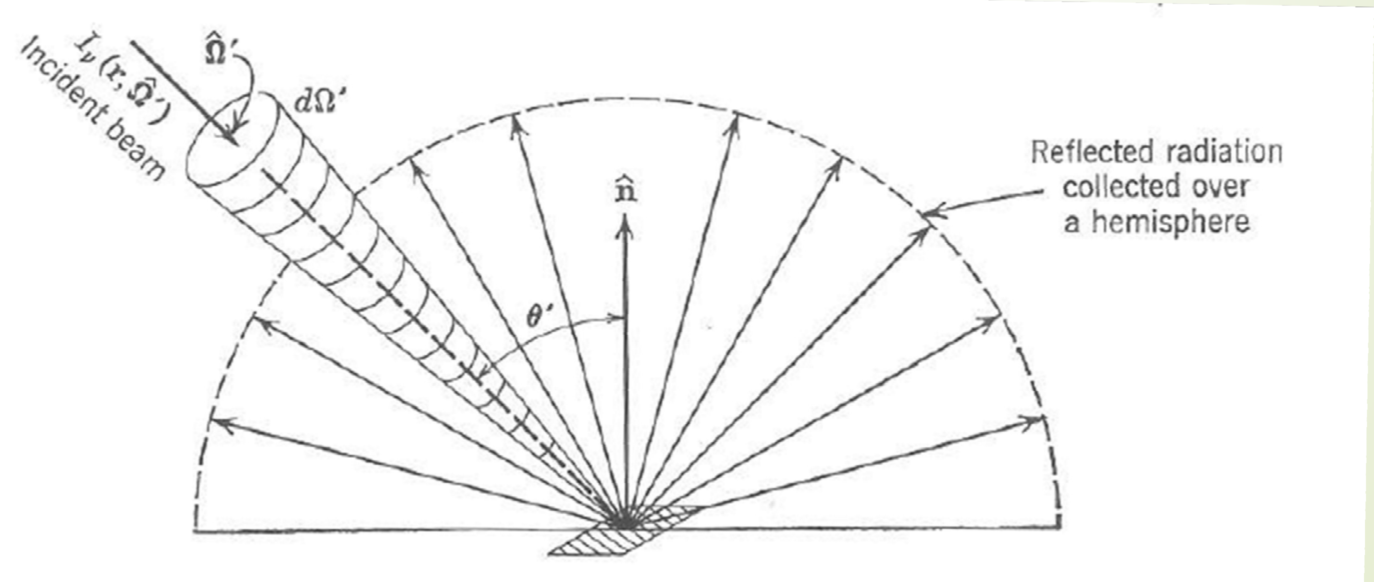


Fig. 1-12. Symbols for the definition of directional-hemispherical reflectivity, $\rho_\nu(\mathbf{r},\hat{\Omega}'\to 2\pi)$.

2. *Radiative properties:* directional dependency

Reflection properties of the surfaces : **directional-hemespherical reflectivity**

$$\rho_\lambda\left(\mathbf{r},2\pi\to\hat{\mathbf{\Omega}}\right)=\frac{\int_{2\pi} f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}'\right)L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}{1/\pi\int_{2\pi} L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}$$

$$\boxed{\rho_\lambda\left(\mathbf{r},2\pi\to\hat{\mathbf{\Omega}}\right)=\int_{2\pi} f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}$$

because :

$$f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}'\right)=\frac{dL_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)}{L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}$$

$$L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)=\int_{2\pi} f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}'\right)L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'$$

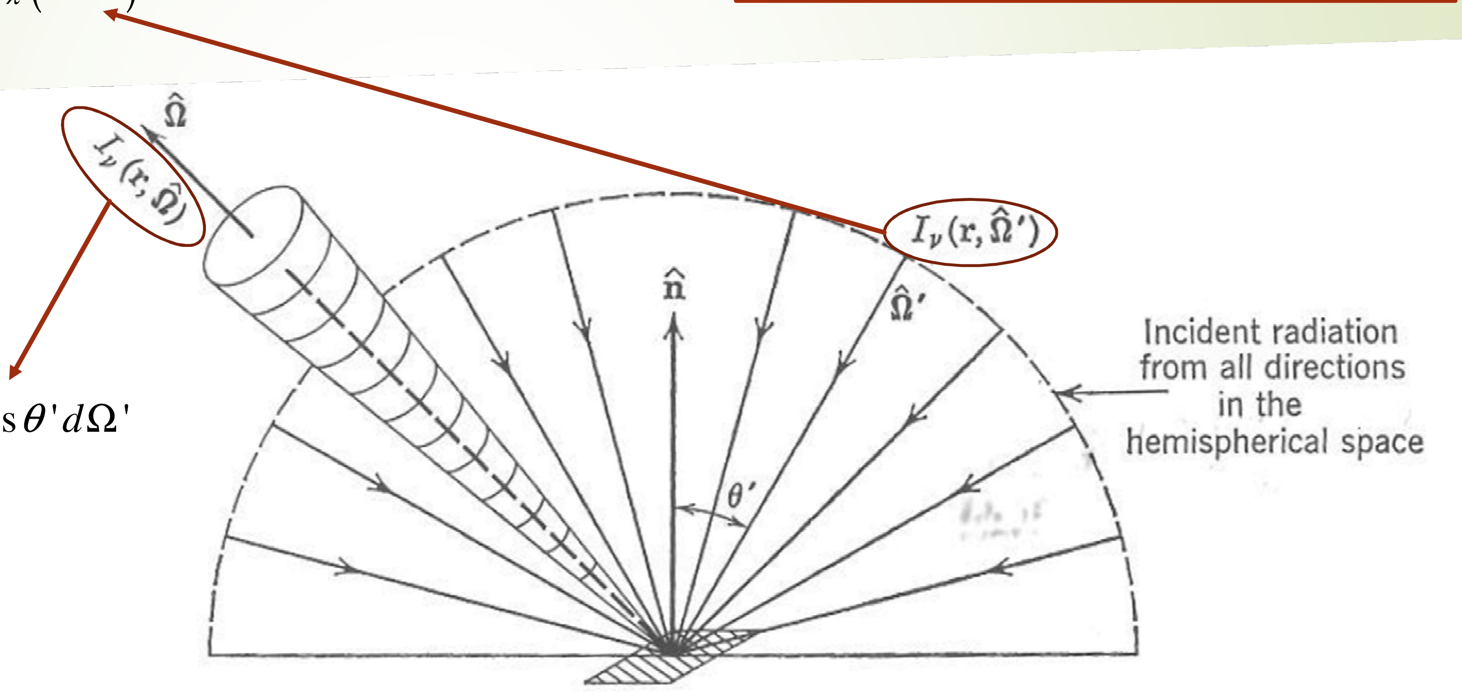


Fig. 1-13. Symbols for the definition of hemispherical-directional reflectivity, $\rho_\nu(\mathbf{r}, 2\pi \to \hat{\Omega})$.

2. Radiative properties: directional dependency

Reflection properties of the surfaces : **hemespherical reflectivity**

Ratio of the reflected flux in the all hemisphere on the flux from this same hemisphere.

$$\rho_\lambda(\mathbf{r}) = \frac{\int\limits_{\Omega'=2\pi} \rho_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}' \rightarrow 2\pi\right) L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}{\underbrace{\int\limits_{\Omega'=2\pi} \underbrace{L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}_{\text{illumination}}}_{\text{Incident flux}}}$$

If the incident luminance $L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)$ is independent of the direction then :

$$\rho_\lambda(\mathbf{r}) = \frac{1}{\pi}\int\limits_{\Omega'=2\pi} \rho_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}' \rightarrow 2\pi\right)\cos\theta' d\Omega'$$

$$\int\limits_{\Omega'=2\pi} \cos\theta' d\Omega' = \pi$$

$$\rho_\lambda(\mathbf{r}) = \frac{1}{\pi}\int\limits_{\Omega'=2\pi}\left[\int\limits_{\Omega=2\pi} f_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}},\hat{\mathbf{\Omega}}'\right)\cos\theta d\Omega\right]\cos\theta' d\Omega'$$

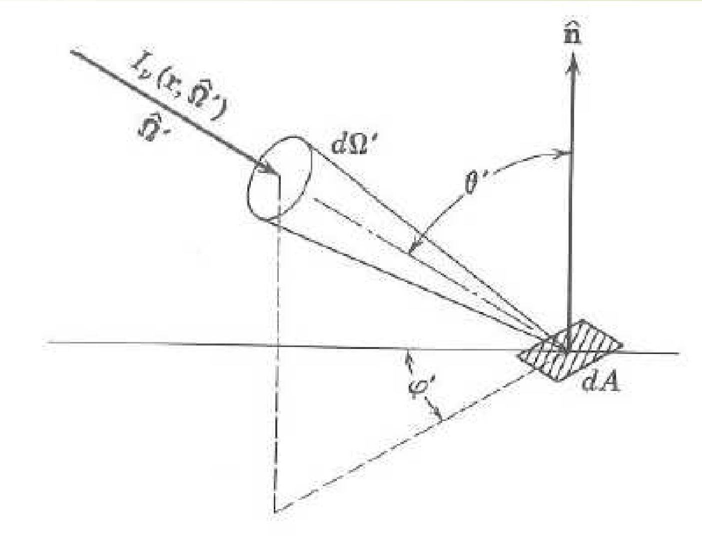
If the bireflection distribution function is independent of the direction :

$$\boxed{\rho_\lambda(\mathbf{r}) = \pi f_\lambda(\mathbf{r})}$$

2. ***Radiative properties:*** directional dependency

- Absorption properties of surfaces: **directional absorptivity**

$$\alpha_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)=\frac{d^3\Phi_a(\lambda)}{L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}$$



surface balance assumed opaque :

$$d^3\Phi_a(\lambda)=\underbrace{L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}_{\text{Incident flux}}-\underbrace{\rho_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\to 2\pi\right)L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}_{\text{Reflected flux}}$$

$$\boxed{\alpha_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)=1-\rho_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\to 2\pi\right)}$$

Hemispherical absorptivity

$$\alpha_\lambda(\mathbf{r})=\frac{\int\limits_{\Omega'=2\pi}\alpha_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}{\int\limits_{\Omega'=2\pi}L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}$$

The incident flux comes from all over the hemisphere

$$\alpha_\lambda(\mathbf{r})=1-\frac{\int\limits_{\Omega'=2\pi}\rho_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\to 2\pi\right)L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}{\int\limits_{\Omega'=2\pi}L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}'\right)\cos\theta' d\Omega'}$$

$$\boxed{\alpha_\lambda(\mathbf{r})=1-\rho_\lambda(\mathbf{r})}$$

2. *Radiative properties:* directional dependency

Surface emission properties: **Directional spectral emissivity**

$$\varepsilon_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)=\frac{L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)}{L_\lambda^0\left(T\right)}$$

Kirchhoff law $\varepsilon_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)=\alpha_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)$

Opaque body $\varepsilon_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)=\alpha_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)=1-\rho_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\to 2\pi\right)$

hemispheric spectral emissivity

$$\varepsilon_\lambda\left(\mathbf{r}\right)=\frac{\int\limits_{\Omega=2\pi} L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)\cos\theta d\Omega}{\int\limits_{\Omega=2\pi} L_\lambda^0\left(T\right)\cos\theta d\Omega}$$

$$\varepsilon_\lambda\left(\mathbf{r}\right)=\frac{1}{\pi}\int\limits_{\Omega=2\pi}\frac{L_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)}{L_\lambda^0\left(T\right)}\cos\theta d\Omega$$

$$\boxed{\varepsilon_\lambda\left(\mathbf{r}\right)=\frac{1}{\pi}\int\limits_{\Omega=2\pi}\varepsilon_\lambda\left(\mathbf{r},\hat{\mathbf{\Omega}}\right)\cos\theta d\Omega}$$

In the same way:

$$\boxed{\varepsilon_\lambda\left(\mathbf{r}\right)=\alpha_\lambda\left(\mathbf{r}\right)}$$

$$\boxed{\varepsilon_\lambda\left(\mathbf{r}\right)=\alpha_\lambda\left(\mathbf{r}\right)=1-\rho_\lambda\left(\mathbf{r}\right)}$$

2. Radiative properties: temperature dependence

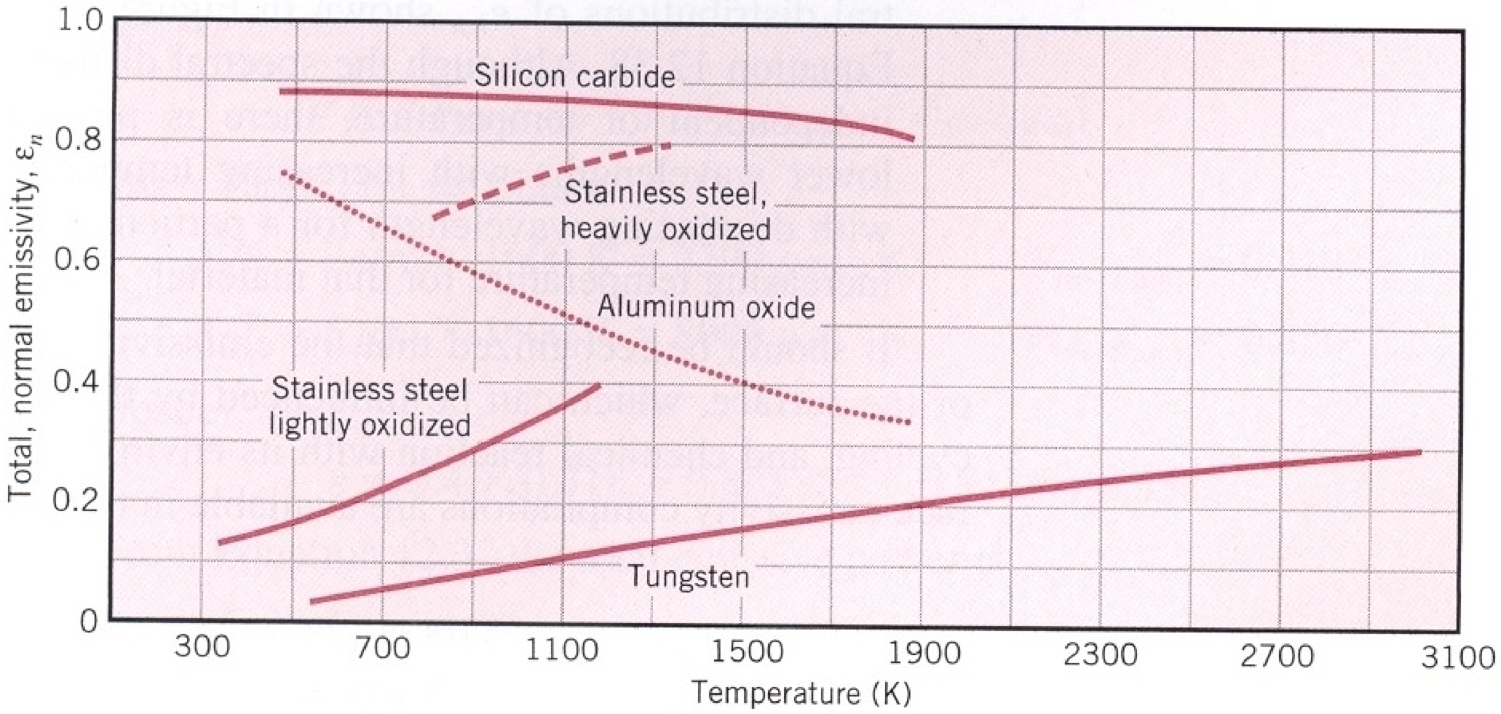


FIGURE 12.19 Temperature dependence of the total, normal emissivity ε_n of selected materials.

1 F.P. Incropera et al., Fundamentals of heat and mass transfer, John Wiley & Sons, New-York, 2002.

2. ***Radiative properties:*** emissivity models

	Model	**Loi** $\varepsilon'_{\lambda,T} =$
Physical	Drude	$\frac{a_0}{\sqrt{\lambda}}$
	Hagen-Rubens	$a_0\sqrt{\frac{T}{\lambda}}$
Mathematical	exponential	$e^{a_0\lambda + a_1 T}$
	temperature, spectral inverse	$e^{a_0\lambda + \frac{a_1}{T}}$
	standardized	$\frac{1}{1 + a_0\lambda^2}$
	polynomial	$\sum_{i=0}^{n} a_i\lambda^i$

2. Radiative properties

Example of radiative properties: spectral transmittance of glass[1]. The greenhouse effect at the terrestrial level: atmospheric gases are assimilated to glass.

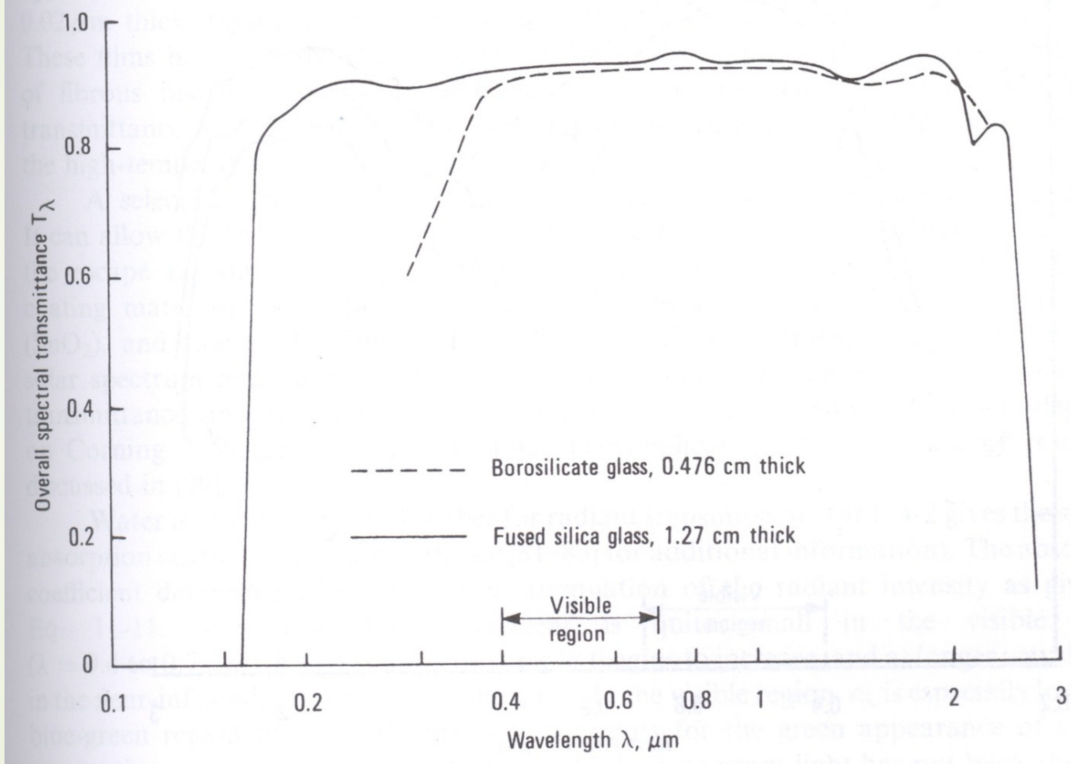


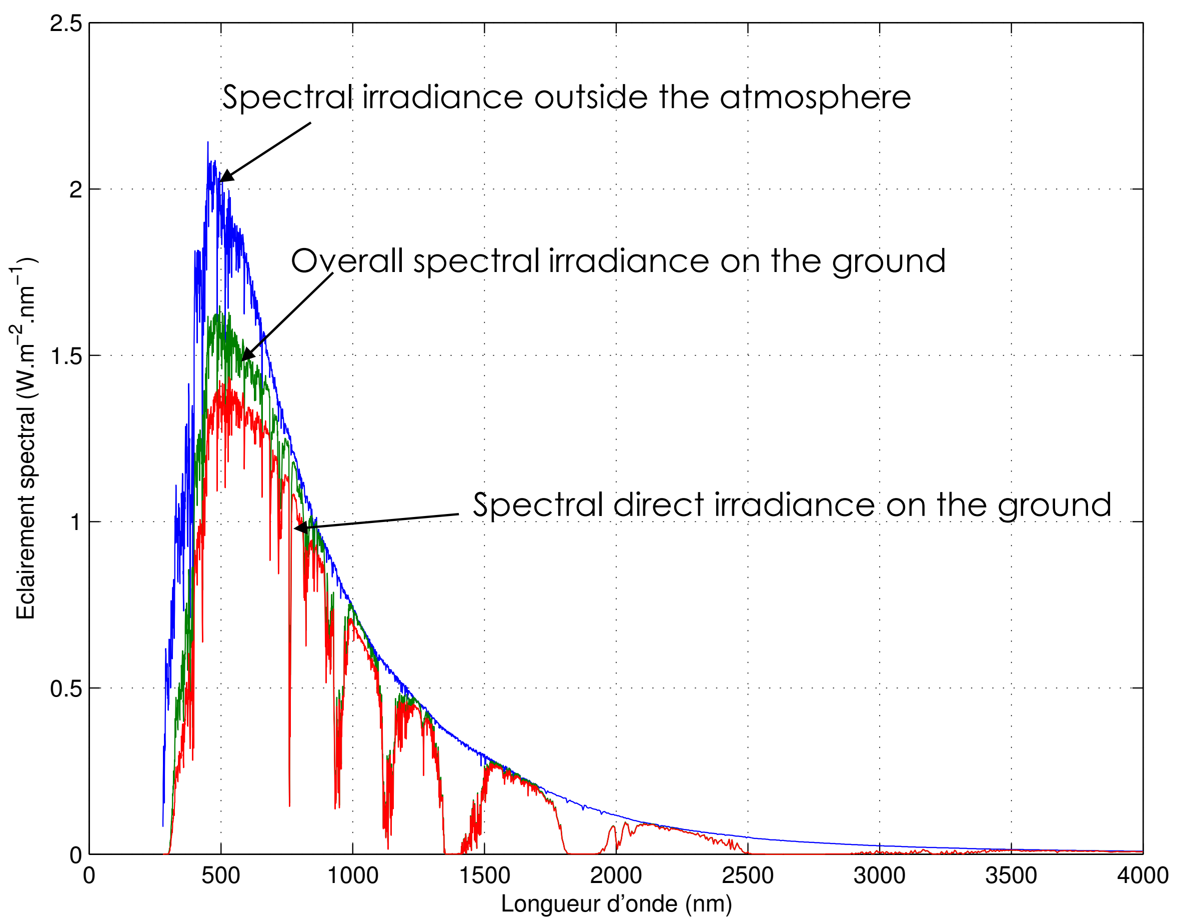
FIGURE 4-38 Normal overall spectral transmittance of a glass plate (includes surface reflections) at 298 K. Replotted from [1].

---

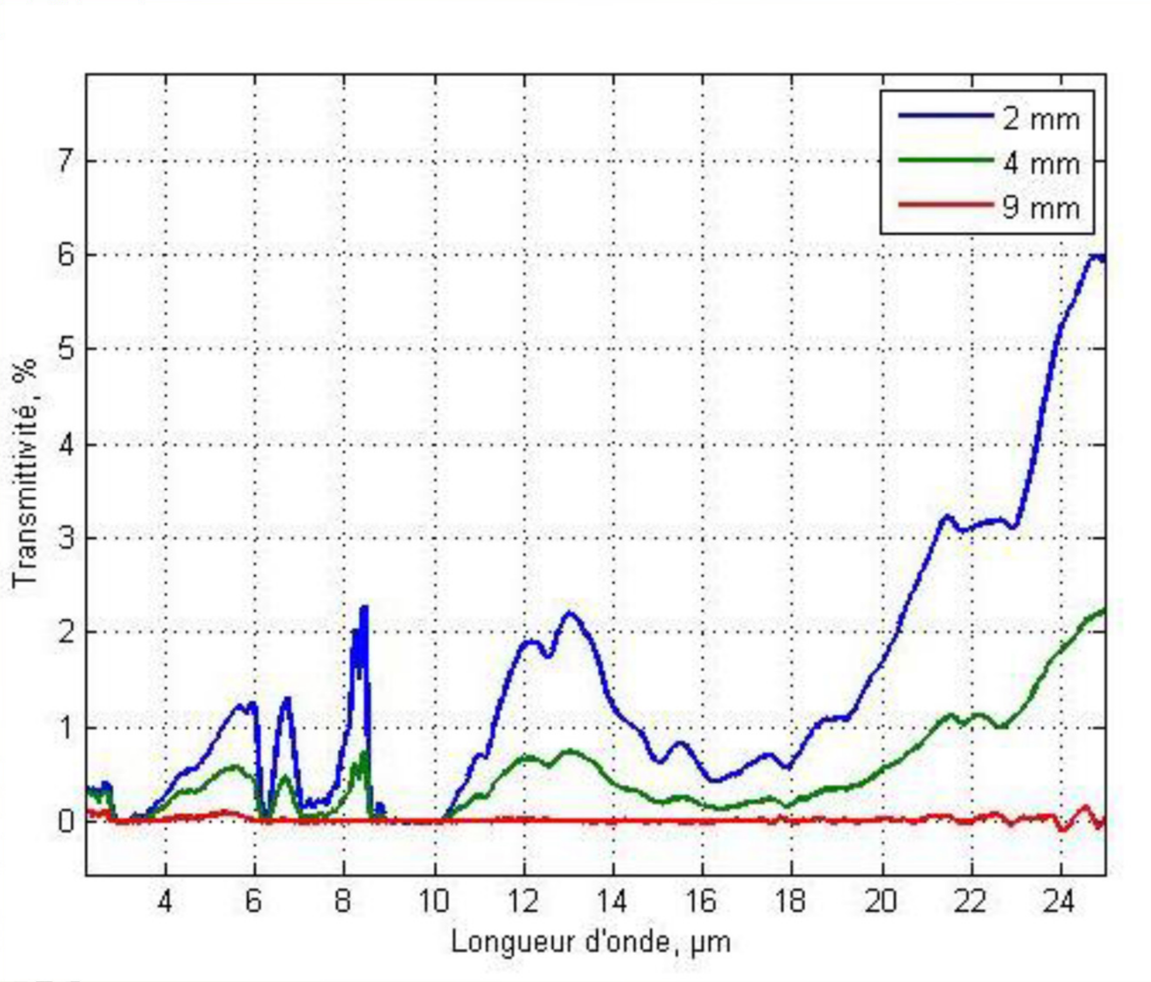
1 R. Siegel, J.R. Howel, Thermal radiation heat transfer, Taylor & Francis Inc., 4th Ed., New-York, 2002.

2. Radiative properties

Example of radiative properties: absorptivity of the atmosphere.

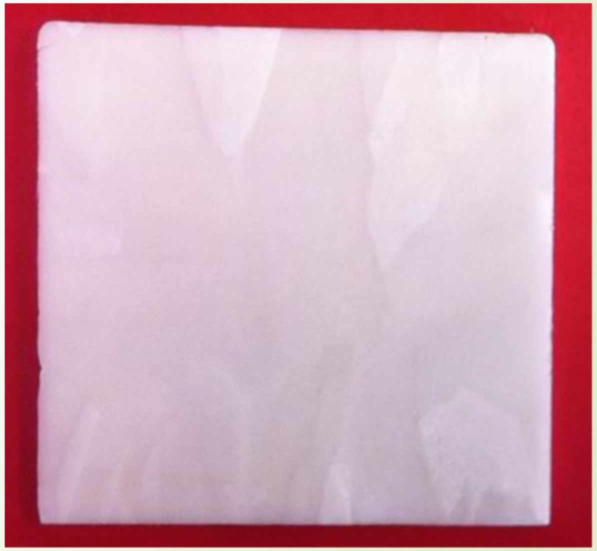


2. Radiative properties



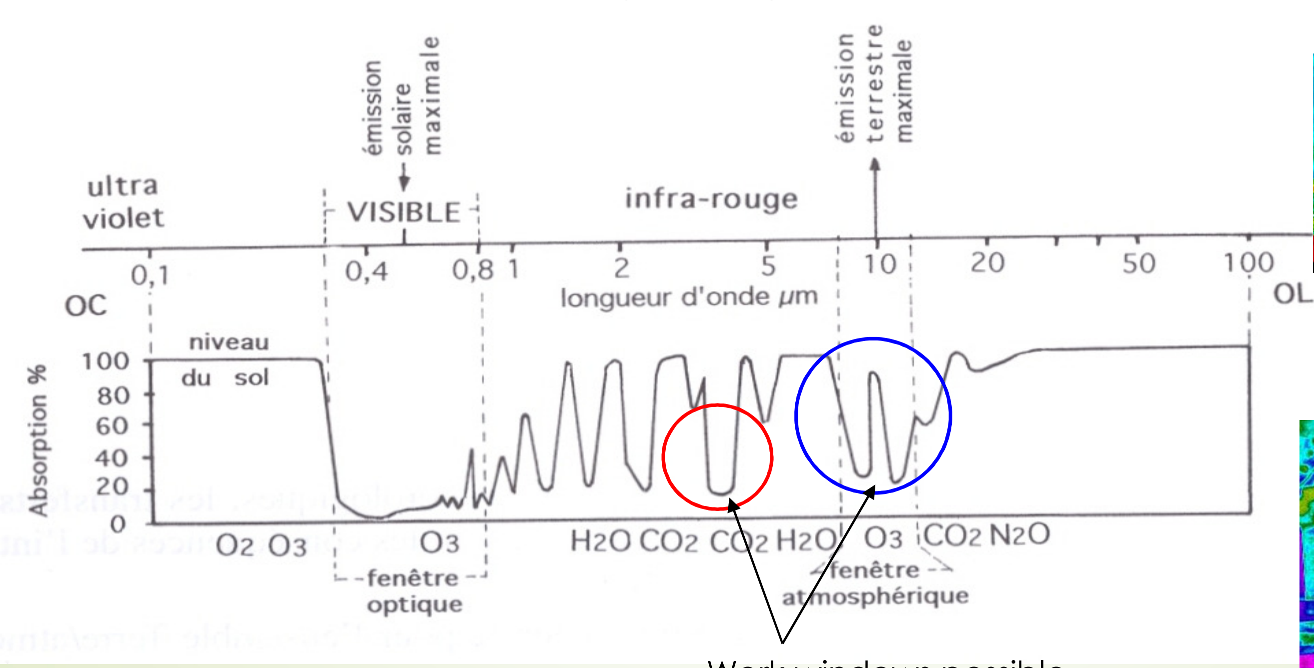
Spectral Transmittivity of Airgel depending on the thickness.

porosity:> 95%
density: 10 kg · m^{-3}

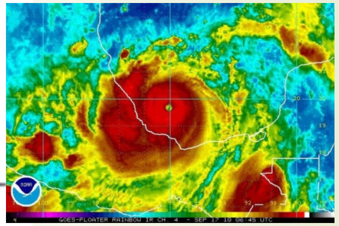


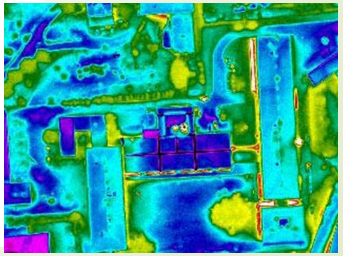
2. Radiative properties

Typical spectral ranges of camera operation:
[3 µm - 5 µm] and [8 µm - 12 µm].
The absorption spectrum of the atmosphere[1] explains it: the radiation is absorbed by the components of the atmosphere H_2O, CO_2, ozone O_3 ... This remark is all the more true as the distance increases between the camera and the target areas: volcano temperatures, thermal wastage imagery by helicopter or satellite ...



Work windows possible





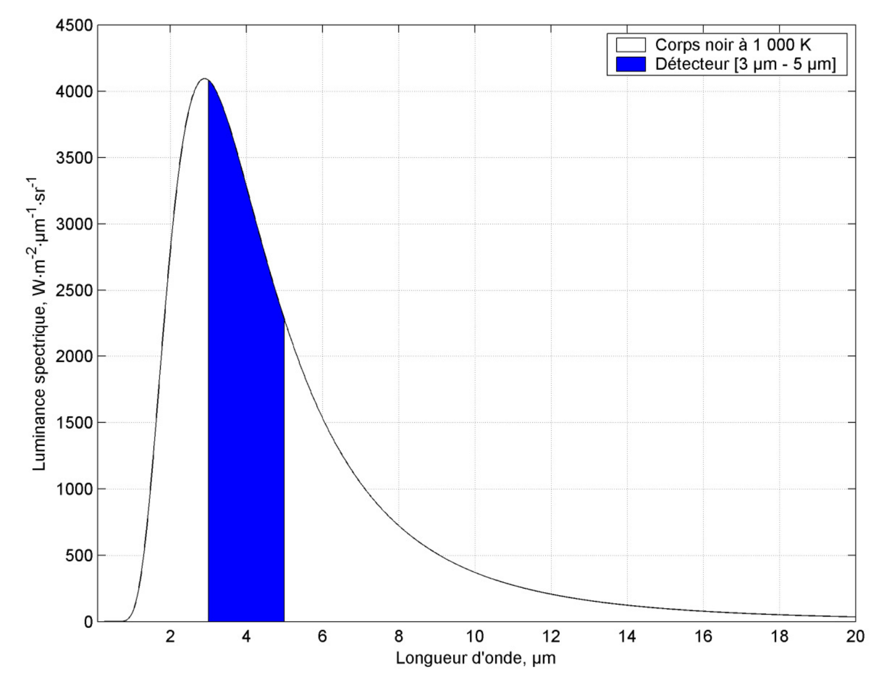
1. M. Leroux, La dynamique du temps et du climat, Dunod, 2004.

3. Radiative Measurements – Principles

Principle : measure a signal Vm or a number of photons proportional to an energy.

Constraints and problems:

- Integration on a known spectral band [l1; l2], working range of the detector. Emissivity often unknown.
- Valid over a range of temperatures: need to calibrate the detector according to the temperature range. Calibration is done by means of a black body.



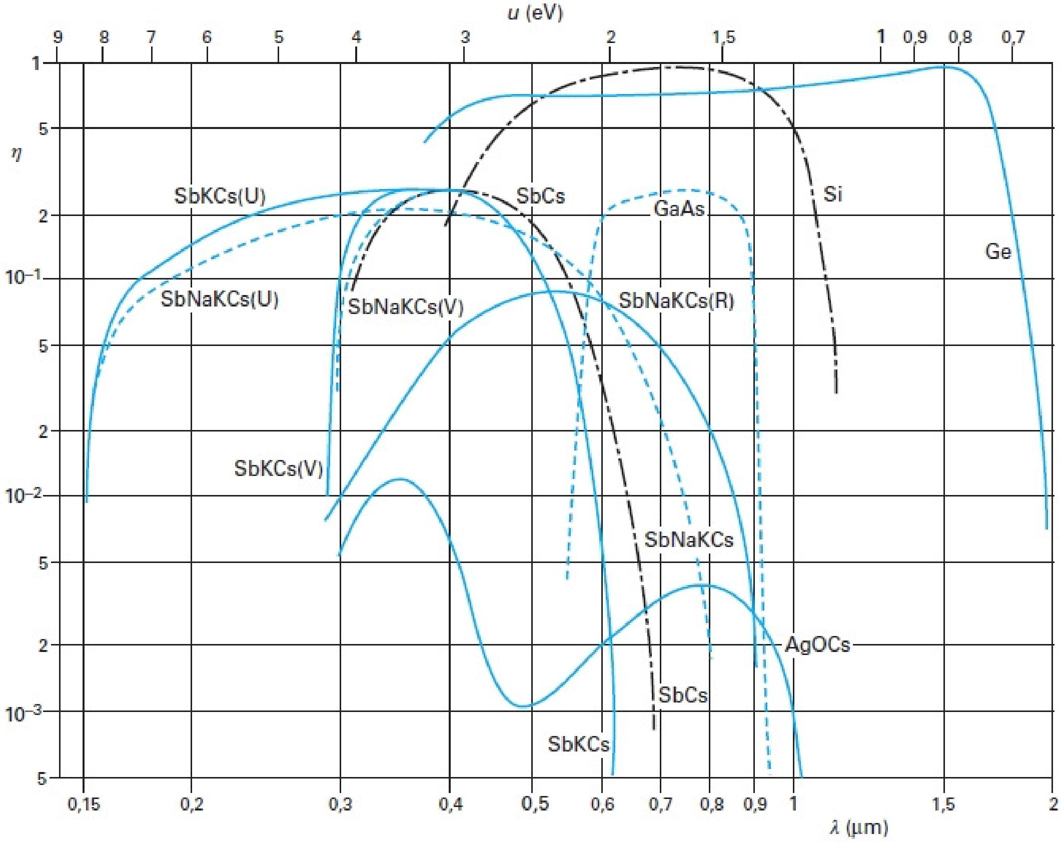
$$\boxed{V_m(T) = s\int_{\lambda_2}^{\lambda_1} d_\lambda L_\lambda(T)d\lambda}$$

sensitive surface of the detector

spectral response of the detector

3. *Radiative Measurements* – Detectors

Examples of spectral responses δ_λ for differents detectors.



Si et Ge : photodiodes et photopiles

Les autres courbes se rapportent aux couches photoémissives, désignées par leur composition chimique (approximative)

(U) dépôt sur fenêtre en quartz fondu

(V) dépôt sur fenêtre en verre

(R) couche «sensibilisée au rouge»

(GaAs) cristal de GaAs avec une couche à affinité électronique négative

Figure 8 – Efficacité quantique *η* de photodétecteurs en fonction de la longueur d'onde *λ* et de l'énergie *u* des photons

Luc Audaire, *Détecteurs de rayonnements optiques,* TI R6450, 2000,

3. Radiative Measurements – How a basic IR camera works

Example with a **thermal camera**. This so-called matrix camera (320 × 240 pixels) performs imaging.

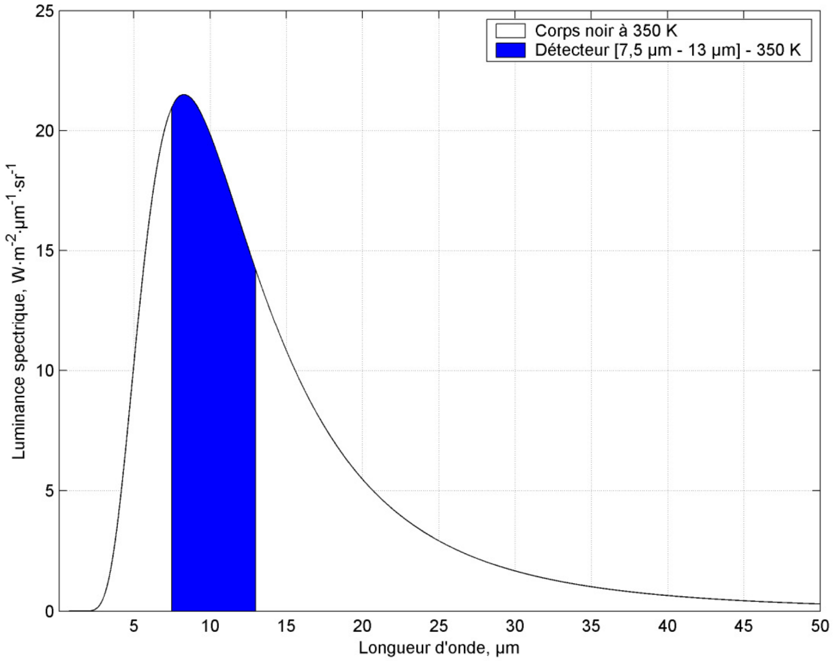
Working range: **[7.5 µm - 13 µm]**.

Input parameters: emissivity ε and ambient temperature T_a.

- Constant effective emissivity on $[\lambda_1 ; \lambda_2]$.
- The environment is taken into account.

$$\boxed{V_m(T) = s\int_{\lambda_2}^{\lambda_1} d_\lambda L_\lambda(T) d\lambda}$$

$$\boxed{L_\lambda(T) = \underset{\underset{\text{Surface}}{\uparrow}}{\varepsilon'_\lambda} L^0_\lambda(T) + (1-\varepsilon'_\lambda)\underset{\underset{\text{Environment}}{\uparrow}}{L^0_\lambda}(T_a)}$$



3. Radiative Measurements – Sensitivity of a IR camera

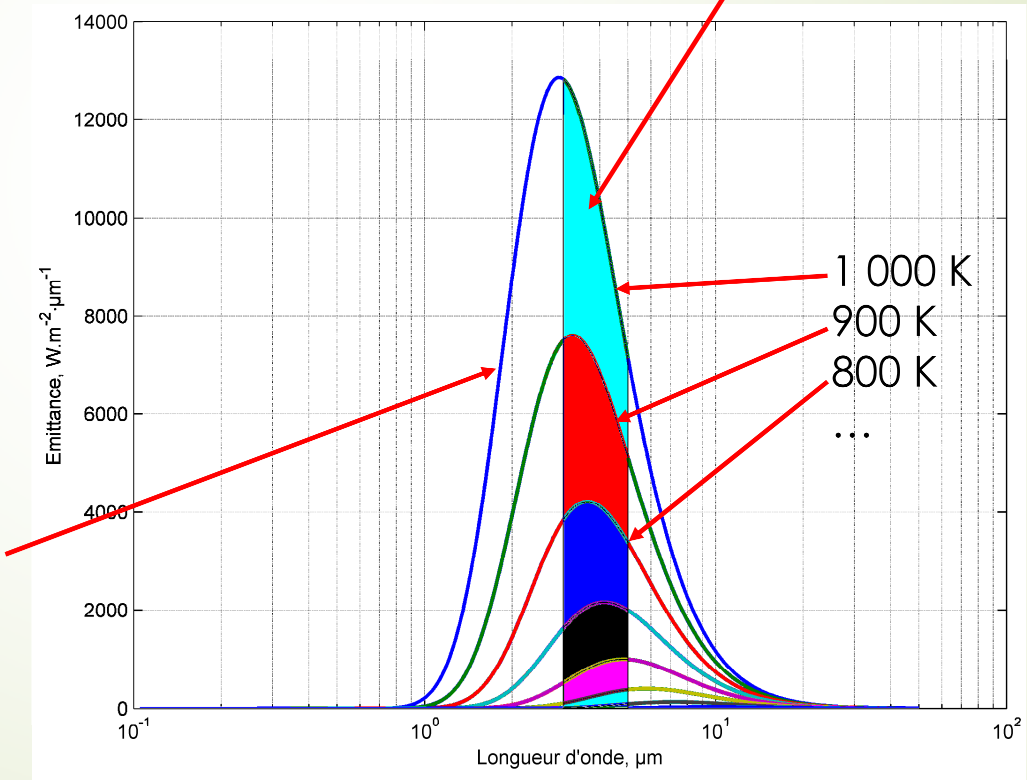
Sensitivity of a camera / radiometer:

- Detector: sensitivity and spectral range
- Temperature range: calibration curves

Example: perfect detector ($d_\lambda = 1$) ; [3 µm – 5 µm]

Emittance traces
between 300 K and 1000 K

Planck's law between 0 and ∞

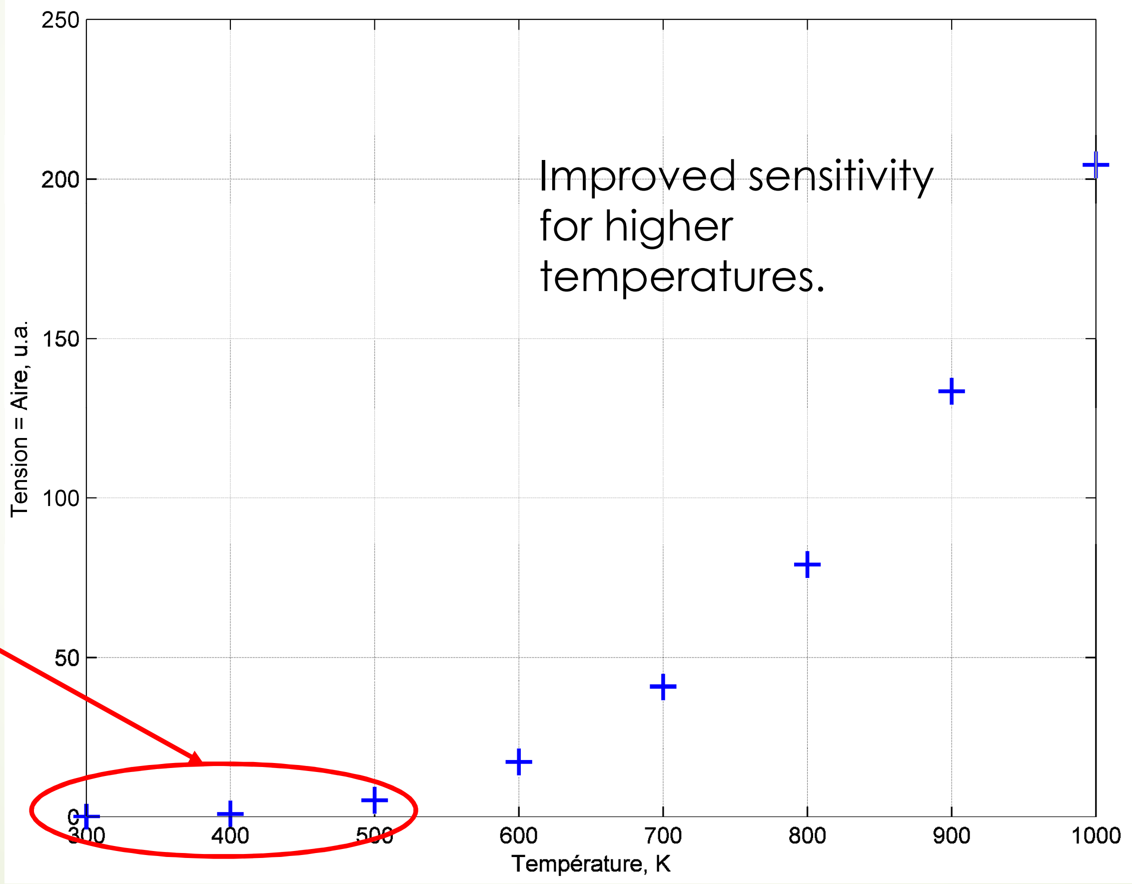


Spectral working area
of the detector

1 000 K
900 K
800 K
…

3. *Radiative Measurements* – Sensitivity of a IR camera

Evolution of "areas" as a function of temperature



Improved sensitivity for higher temperatures.

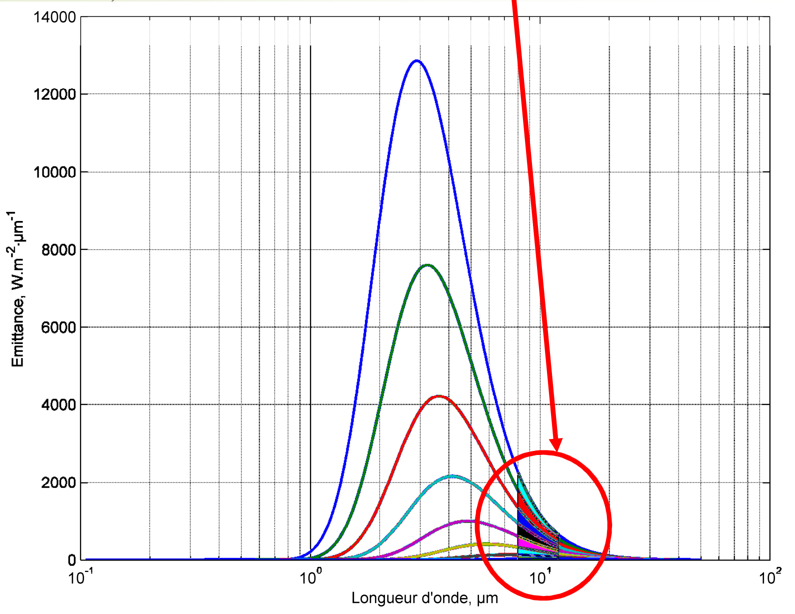
Variation of temperature not very visible at low temperatures.

3. Radiative Measurements – Sensitivity of a IR camera

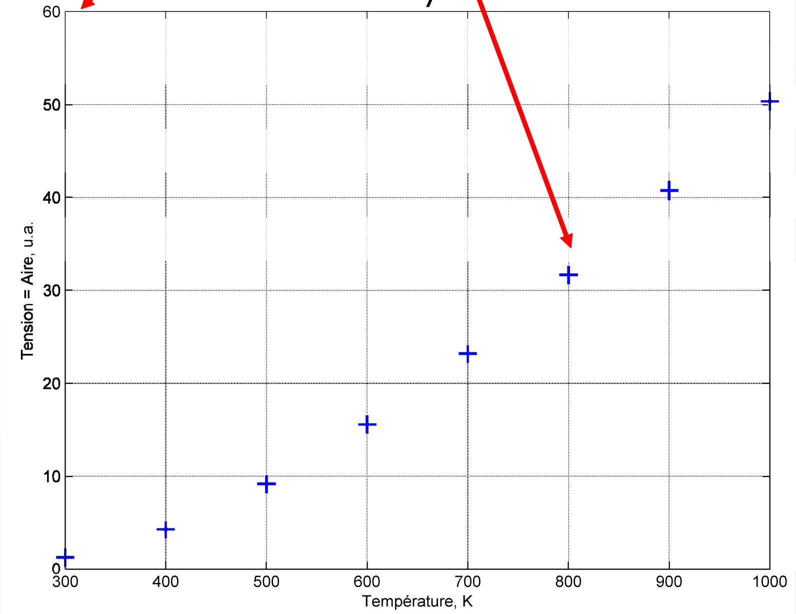
Example : perfect detector (d_λ = 1) ; [8 µm – 12 µm]

Emittance traces between 300 K and 1000 K

Work areas of the detector



Less signal than for detector [3 µm - 5 µm] but more interesting sensitivity

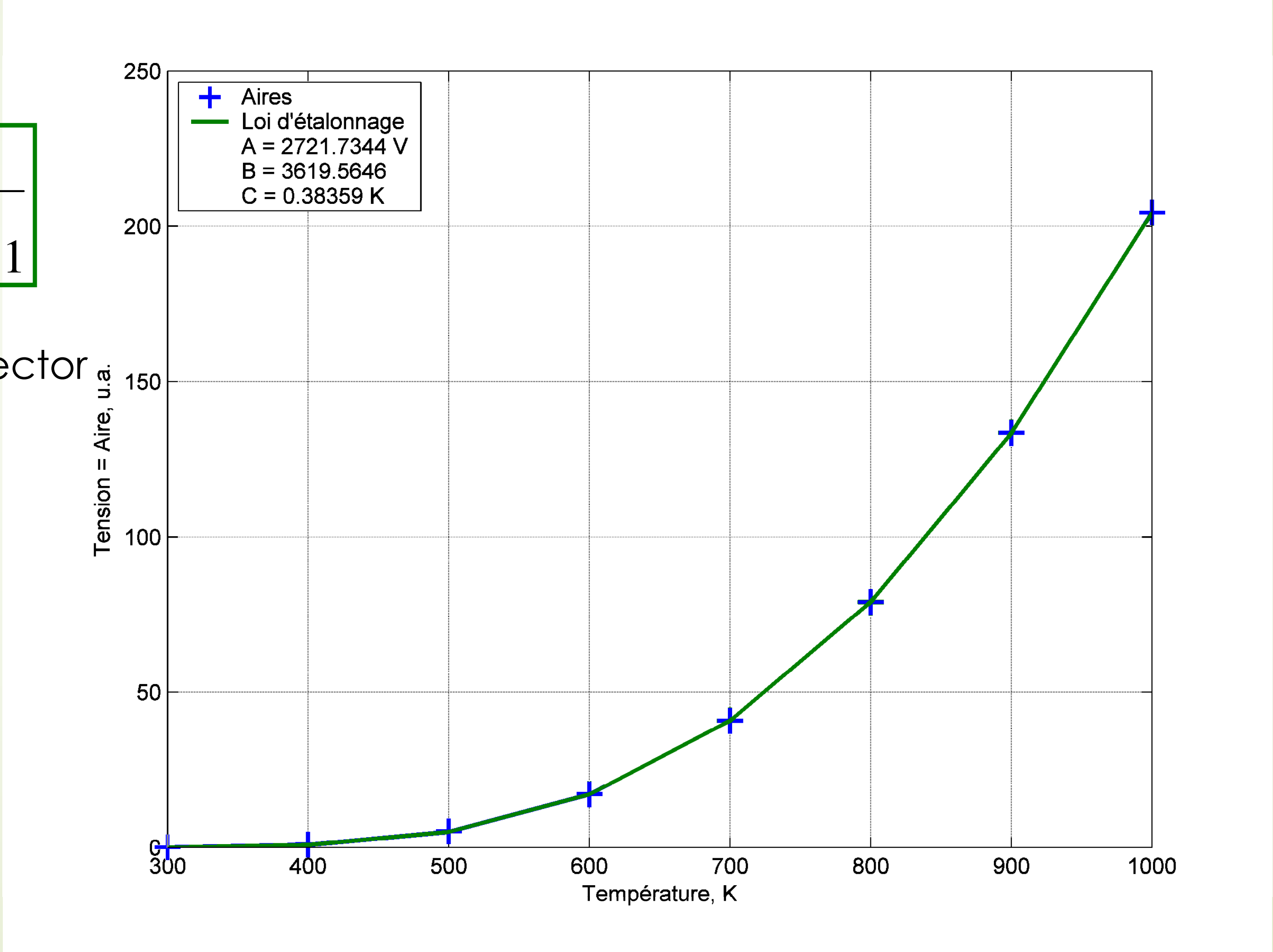


3. Radiative Measurements – Sensitivity of a IR camera

Calibration of the detector : to know the luminance from a measurement of voltage Vm.

$$\boxed{V_m = \frac{A}{Ce^{\frac{B}{T}} - 1}}$$

With the detector [3 µm – 5 µm]

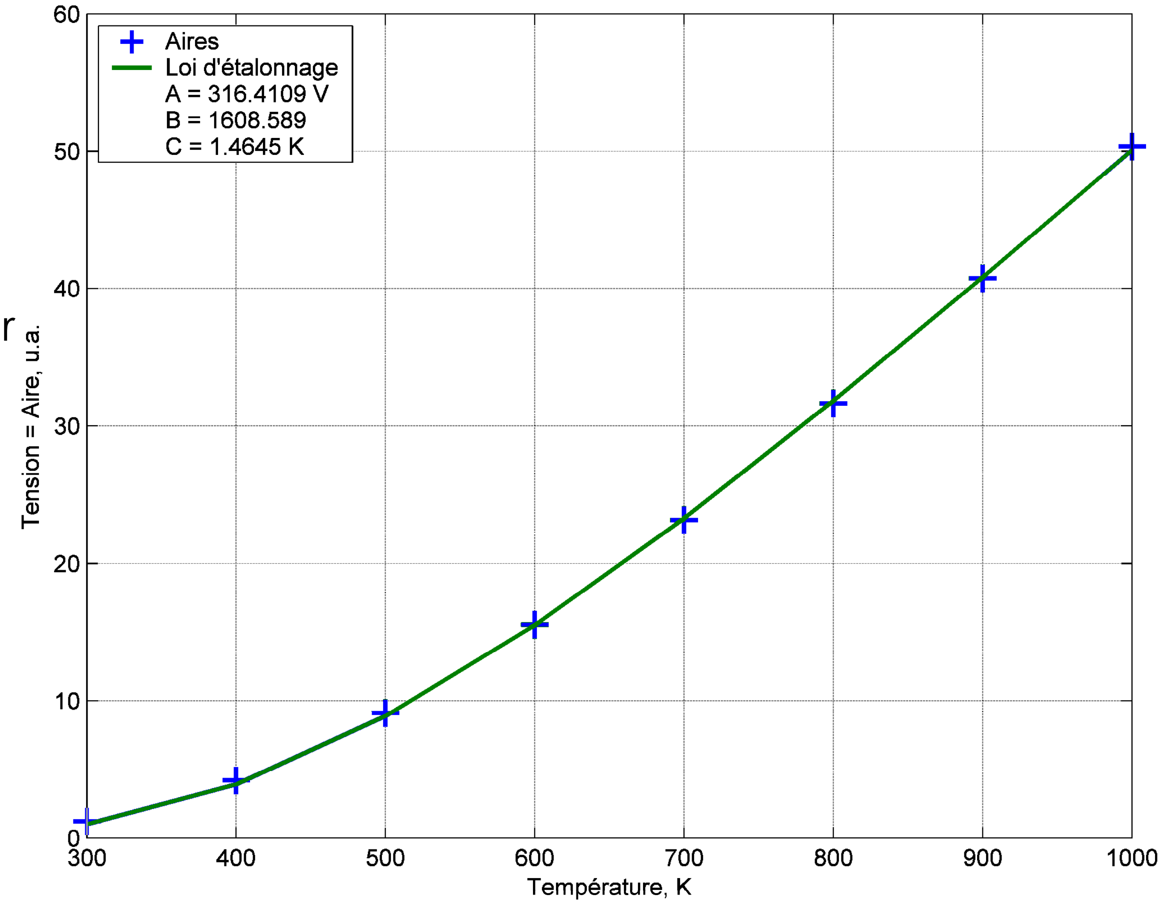


3. ***Radiative Measurements –*** Sensitivity of a IR camera

Calibration of the detector : to know the luminance from a measurement of voltage Vm.

$$\boxed{V_m = \frac{A}{Ce^{\frac{B}{T}} - 1}}$$

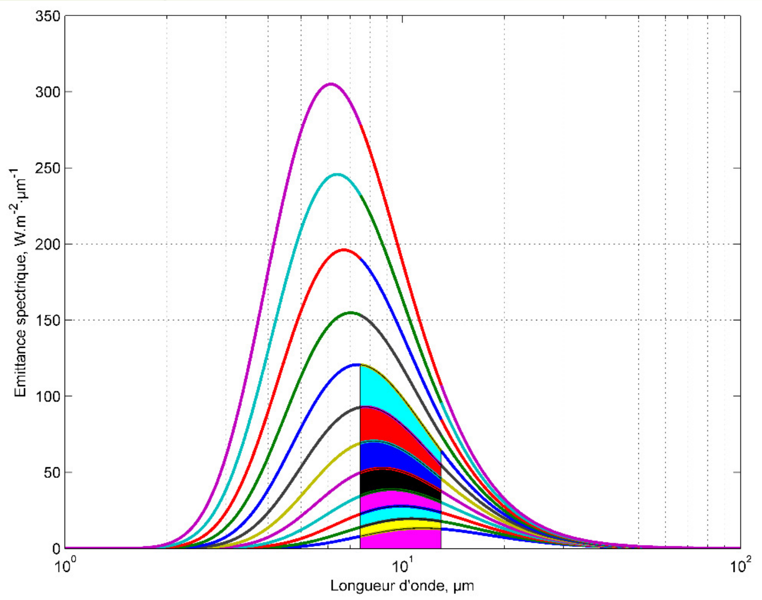
With the detector [8 µm – 12 µm]



3. Radiative Measurements – Sensitivity of a IR camera

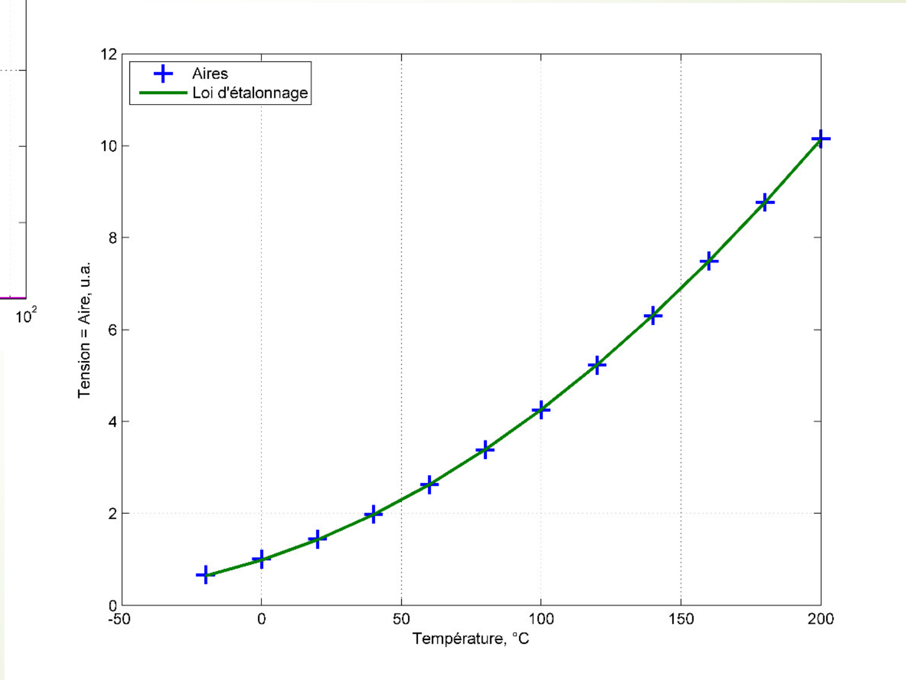
Example : perfect detector (d_λ = 1) ; [7,5 µm – 13 µm] ; no filter.

Emittance traces between -20 °C and 200 °C.

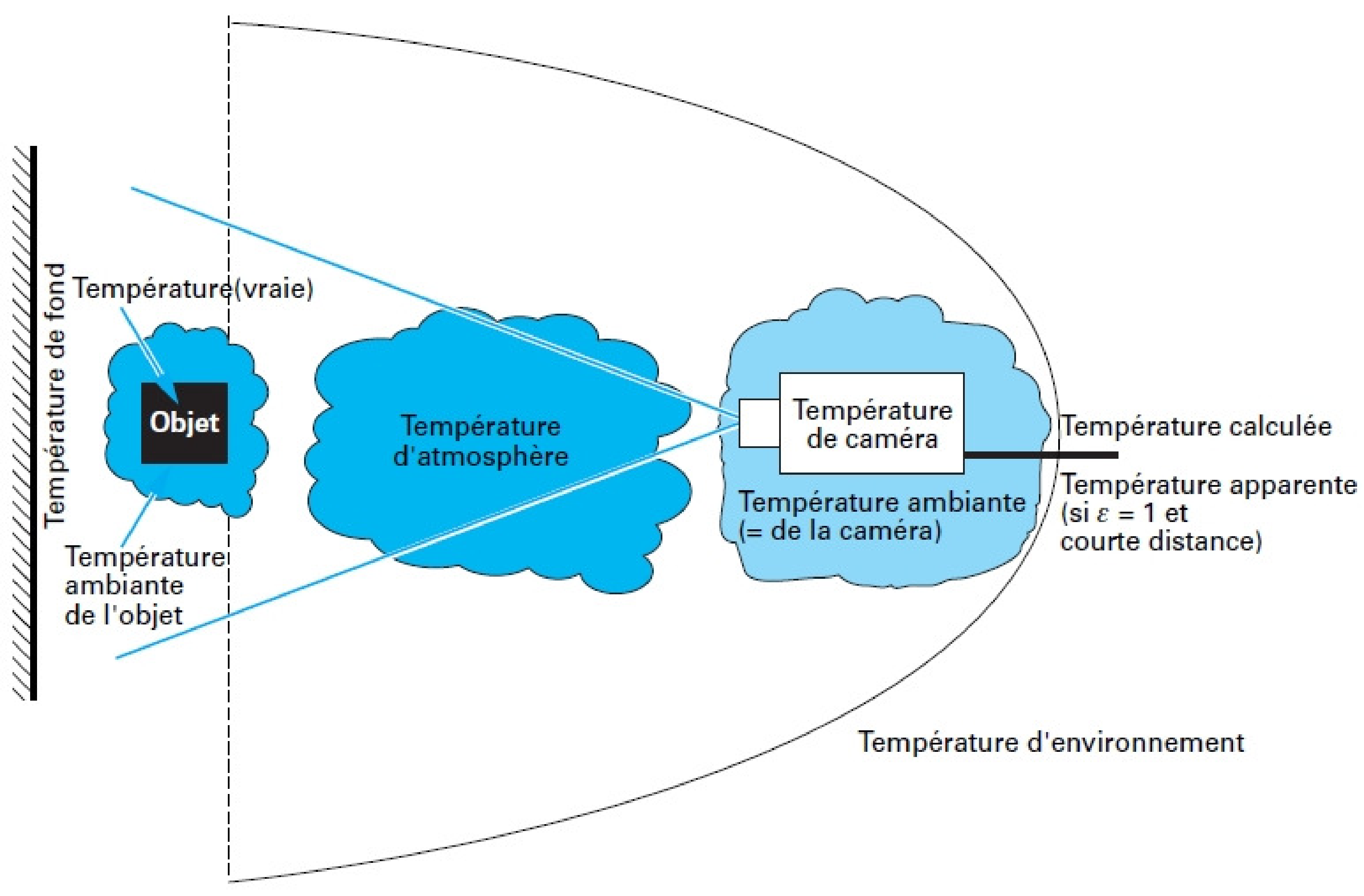


$$\boxed{V_m = \frac{A}{Ce^{\frac{B}{T}} - 1}}$$

A = 165
B = 1 470
C = 0,7769

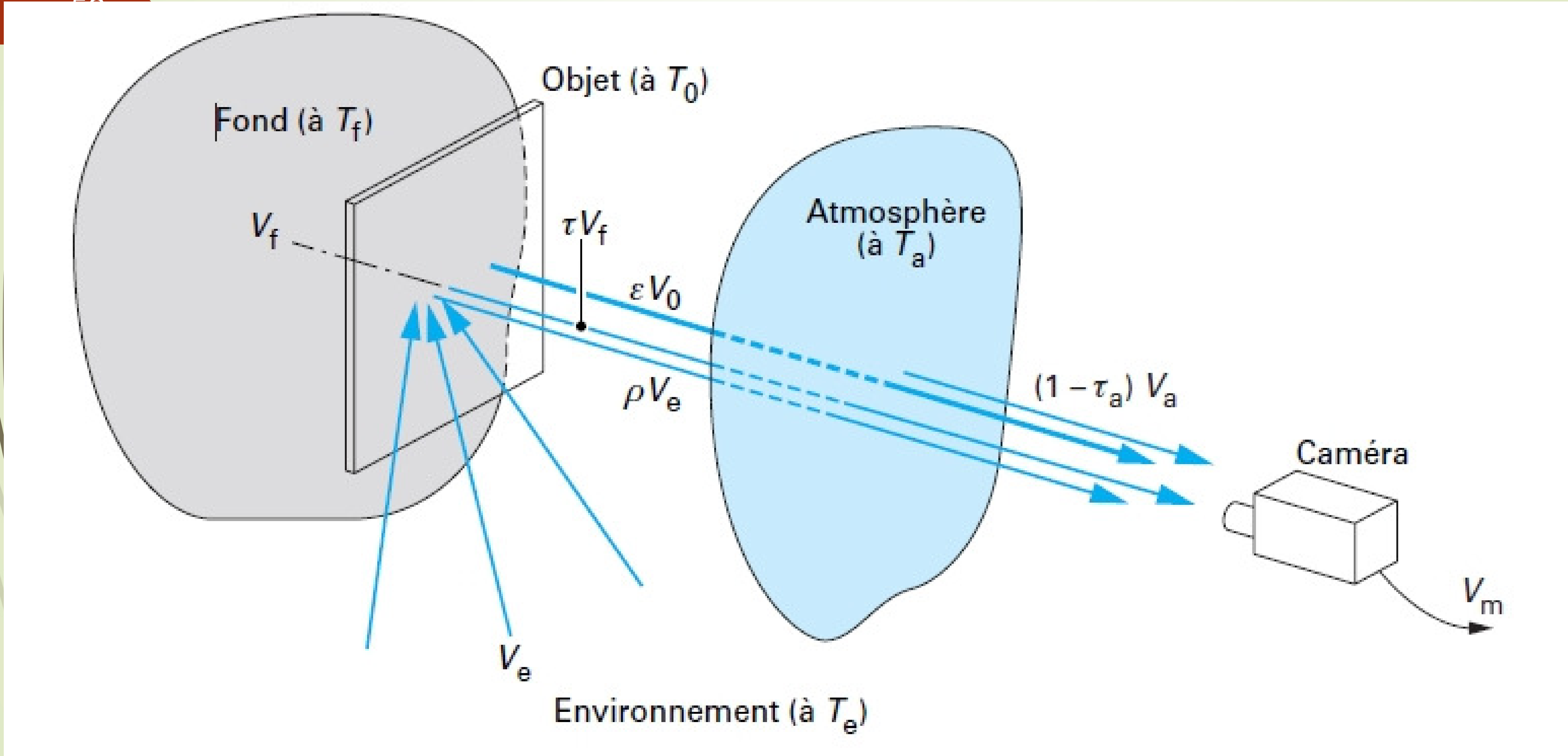


3. *Radiative Measurements* – Experimental configuration



D. Pajani, Thermographie, principes et mesure, Technique de l'Ingénieur, TI-r2740.

3. *Radiative Measurements* – Experimental configuration



D. Pajani, Thermographie, principes et mesure, Technique de l'Ingénieur, TI-r2740.

3. Radiative Measurements – Influence of the environment

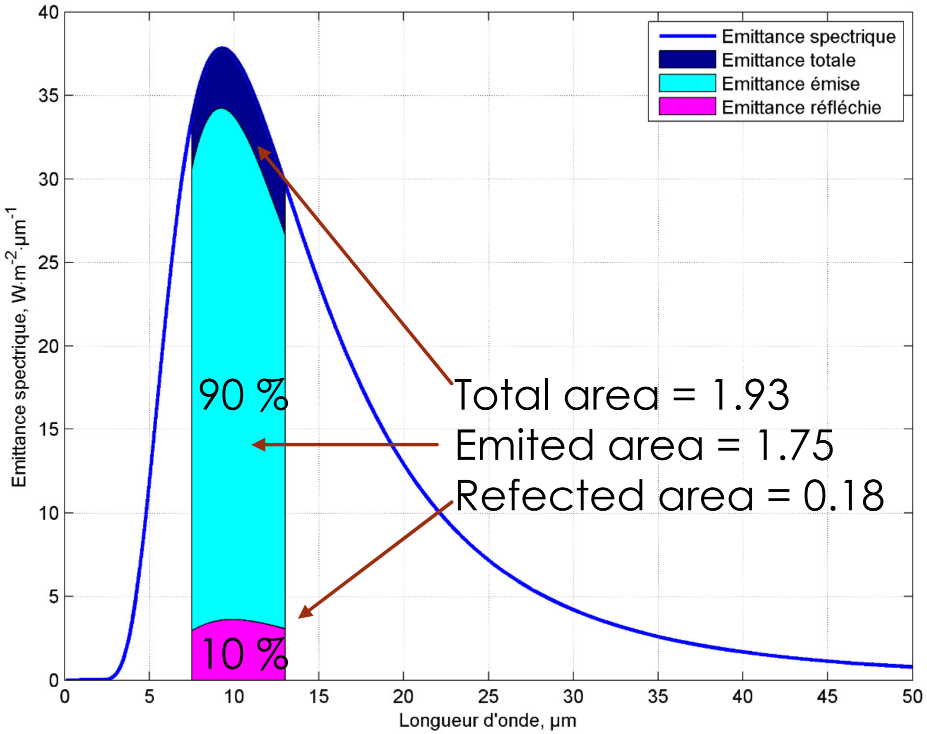
Aluminium Plate with black paint.

- Surface temperature θ = 41 °C
- *Environment* Temperature θ_a = 20 °C
- Total emissivity area ε = 0.87

What is the error if the environment is not taken into account?

$$T = \frac{B}{\ln\left[\left(\frac{A}{V_m}+1\right)\frac{1}{C}\right]}$$

T = 38.8 °C then 5.4 %.

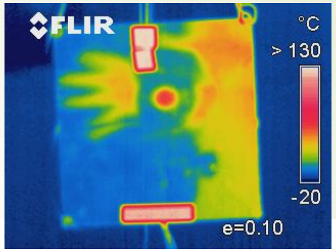


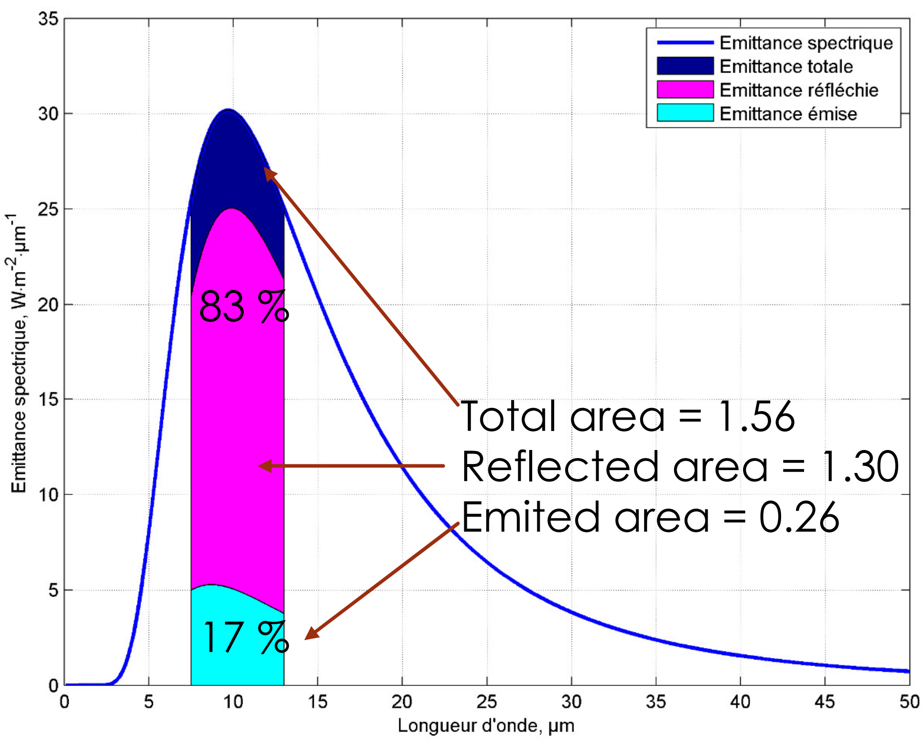
Total area = 1.93
Emited area = 1.75
Refected area = 0.18

3. Radiative Measurements – Influence of the environment

Aluminium plate without black paint.

- Surface temperature $\theta = 60$ °C
- *Environment* $\theta_a = 20$ °C
- Total emissivity area $\varepsilon = 0.10$





Total area = 1.56
Reflected area = 1.30
Emited area = 0.26

What is the error if the environment is not taken into account?

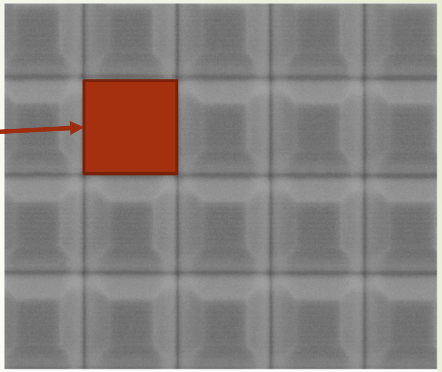
T = 25.4 °C then error 58 %

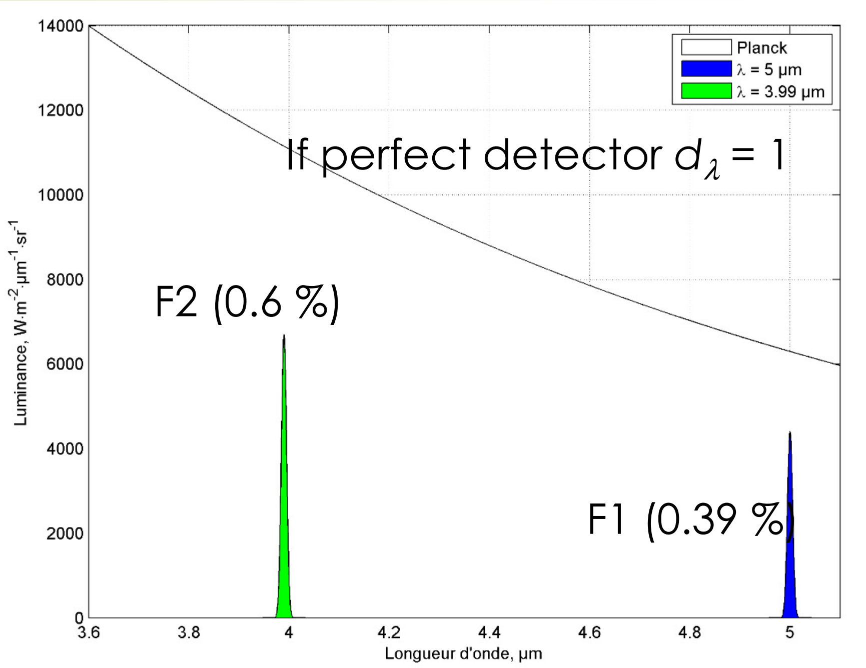
The environment has an important influence on the measurement, especially if the emissivity of the surface is low.

3. Radiative Measurements – Operation of a research type camera

Matrix camera - FLIR Jade III MW type [3.6 µm - 5.1 µm]
Detector: InSb
Matrix: 340 x 240 detectors
Edge of a detector: 30 µm
Three compartment filter wheel:

- empty
- F1 : [4.98 µm – 5.02 µm]
- F2 : [3.97 µm – 4.01 µm]



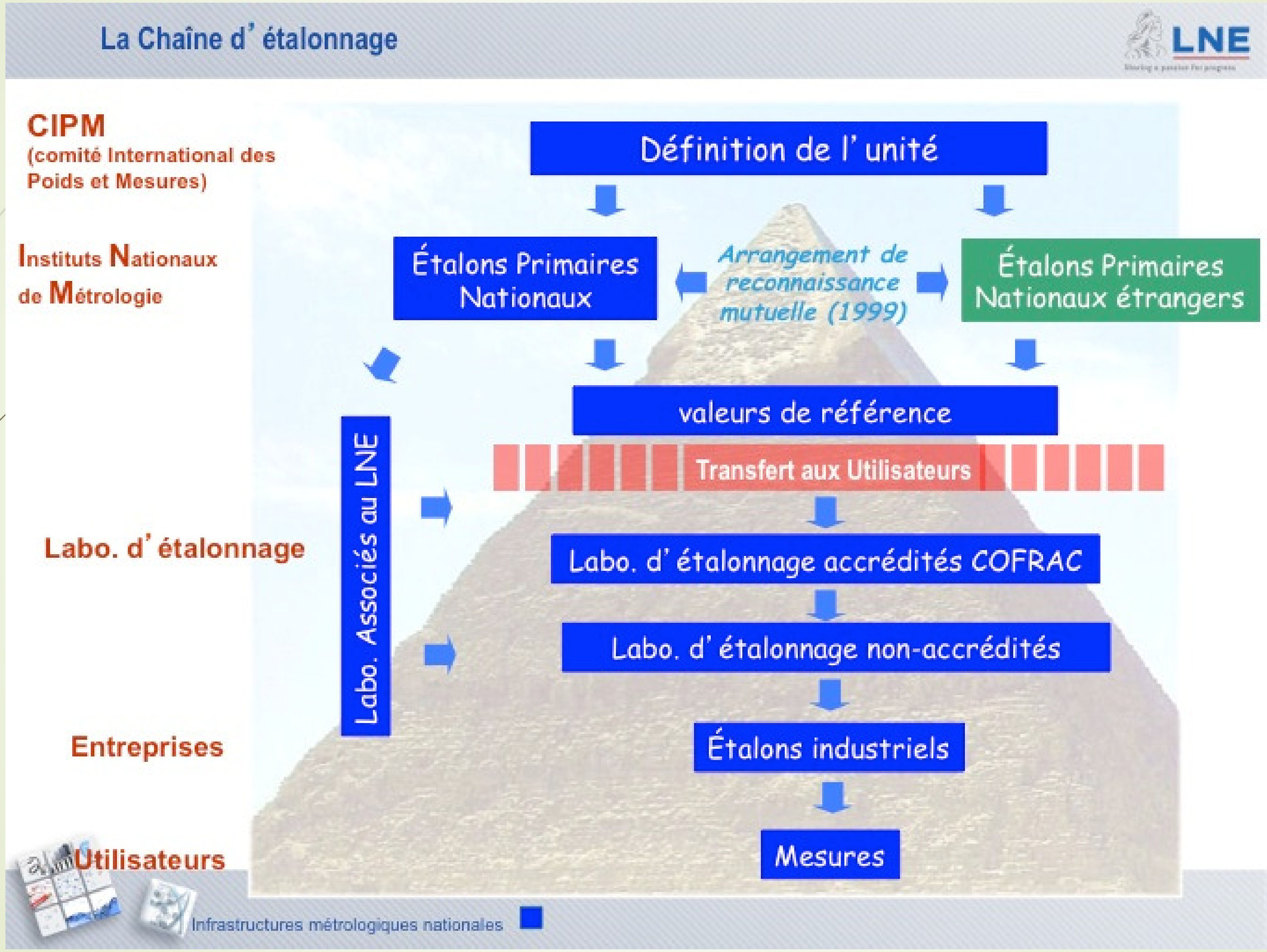


- 14 bits for the resolution (16 000 DL)
- Integration time from 3 µs to 10 ms

filter	**Opening time (µs)**	**Temperature range (°C)**
whitout	1 900	5 – 40
	540	20 – 90
	67	90 – 200
F1	1 600	100 – 200
	305	200 – 490
	60	490 – 1 000
F2	290	200 – 430
	57	430 - 800

- *1. Thermal Radiation*
- 2. Radiatives Properties
- 3. Radiative measurements
- **4. Calibration**
- 5. Other techniques
- 6. Applications
- 7. Inverse method
- 8. Prospect…

4. Calibration



La Chaîne d'étalonnage

CIPM
(comité International des Poids et Mesures)

Instituts Nationaux de Métrologie

Labo. d'étalonnage

Entreprises

Utilisateurs

Définition de l'unité

Étalons Primaires Nationaux

Arrangement de reconnaissance mutuelle (1999)

Étalons Primaires Nationaux étrangers

valeurs de référence

Transfert aux Utilisateurs

Labo. Associés au LNE

Labo. d'étalonnage accrédités COFRAC

Labo. d'étalonnage non-accrédités

Étalons industriels

Mesures

Infrastructures métrologiques nationales ■

source : http://culturesciencesphysique.ens-lyon.fr/ressource/metrologie-SI

4. Calibration

Fixed points are defined corresponding to equilibrium states of pure bodies. Limited for the moment at the freezing point of copper at 1084.62 ° C!
And beyond?

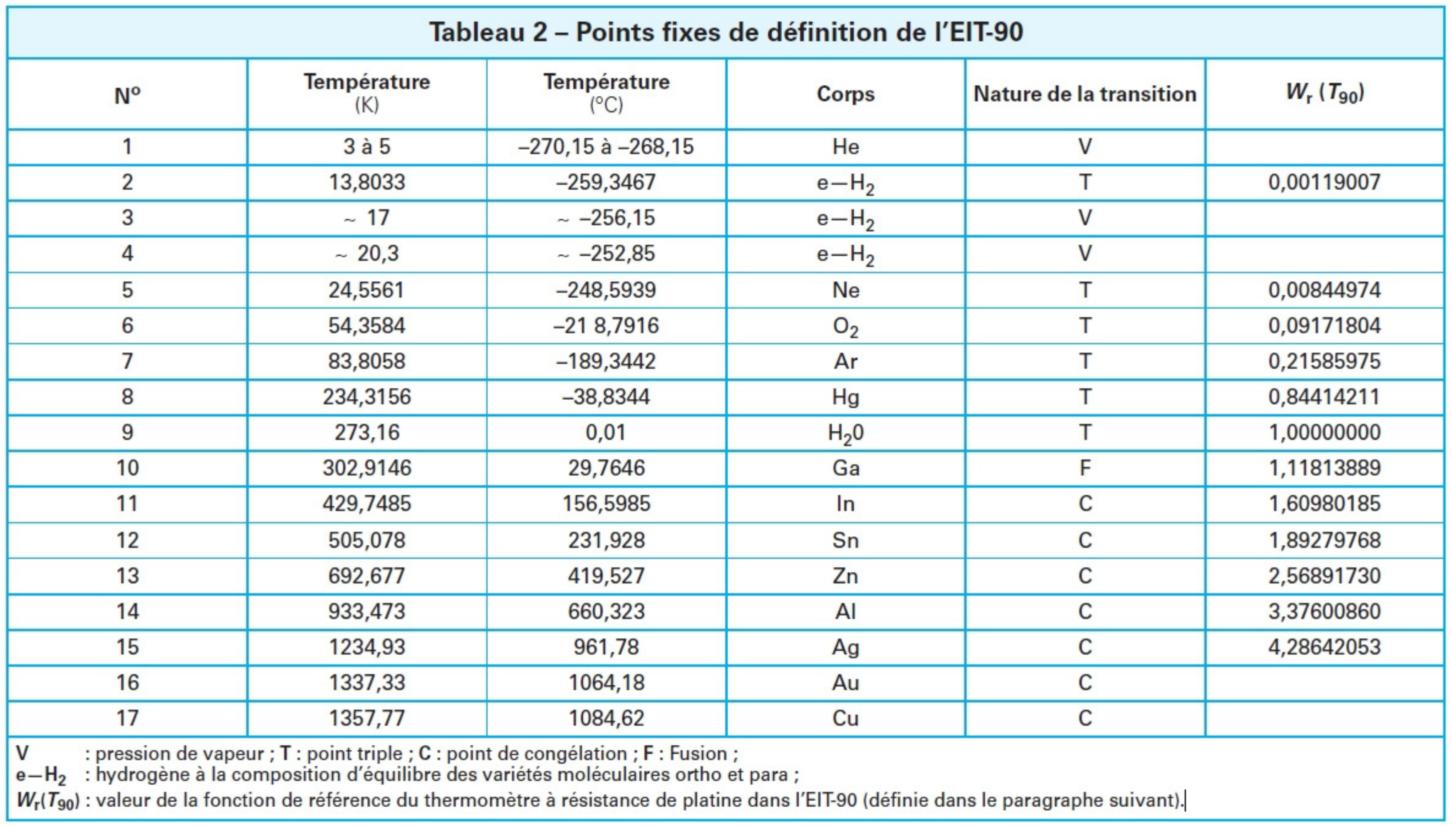


Tableau 2 – Points fixes de définition de l'EIT-90

N°	Température (K)	Température (°C)	Corps	Nature de la transition	$W_r\ (T_{90})$
1	3 à 5	–270,15 à –268,15	He	V	
2	13,8033	–259,3467	$e-H_2$	T	0,00119007
3	~ 17	~ –256,15	$e-H_2$	V	
4	~ 20,3	~ –252,85	$e-H_2$	V	
5	24,5561	–248,5939	Ne	T	0,00844974
6	54,3584	–21 8,7916	O_2	T	0,09171804
7	83,8058	–189,3442	Ar	T	0,21585975
8	234,3156	–38,8344	Hg	T	0,84414211
9	273,16	0,01	H_20	T	1,00000000
10	302,9146	29,7646	Ga	F	1,11813889
11	429,7485	156,5985	In	C	1,60980185
12	505,078	231,928	Sn	C	1,89279768
13	692,677	419,527	Zn	C	2,56891730
14	933,473	660,323	Al	C	3,37600860
15	1234,93	961,78	Ag	C	4,28642053
16	1337,33	1064,18	Au	C	
17	1357,77	1084,62	Cu	C	

V : pression de vapeur ; **T** : point triple ; **C** : point de congélation ; **F** : Fusion ;
$e-H_2$: hydrogène à la composition d'équilibre des variétés moléculaires ortho et para ;
$W_r(T_{90})$: valeur de la fonction de référence du thermomètre à résistance de platine dans l'EIT-90 (définie dans le paragraphe suivant).

Mohamed Sadli, L'échelle internationale de température : EIT-90, Techniques de l'ingénieur, r2510

4. Calibration

Research of new fixed points for about 20 years by the national calibration and metrology laboratories:

Germany

England

France

Italy

Russia

United state

Brazil

Japan

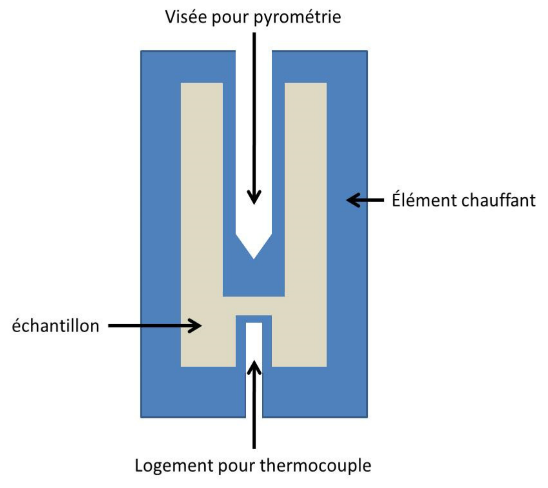
South Korea

Thaïlande

Fixed points: eutectic type metal-carbon. A eutectic is a mixture of two pure bodies that melts and solidifies at a constant temperature:

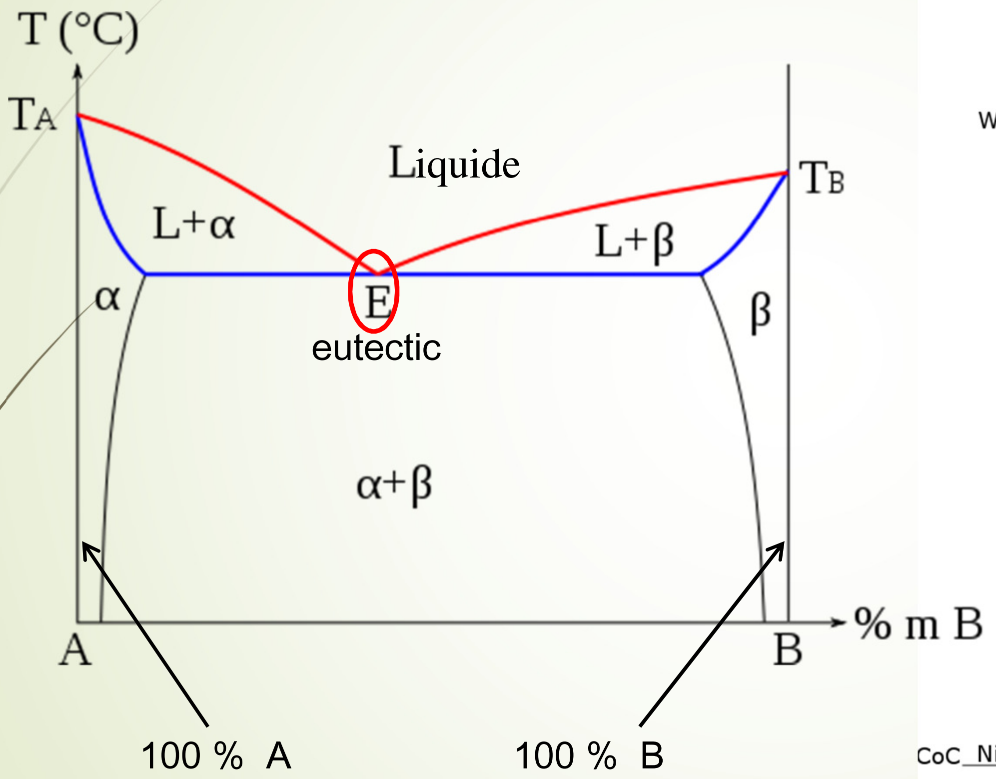
CoC / **NiC** / **FeC** / **ReC** / **TiC** / **ZnC** / **PdC** / **IrC** / **PtC** / **RuC** / **WC…**

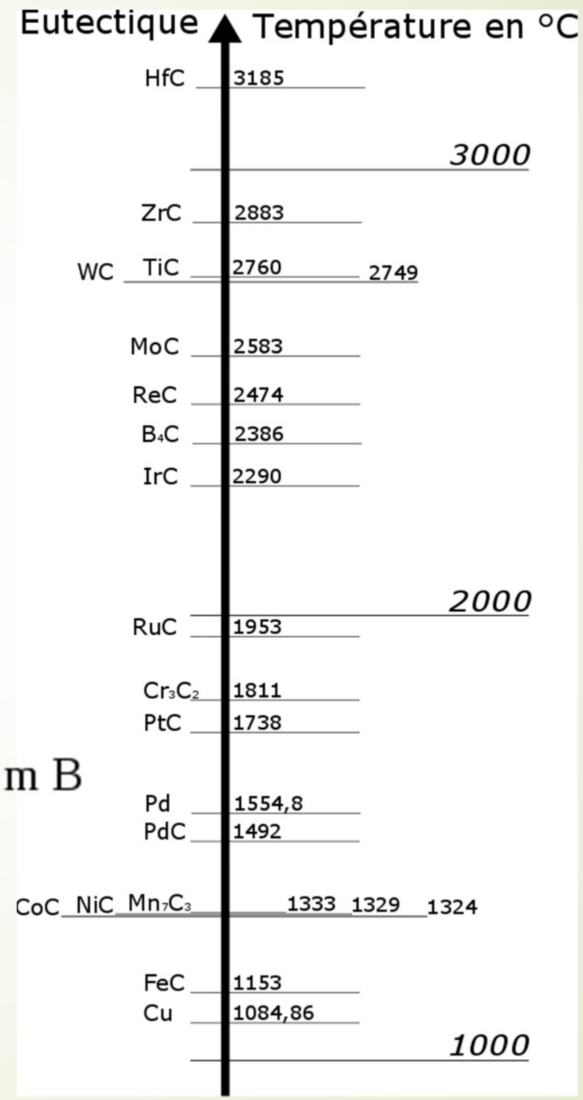
Measurements made with Pt / Pd type thermocouple and radiation detectors (pyrometers).
Stabilization and regulation of several hours.
Precision of some millikelvins.



4. Calibration

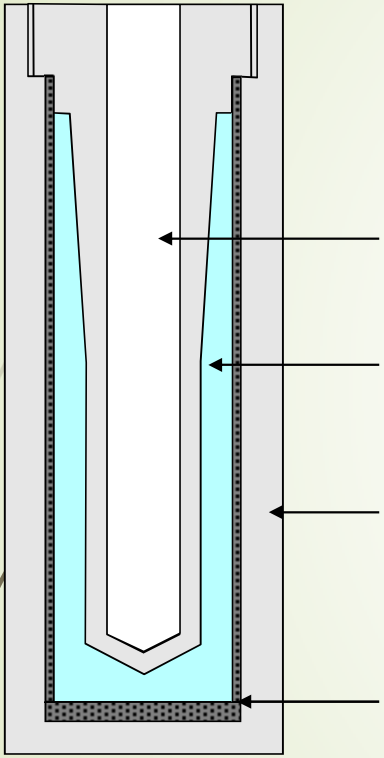
Phase Diagram (binary)





4. Calibration

Fixed points HT ; study from LNE-Cnam

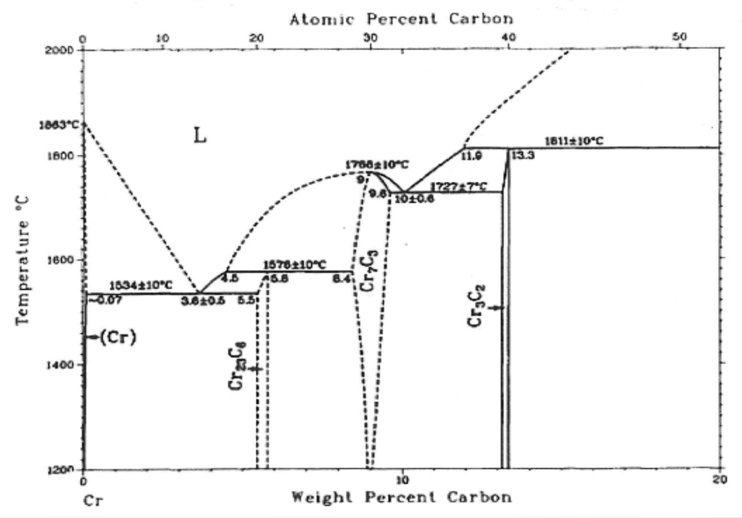


glove finger

Eutectic alloy

Crucible graphite

Graphite sheet



Ru-C
1953 ℃

Pt-C
1738 ℃

Pd-C
1492 ℃

Co-C
1324 ℃

Cu
1084 ℃

1. Thermal Radiation

2. Radiatives Properties

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8. Prospect…

6. *Others techniques* – The multispectral method

The cameras "broad spectral bands" only allow to obtain the temperature or the emissivity. How to get both?
Possible solution: the multispectral method.
Principle: work on several extremely narrow spectral ranges, even monochromatic.

- First theoretical approach-

Planck's law

$$L_\lambda(T) = \varepsilon'_\lambda \frac{2hc^2\lambda^{-5}}{e^{\frac{hc}{k\lambda T}}-1} \approx 2\varepsilon'_\lambda hc^2\lambda^{-5}e^{-\frac{hc}{k\lambda T}}$$

Wien approximation
$\lambda T << 14\,000\mu m \cdot K$

1-/ Two measurements at wavelengths λ_1 and λ_2 :

$$L_{\lambda_1}(T) = 2\varepsilon'_{\lambda_1} hc^2\lambda_1^{-5}e^{-\frac{hc}{k\lambda_1 T}}$$

$$L_{\lambda_2}(T) = 2\varepsilon'_{\lambda_2} hc^2\lambda_2^{-5}e^{-\frac{hc}{k\lambda_2 T}}$$

2-/ Signal ratio :

$$T = \frac{hc\left(\lambda_2^{-1}-\lambda_1^{-1}\right)}{k\ln\left[\frac{L_{\lambda_1}}{L_{\lambda_2}}\frac{\varepsilon'_{\lambda_1}}{\varepsilon'_{\lambda_2}}\left(\frac{\lambda_2}{\lambda_1}\right)^{-5}\right]}$$

3-/ if λ_1 and λ_2 are close, then:

$$\varepsilon'_{\lambda_1} \approx \varepsilon'_{\lambda_2}$$

and T and ε'_λ are easily calculated.

6. *Others techniques* – The multispectral method

- First theoretical approach-

- Constant temperature during the measure. ← steady regime
- Constant emissivity between λ_1 and λ_2. ← strong hypothesis or a priori knowledge of the radiative properties of the material.
- Theoretical model without noise.
- Perfect detector.
- Perfectly monochromatic measurements.

- How to choose wavelengths?
- What is the minimum interval? maximum? between two wavelengths?
- And if the emissivity varies?
- And if the temperature varies?
- And if the criterion λT >> 14 000 µm·K ?
- What is the main source of error?
- How many wavelengths to use?
- Influence of measurement noise?

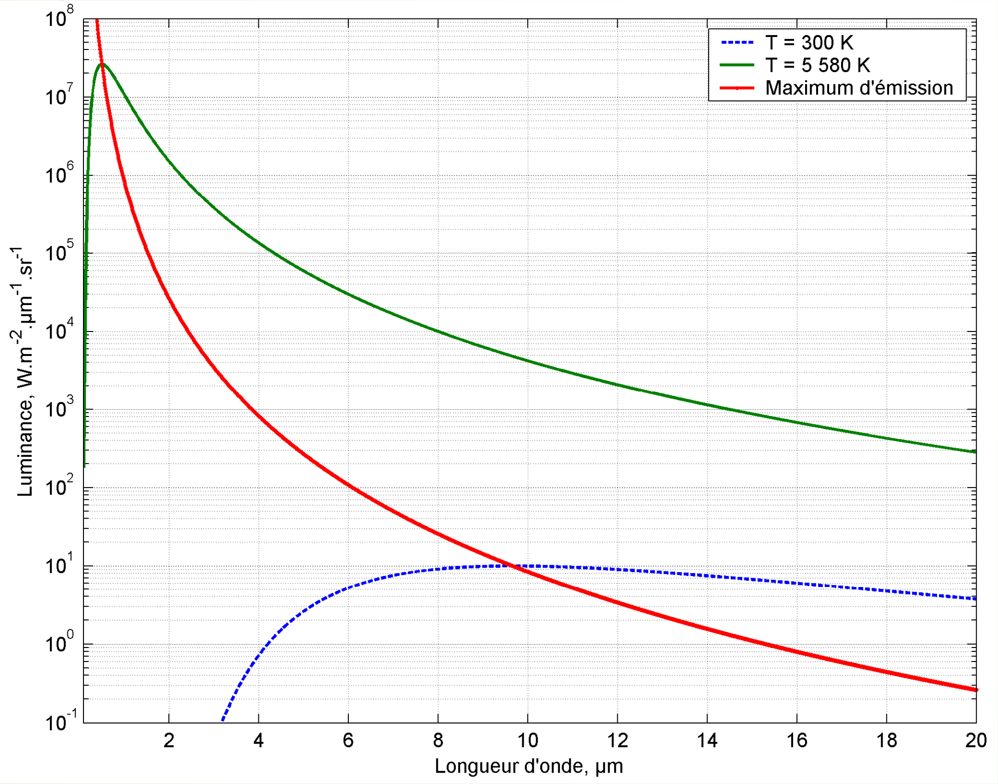
...

before the second experimental approach.

6. *Others techniques* – The multispectral method

- How to choose wavelengths?

In monochromatic, in the case of a black body: $L_\lambda^0(T) = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}$



Sensitivity : $\chi_\lambda = \frac{\partial L_\lambda(T)}{\partial \lambda}$

$$\chi_\lambda = L_\lambda(T)\left(\frac{5}{\lambda} - \frac{C_2}{\lambda^2 T}\right)$$

Maximum for : $\boxed{\lambda_M T = C_2 / 5}$

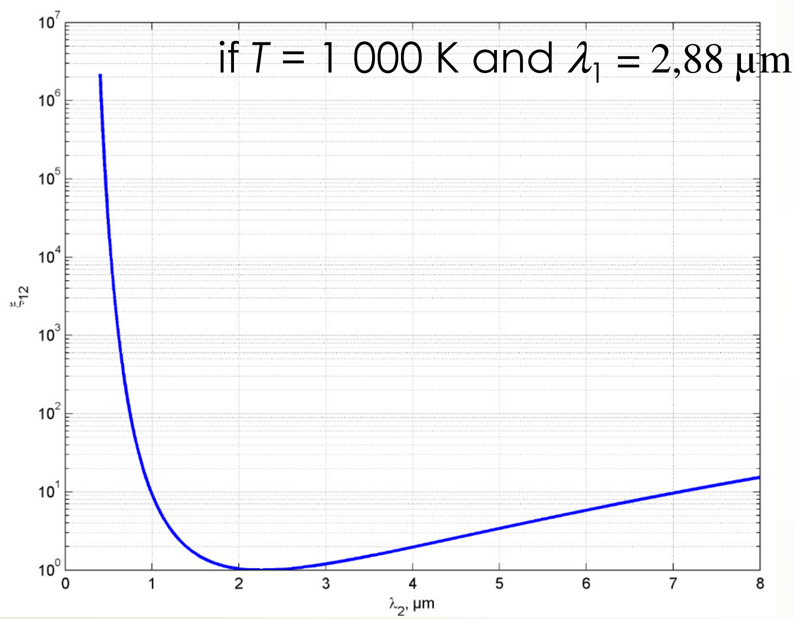
if T = 1 000 K, then $\lambda_M \approx 2{,}88\ \mu\text{m}$.

T is unknown, but it's a starting point!

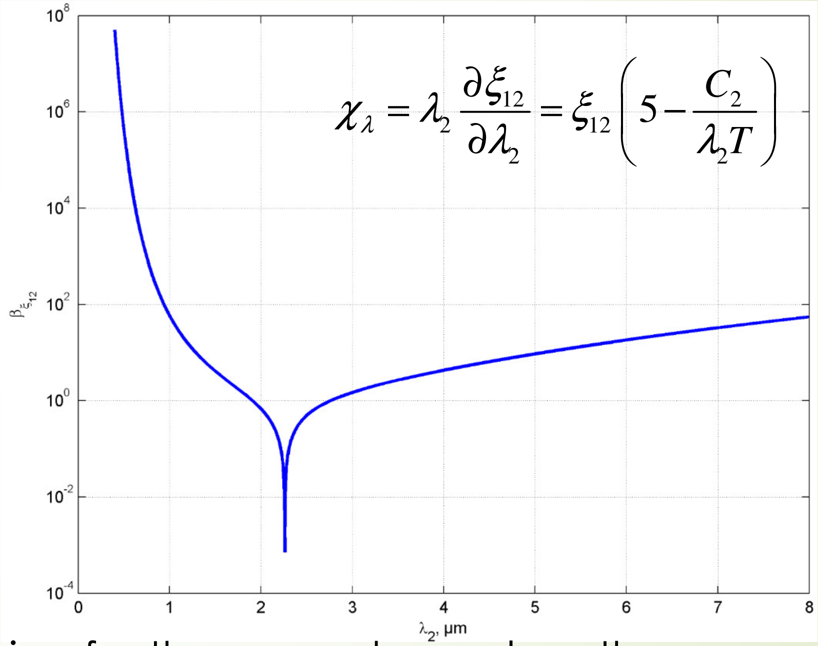
6. *Others techniques* – The multispectral method

How to choose wavelengths in bispectral ?

In the case of a gray body : $\xi_{12} = \dfrac{L_1(T)}{L_2(T)} = \dfrac{\varepsilon_1^{'}}{\varepsilon_2^{'}}\left(\dfrac{\lambda_1}{\lambda_2}\right)^{-5} e^{-\frac{C_2}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)}$ and $\varepsilon_1^{'} \approx \varepsilon_2^{'}$



if T = 1 000 K and λ_1 = 2,88 µm.



$$\chi_\lambda = \lambda_2 \frac{\partial \xi_{12}}{\partial \lambda_2} = \xi_{12}\left(5 - \frac{C_2}{\lambda_2 T}\right)$$

Obviously, no obvious choice for the second wavelength.

Establishment of criteria:
- minimum ratio ξ_{12min}.
- Minimum difference $\Delta\lambda_{min}$ between two wavelengths.

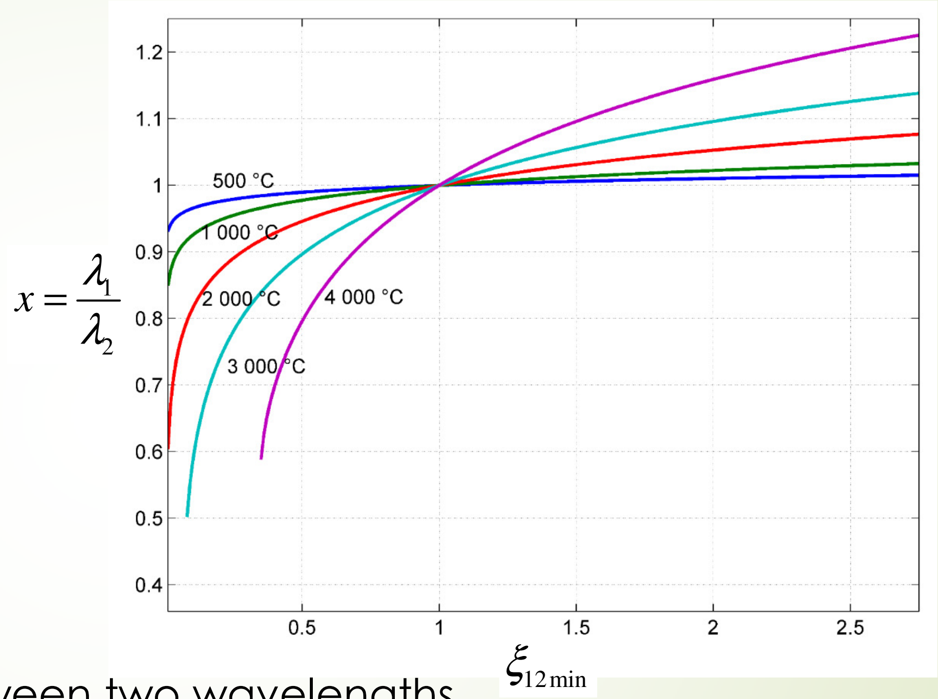
6. *Others techniques* – The multispectral method

How to choose wavelengths in bispectral ?

- minimum ratio ξ_{12min}.

$$\xi_{12\min} = \left(\frac{\lambda_1}{\lambda_2}\right)^{-5} e^{-\frac{C_2}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)}$$

$$\boxed{\xi_{12\min} = x^{-5} e^{-\frac{C_2}{\lambda_1 T}(1-x)}}$$



- Minimum difference $\Delta\lambda_{min}$ between two wavelengths

Relative temperature error

$$\frac{e_T}{T} = \frac{\frac{e_{L_1}}{L_1} + \frac{e_{L_2}}{L_2} + \frac{e_{\lambda_1}}{\lambda_1}\left|\frac{C_2}{\lambda_1 T} - 5\right| + \frac{e_{\lambda_2}}{\lambda_2}\left|\frac{C_2}{\lambda_2 T} - 5\right|}{\boxed{\frac{C_2}{T}\left|\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right|}}$$

To not amplify the error :

$$\boxed{\Delta\lambda_{\min} = \lambda_2 - \lambda_1 > \frac{T\lambda_2^2}{C_2}}$$

6. *Others techniques* – The multispectral method

How to choose wavelengths in polychromatic ?

minimisation of the fluxes : $J=\sum_{i=1}^{n}\left|\varphi_i^{th}(\beta)-\varphi^{\exp}\right|^2$ with $\beta=(T,a_0,a_1...,a_n)$

flux approximated by the approximation of Wien $\varphi_i^{th}(\beta)=\varepsilon(\lambda)C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T}}$ with $\varepsilon(\lambda)=\sum_{i=1}^{n}a_i\lambda^i$

temperature : 2 000 °C

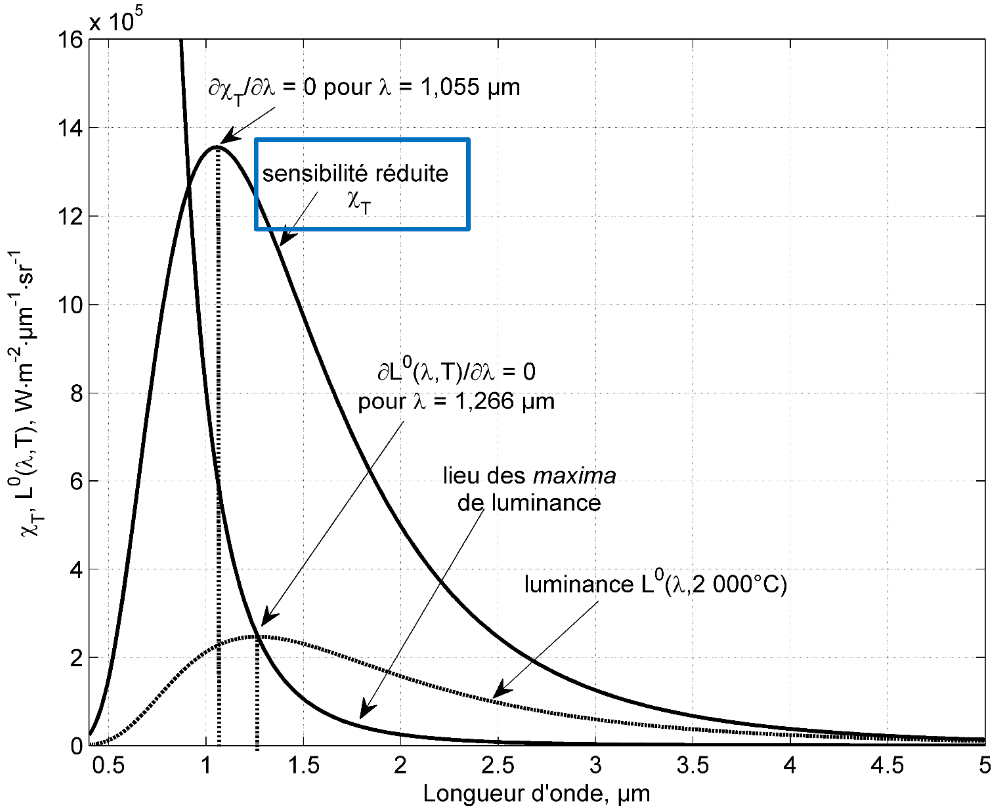
fonctionnelle $J=\sum_{i=1}^{1}\left|\varphi_i^{th}(\beta)-\varphi^{\exp}\right|^2$

with $\beta=T$ (case of the black body)

result $\mathrm{cov}(\beta)=\left|\sigma_T^2\right|=\left(X^TX\right)^{-1}=\frac{\partial\varphi_1}{\partial T}$

reduced sensitivity $\boxed{\chi_T=T\frac{\partial\varphi}{\partial T}=\varphi\frac{C_2}{\lambda T}}$

χ_T goes through a maximum : $\lambda_{1opt}=1,055\,\mu m$



6. *Others techniques* – The multispectral method

minimisation of the fluxes: $J = \sum_{i=1}^{n} \left|\varphi_i^{th}(\beta) - \varphi^{\exp}\right|^2$ with $\beta = \left(T, a_0, a_1 ..., a_n\right)$

flux approximated by the approximation of Wien : $\varphi_i^{th} = \varepsilon(\lambda) C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}$ with $\varepsilon(\lambda) = \sum_{i=1}^{n} a_i \lambda^i$

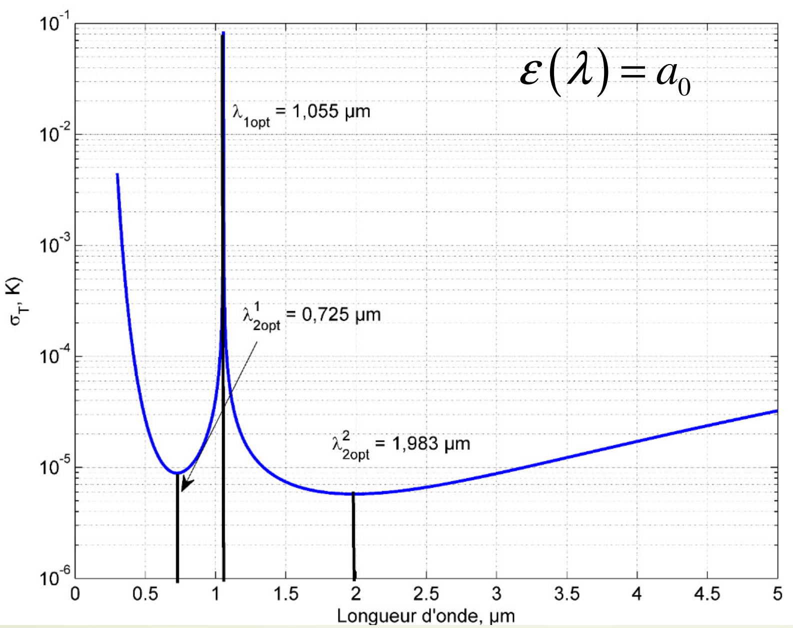
covariance matrix

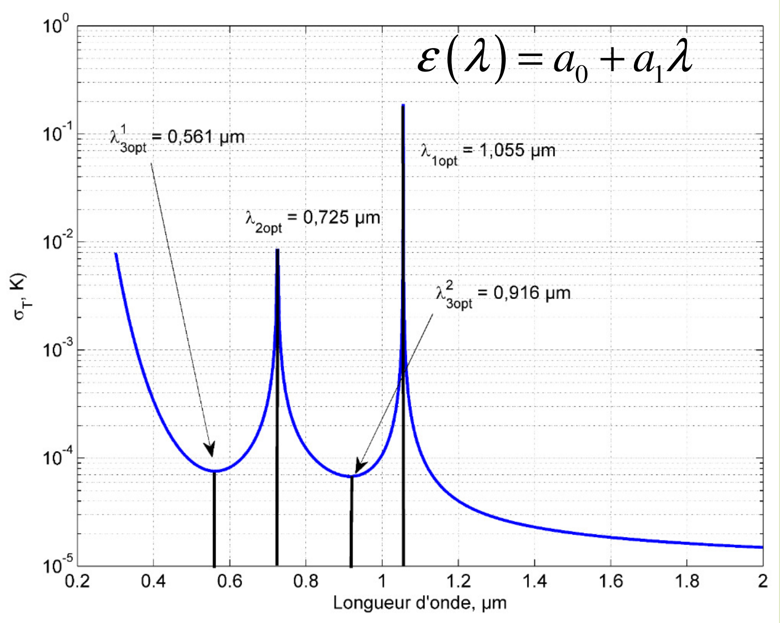
minimizing the standard deviation on temperature σ_T

$$\mathrm{cov}(\beta) = \begin{vmatrix} \sigma_T^2 & \mathrm{cov}(T, a_0) & \cdots & \mathrm{cov}(a_n, T) \\ \mathrm{cov}(T, a_0) & \sigma_{a_0}^2 & \cdots & \mathrm{cov}(a_0, a_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(T, a_n) & \mathrm{cov}(a_n, a_0) & \cdots & \sigma_{a_n}^2 \end{vmatrix} = \left(X^T X\right)^{-1} \sigma_{bruit}^2$$

jacobian

$$X = \begin{vmatrix} \frac{\partial \varphi_1}{\partial T} & \frac{\partial \varphi_1}{\partial a_0} & \cdots & \frac{\partial \varphi_1}{\partial a_n} \\ \frac{\partial \varphi_2}{\partial T} & \frac{\partial \varphi_2}{\partial a_0} & \cdots & \frac{\partial \varphi_2}{\partial a_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \varphi_n}{\partial T} & \frac{\partial \varphi_n}{\partial a_0} & \cdots & \frac{\partial \varphi_n}{\partial a_n} \end{vmatrix}$$





6. *Others techniques* – The multispectral method

- How to choose wavelengths ?
  - Objective and arbitrary criteria
  - What is the minimum interval? maximum? between two wavelengths?
  - Arbitrary criteria

- And if the criterion λT >> 14 000 µm·K ?
  - Using Planck's Law and Mean Square minimisation
  - What is the main source of error?
  - Emissivity, measurement noise

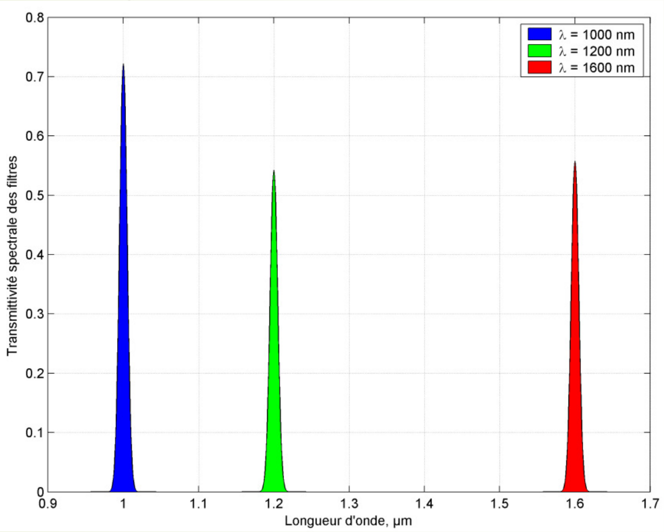
- How many wavelengths to use?
  - One wavelength more than the number of parameters to estimate

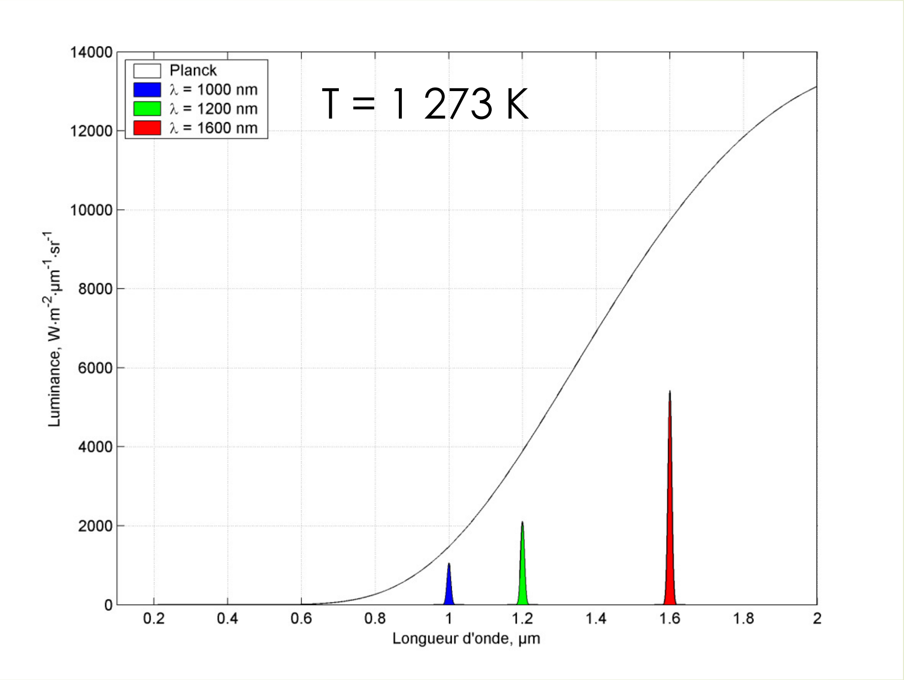
- Influence of measurement noise ?
- Difficulty estimating for low signal-to-noise ratios

And if the temperature varies? And if the emissivity varies? How to take into account the temperature range, the working range of the detector.

6. *Others techniques* – The multispectral method

Technically, it is necessary to use monochromatic filters, which classically have the appearance of Gaussian.





maximum transmittivity

average wavelength

$$\boxed{\tau_\lambda = \tau_{\max} e^{-\frac{1}{2}\left(\frac{\lambda-\bar{\lambda}}{\sigma_\lambda}\right)^2}}$$

Standard deviation

1. *Thermal Radiation*
2. Radiatives Properties
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6. **Applications**
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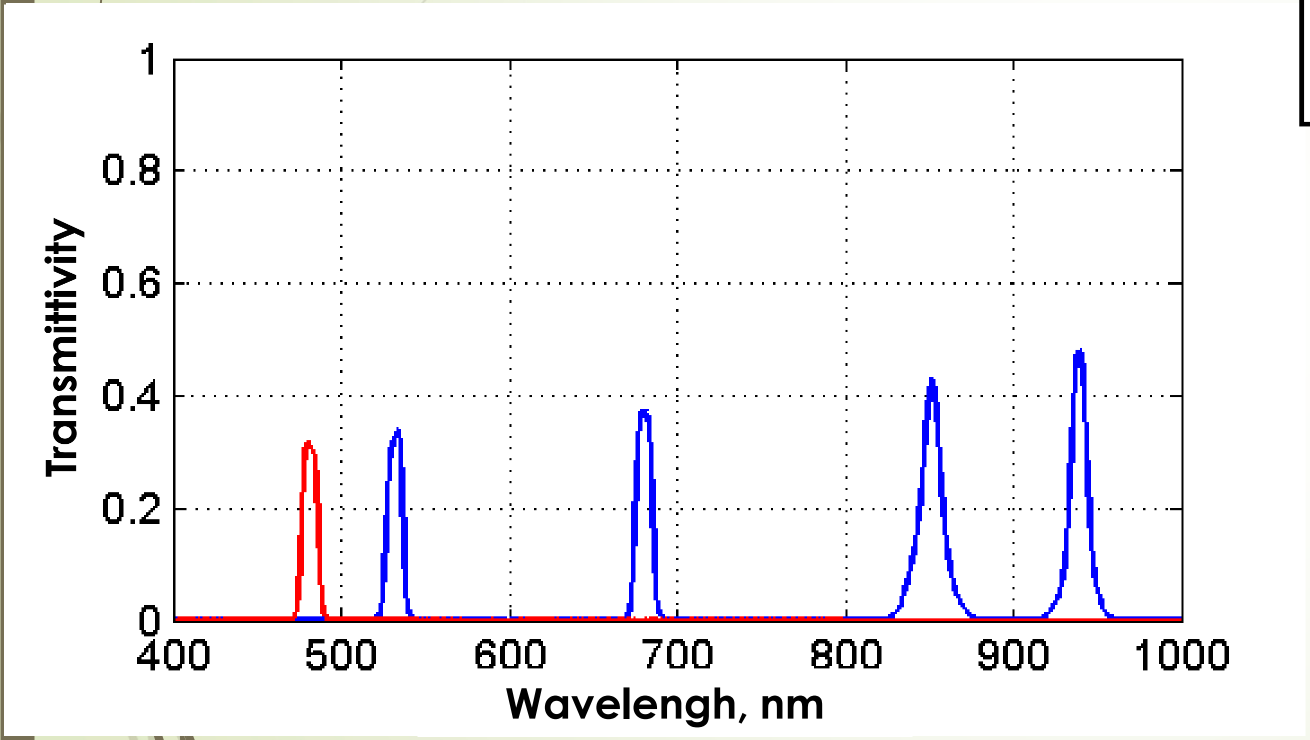
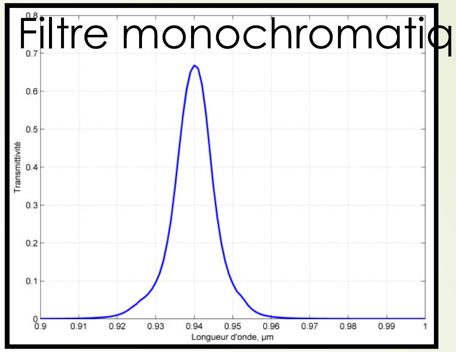
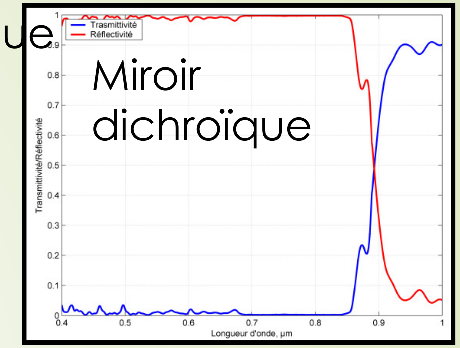
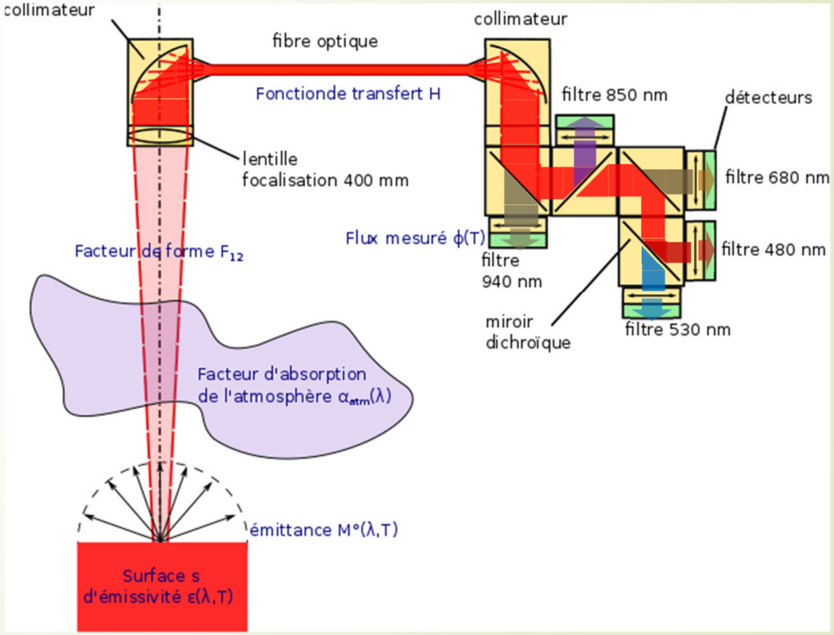
8. Prospect…

6. *Others techniques* – The multispectral method









6. *Others techniques* – The multispectral method

Experimental device for simultaneously measuring temperature and emissivity: multispectral method.

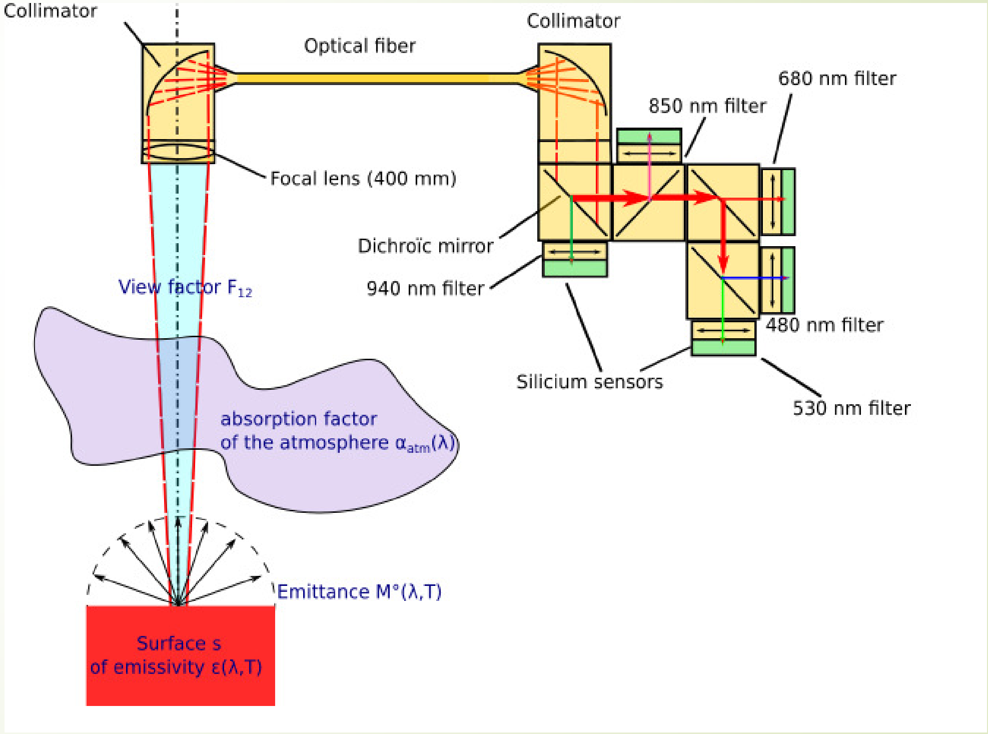
Application during a welding operation:

fast kinetics,

high temperatures (≈ 2500 ° C), microscopic scale.

θ(°C)	λ_1 = 480 nm	$\lvert\lambda_1-\lambda_2\rvert$ = 50 nm	$\lvert\lambda_1-\lambda_3\rvert$ = 200 nm	$\lvert\lambda_1-\lambda_4\rvert$ = 370 nm	$\lvert\lambda_1-\lambda_5\rvert$ = 560 nm
	$\Delta\lambda_{mini}$ (nm)	Φ_2/Φ_1	Φ_3/Φ_1	Φ_4/Φ_1	Φ_5/Φ_1
1 000	21	5,6	178	1 622	3 502
2 000	37	2,1	8,5	17	22
3 000	53	1,45	2,6	3,1	3,1

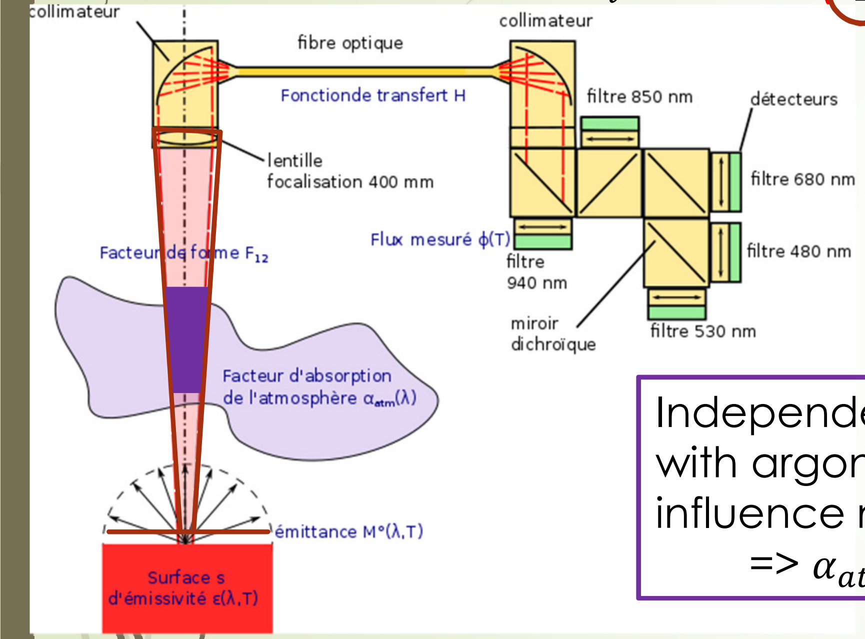
θ(°C)	λ_2 = 530 nm	$\lvert\lambda_2-\lambda_1\rvert$ = 50 nm	$\lvert\lambda_2-\lambda_3\rvert$ = 150 nm	$\lvert\lambda_2-\lambda_4\rvert$ = 320 nm	$\lvert\lambda_2-\lambda_5\rvert$ = 410 nm
	$\Delta\lambda_{mini}$ (nm)	Φ_2/Φ_1	Φ_3/Φ_2	Φ_4/Φ_2	Φ_5/Φ_2
1 000	25	5,6	31	289	623
2 000	45	2,1	4,0	8,5	10
3 000	64	1,45	1,8	2,2	2,1



6. *Others techniques* – The multispectral method

$$\Phi_i^{th}(\lambda,T,\varepsilon) = F_{12}\, s \int_0^{\infty} \varepsilon(\lambda,T)\,\alpha_{atm}(\lambda,T)\,M^0(\lambda,T)\,H_{M_i}\, e^{-\frac{1}{2}\left(\frac{\bar{\lambda}_i-\lambda}{\sigma_i}\right)^2} d\lambda$$

unknowns



- F_{12}: view factor
- s: aimed surface
- $\varepsilon(\lambda,T)$: emissivity
- $\alpha_{atm}(\lambda,T)$: atmosphere
- $M^0(\lambda,T)$: emittance
- H_{mi}: amplitude of Gaussian transfer function

Independent tests with argon, helium : influence negligible
=> $\alpha_{atm}(\lambda,T)$ = 1

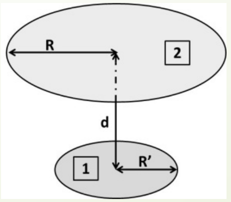
$$F_{12} = \frac{1}{2}\left[X - \sqrt{X^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right]$$

$$X = 1 + \frac{1+R_2^2}{R_1^2}$$

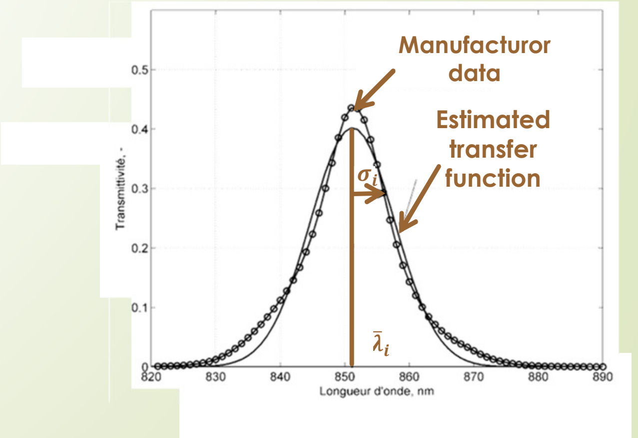
$$R_2 = R/d \qquad R_1 = R'/d$$

$$s = \pi R'^2$$

Lens

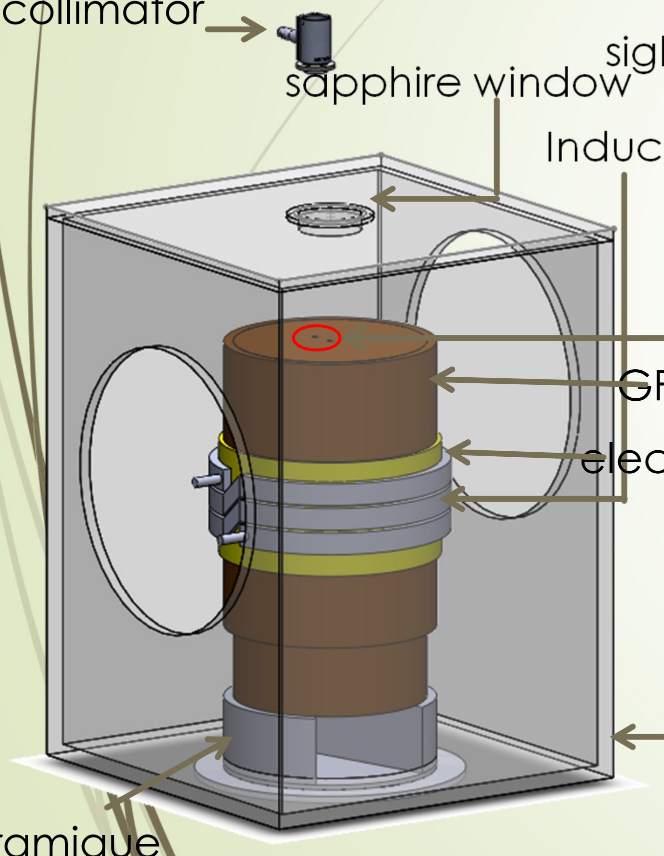


Sample



6. Others techniques – The multispectral method

Pyrometer calibration with a high temperature apparatus.



collimator

sapphire window

sighting holes

Inductor

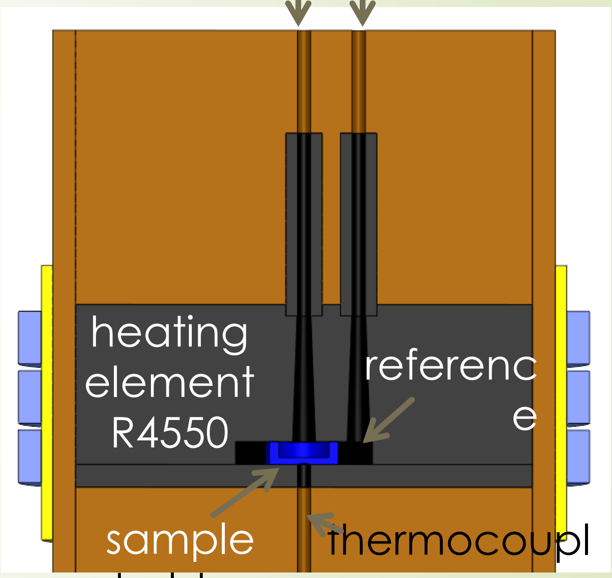
GFA insulating

electric insulating

aluminum chamber

ceramique

sighting holes: reference, sample



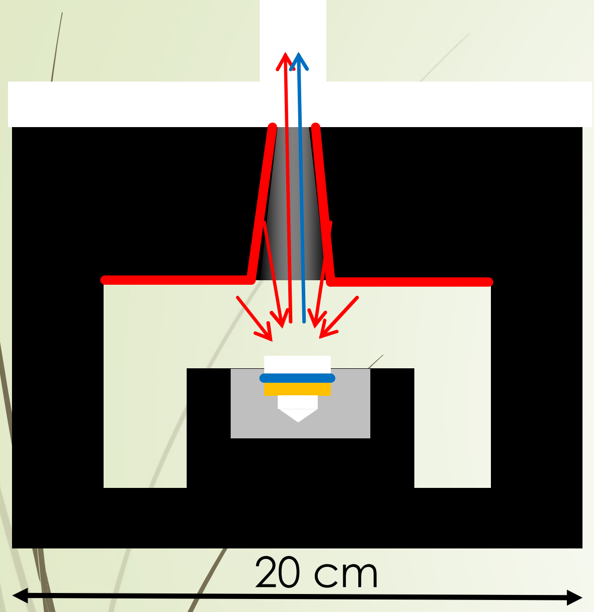
heating element R4550

reference

sample holder

thermocouple

6. *Others techniques* – The multispectral method



Pyrometer calibration with a high temperature apparatus.

$$\Phi_i^{th}(T) = sF_{12}\int_0^\infty \Big[\underbrace{\varepsilon'_{\lambda,s}M^0(\lambda,T_s)}_{\text{sample emission}} + \underbrace{\rho'_{\lambda,s}\varepsilon'_{\lambda,env}M^0(\lambda,T_{env})\Big]H_{M_i}}_{\text{environment (carbon) emission reflected by the sample}} e^{-\frac{1}{2}\left(\frac{\overline{\lambda_i}-\lambda}{\sigma_i}\right)^2} d\lambda$$

$$T_{env} \approx T_s = T_{eq}$$

$$=> \underbrace{(\varepsilon'_{\lambda,s} + \rho'_{\lambda,s}\varepsilon'_{\lambda,env})}_{\boldsymbol{\varepsilon_{éq}}} M^0(\lambda,T)$$

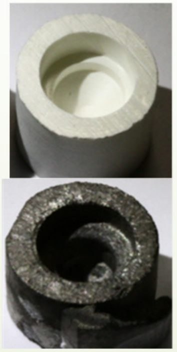
$$\boxed{\boldsymbol{\varepsilon_{eq} = 0.96 \pm 0.03.}}$$

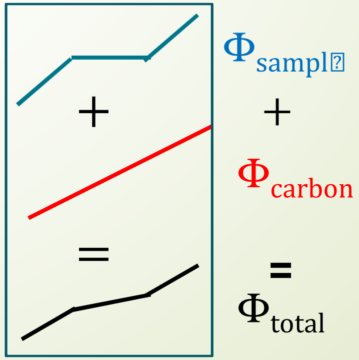
Laboratory measurement

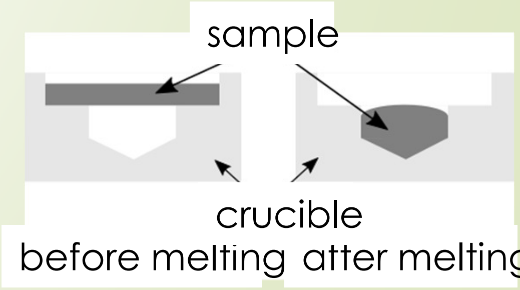
$$\boldsymbol{\varepsilon'_{\lambda,env} < \varepsilon_{eq} < 1} \quad \leftrightarrow \quad 0{,}93 < \boldsymbol{\varepsilon_{éq}} < \mathbf{1}$$

Pure substances:

- <u>nickel ($\theta_f = 1\,455\ °C$)</u>
- iron ($\theta_f = 1\,538\ °C$) → diffusion problem
- chromium ($\theta_f = 1\,907\ °C$) → diffusion problem
- <u>niobium ($\theta_f = 2\,477\ °C$)</u>





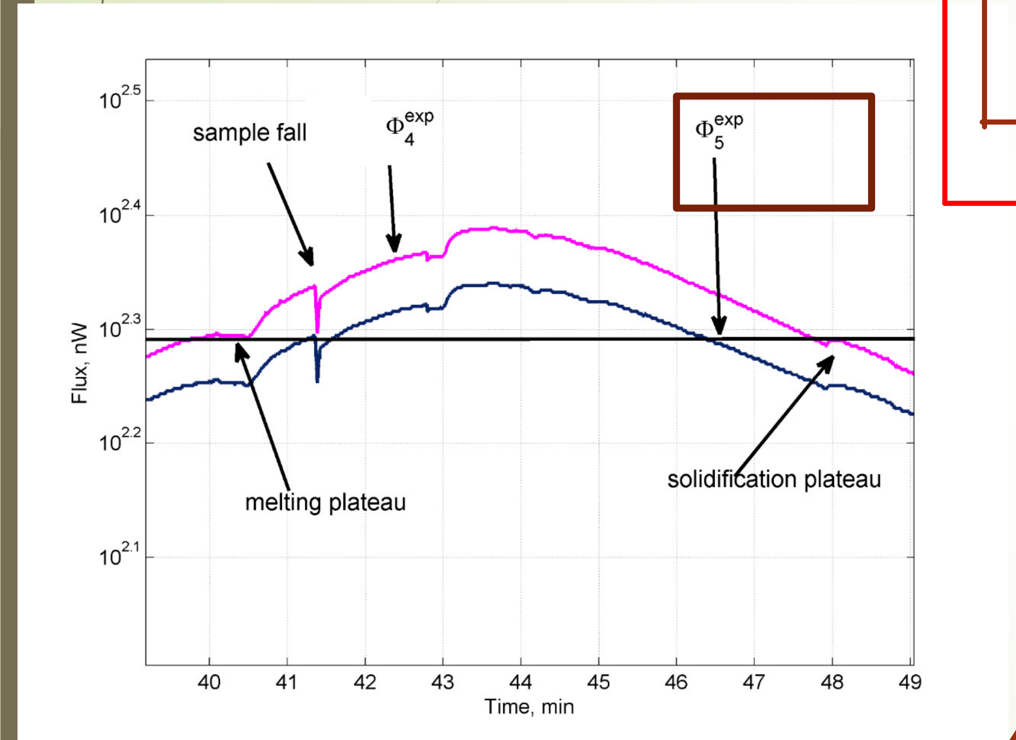


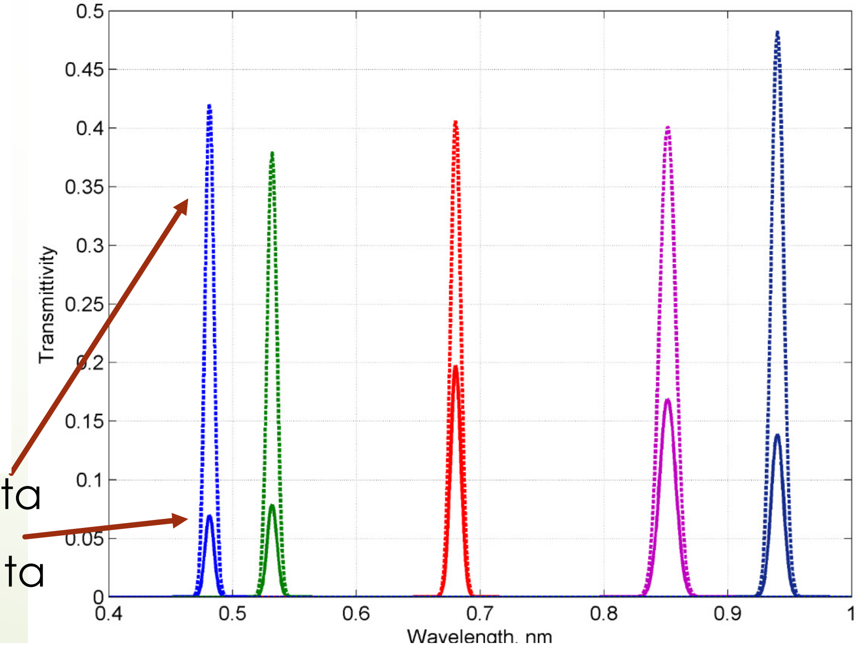
6. *Others techniques* – The multispectral method

Pyrometer calibration with a high temperature apparatus.

$$\left[\Phi_i^{th}\left(H_{M_i}\right) - \Phi_i^{exp}\right]^2 = 0$$

Optical path	**1**	**2**	**3**	**4**	**5**
$\bar{\lambda}_i$ (nm)	481.35	531.87	680.22	851.39	940.00
σ_i *(nm)*	3.55	3.85	4.09	6.41	4.92
Φ_i^{exp} (nW) melting plateau	0.4	1.5	35.7	197	179
H_{Mi}	0.070	0.079	0.198	0.169	0.139





supplier data

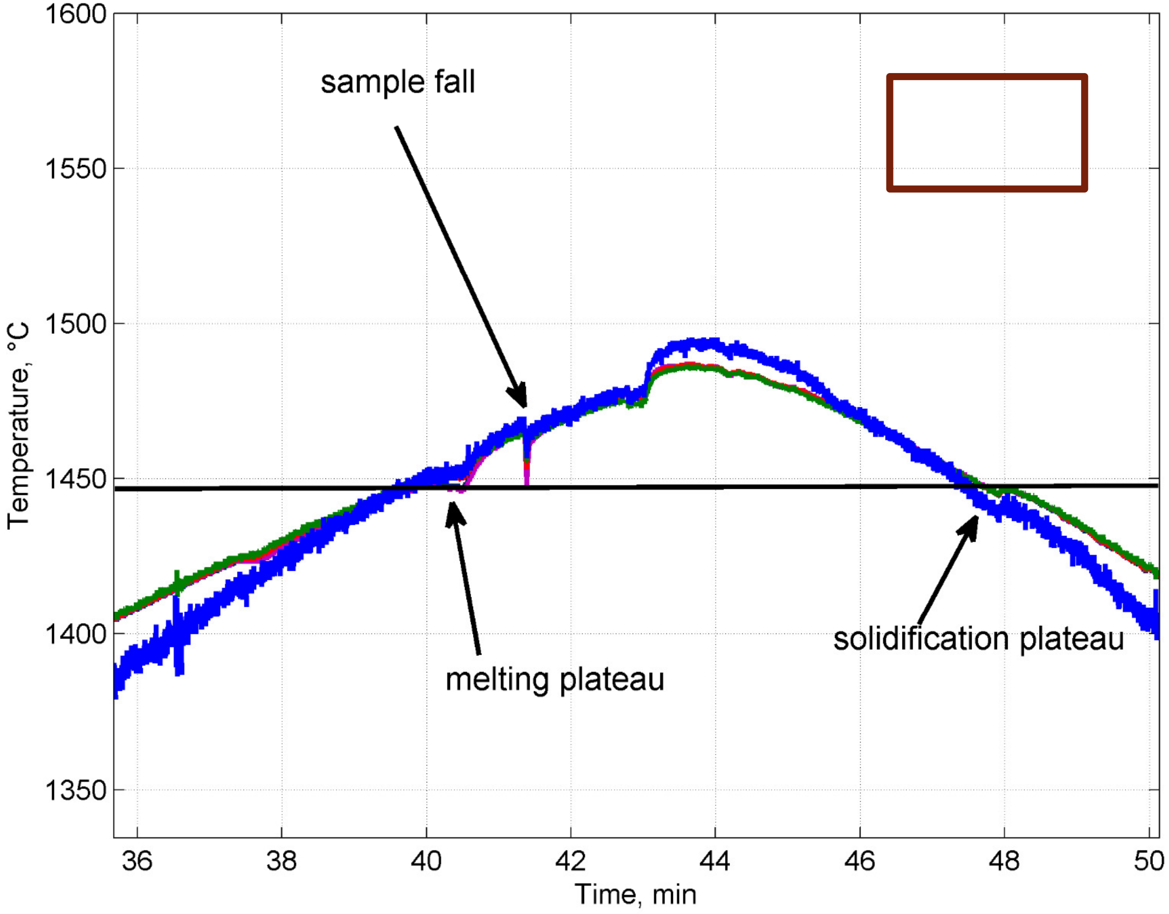
calibrated data

Systematic error 11 %.

$$\frac{\Delta F_{12}}{F_{12}} + \frac{\Delta s}{s} + \frac{\Delta \varepsilon_{éq}}{\varepsilon_{éq}}$$

6. *Others techniques* – The multispectral method

Pyrometer calibration with a high temperature apparatus.



Function to minimize $\left[\Phi_i^{th}(T_s) - \Phi_i^{exp}\right]^2 = 0$

assuming ε_{eq} = 0.96.

Lower temperature limit of use:
  optical path 1: > 1 500 °C
  op2: 1 300 °C
  op3: 1 100 °C
  op4 and op5: 900 °C → bispectral temperature measurement.

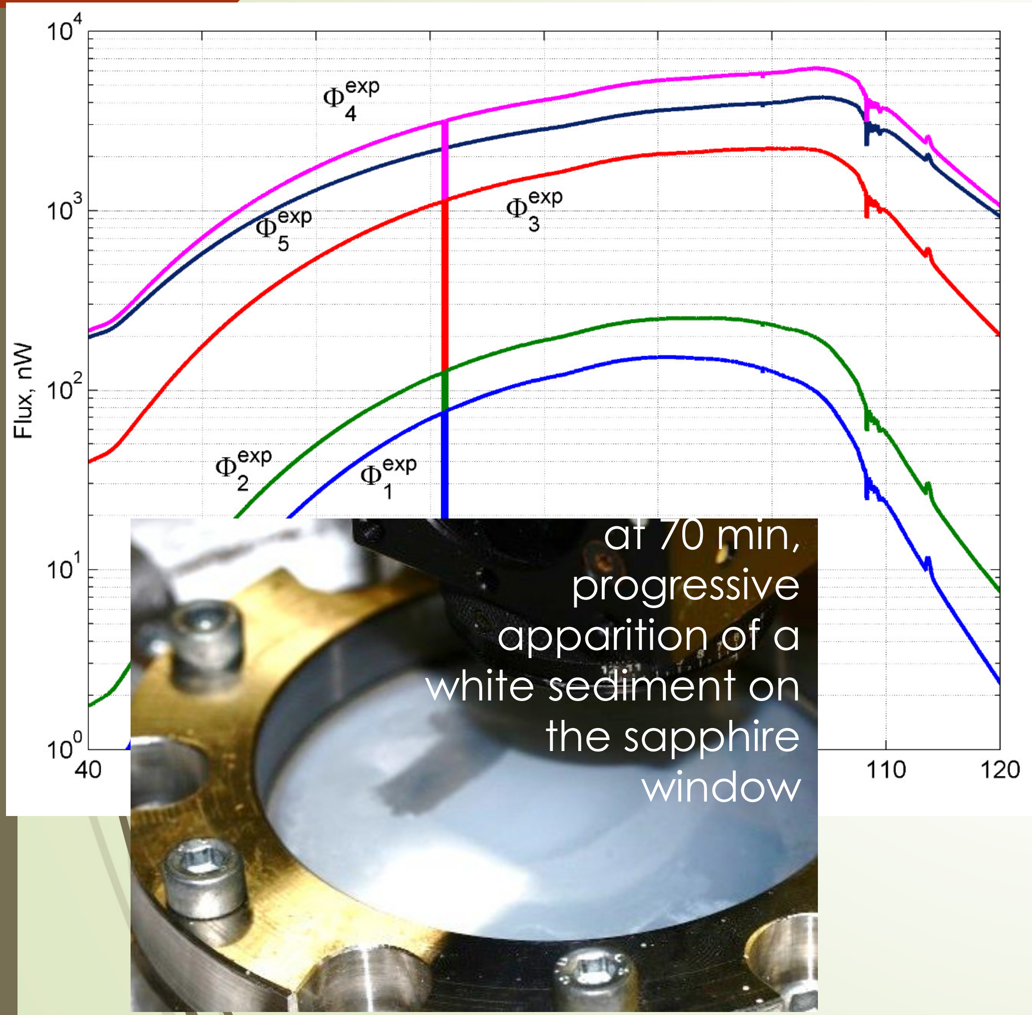
Optical paths 3 to 5 : good calibration
op1 and 2: need new calibration.

6. *Others techniques* – The multispectral method

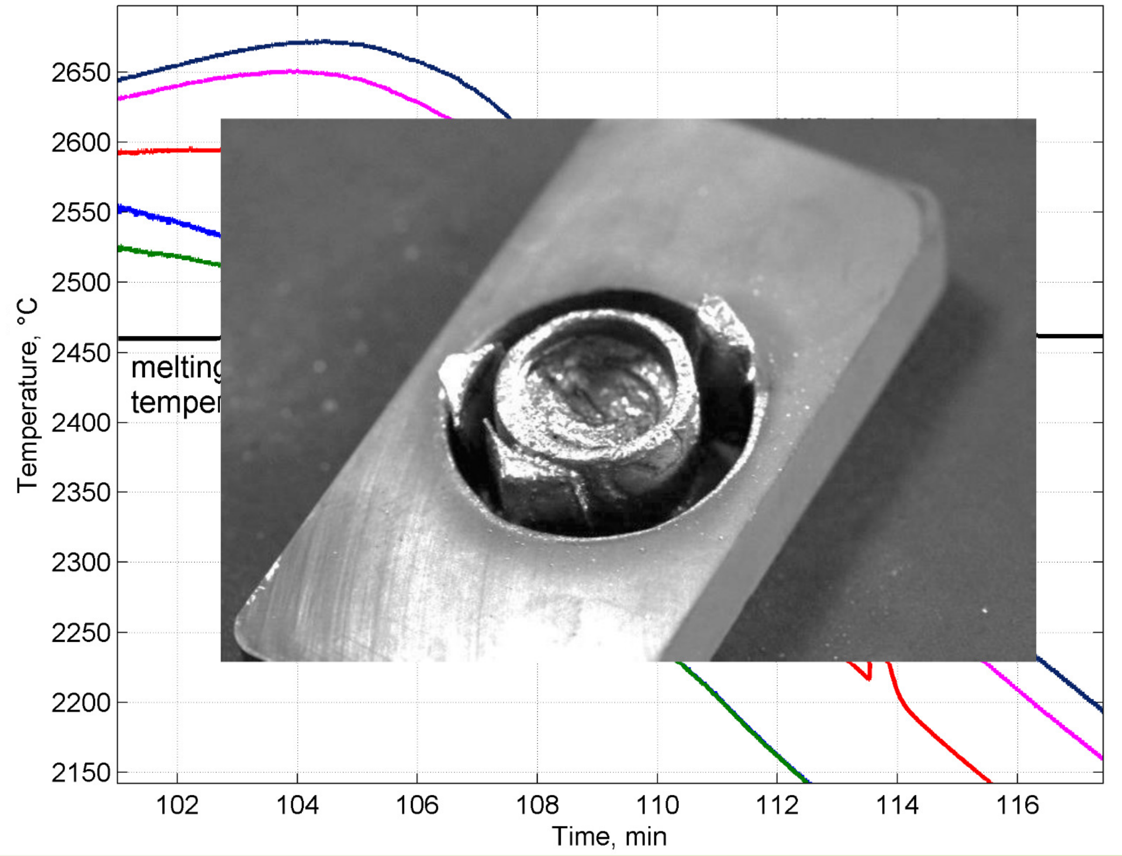
Pyrometer calibration with a high temperature apparatus.

Function to minimize $\left[\Phi_i^{th}(T_s) - \Phi_i^{exp}\right]^2 = 0$

with H_{Mi} estimated previously and ε_{eq} = 0.96.



at 70 min, progressive apparition of a white sediment on the sapphire window



1. *Thermal Radiation*
2. Radiatives Properties
3. Radiative measurements
4. Calibration
5. Other techniques
6. Applications
7. **Inverse method**
8. Prospect…

9th International Conference on
Inverse Problems in Engineering
University of Waterloo, May 23-26, 2017

ICIPE 2017

Session 3 : Inverse Heat Transfer II

Wednesday, May 24th

Definition of a methodology to estimate the thermal diffusivity of solid and liquid metal materials

Thomas PIERRE*[1], Mickaël COURTOIS[1], Muriel CARIN[1], Philippe LE MASSON[1], Helcio R.B. ORLANDE[2]

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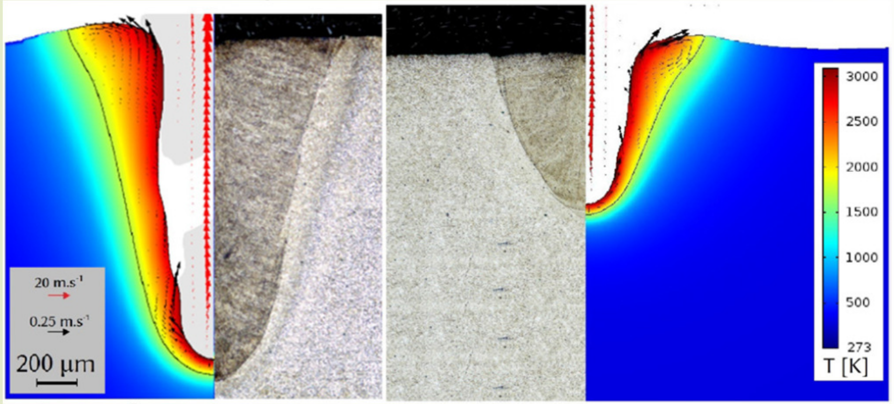
[2] Federal University of Rio de Janeiro, Rio de Janeiro, Brazil

Context and motivation

Example of study: calculation of the weld bead size and welding defects

(Heat transfer and fluid flow simulation and modelling in welding Courtois et al, J. Phys. D: Appl. Phy., 2013)



Involved conservation equations of heat, mass, and momentum:

$$\rho c_p^m \left[\frac{\partial T}{\partial t} + \vec{\nabla}\cdot(\vec{u}T)\right] = \vec{\nabla}\cdot\left(k\vec{\nabla}T\right) + S_{laser} + Q_{vap}$$

$$\vec{\nabla}\cdot\vec{u} = 0$$

$$\rho\left[\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\left(\vec{\nabla}\cdot\vec{u}\right)\right] = \vec{\nabla}\cdot\left\{-\rho I + \mu\left[\vec{\nabla}\vec{u} + \left(\vec{\nabla}\vec{u}\right)^T\right]\right\} + \rho\vec{g} - \rho_l\beta_l\left(T - T_{melting}\right)\vec{g}\phi + K\vec{u} + \gamma\vec{n}\kappa\delta(\phi)$$

Necessary physical properties:

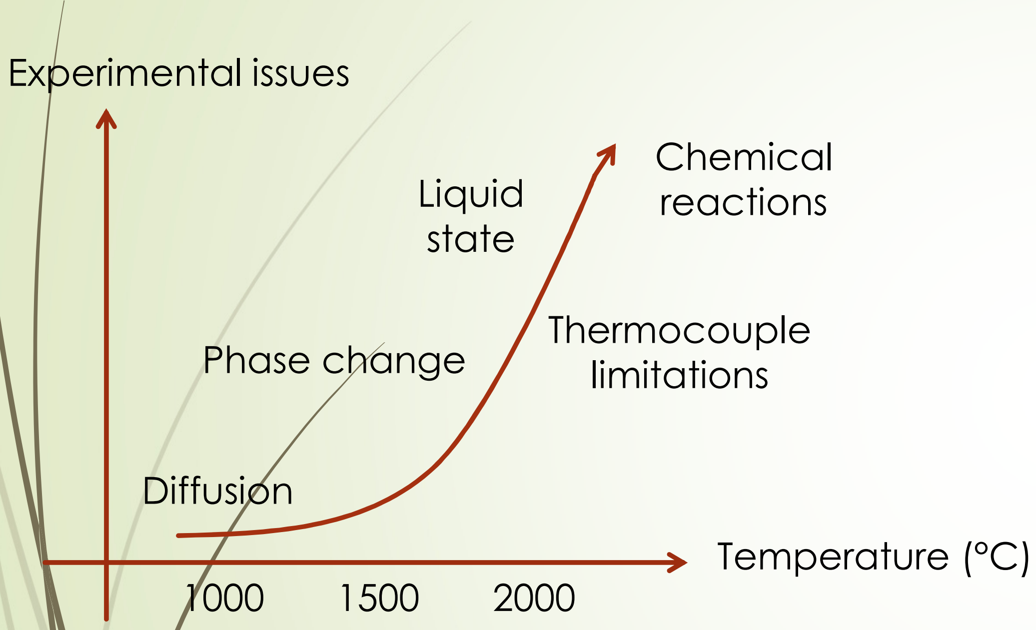
- ● density
- ● heat capacity
- ● thermal conductivity
- ● dynamic viscosity
- ● volume expansion coefficient
- ● surface tension

of the matter at **solid**, **liquid states** (gaseous?)

in a wide temperature range --> **2 500 °C**.

Context and motivation

Some problems encountered at high temperatures



Experimental issues

Chemical reactions

Liquid state

Thermocouple limitations

Phase change

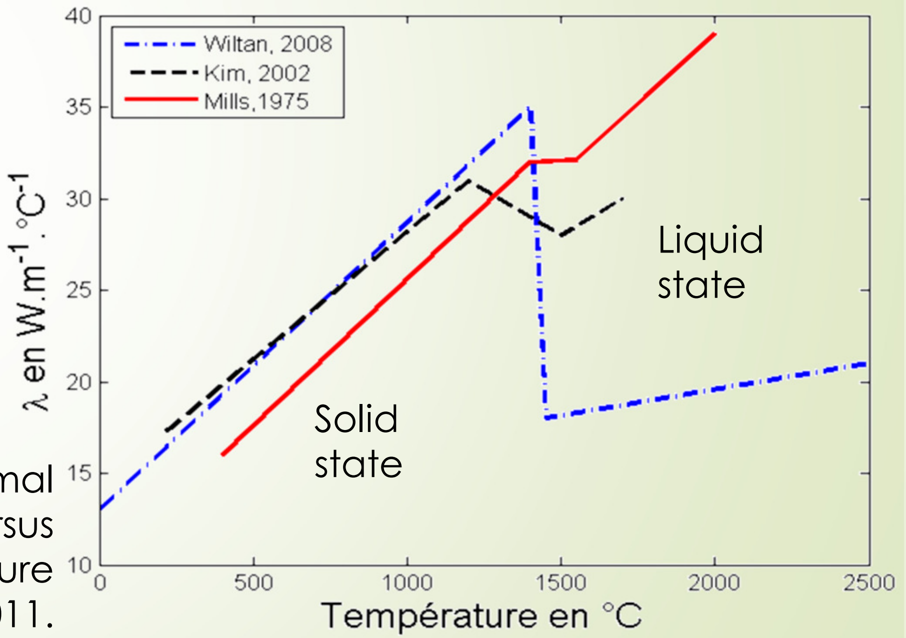
Diffusion

Temperature (°C)

1000 1500 2000

The literature presents:

- **scarce high temperature thermal properties** compared to ambient temperature's ones
- **discrepancies** of the thermophysical properties between authors



316L steel thermal conductivity versus temperature Dal, 2011.

--> Development of high temperature apparatuses dedicated to thermal characterization of solid and liquid materials.

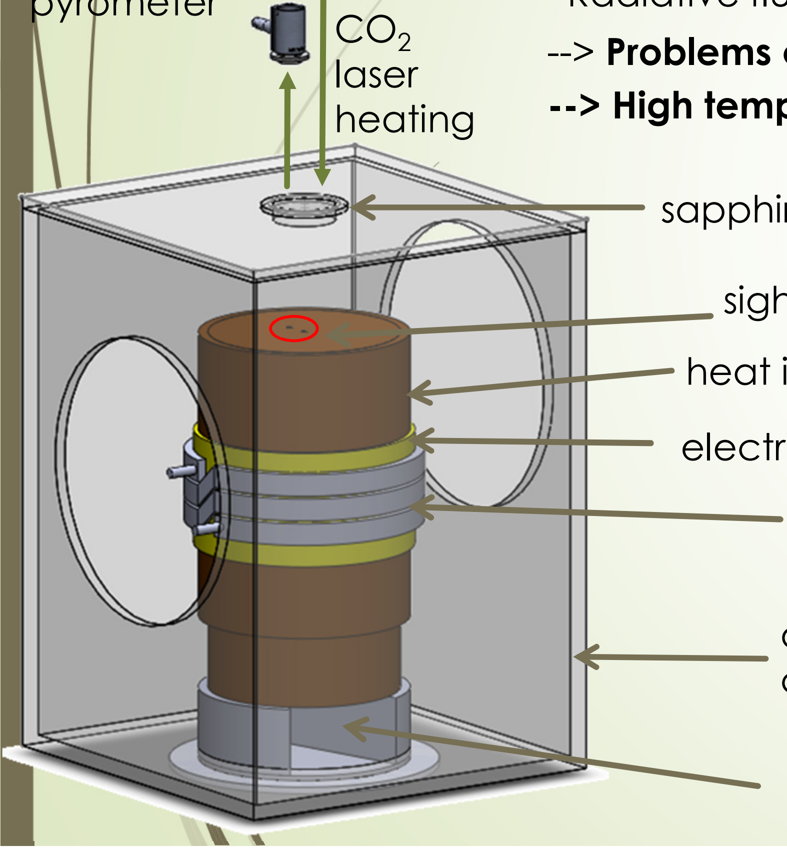
Development of two high temperature apparatuses

First apparatus (existing)

- Temperature level: 2 500 °C (successful niobium melting)
- Steady-state regime
- Stable sample in the crucible: no fluid flow, no magnetic field, controlled atmosphere
- Temperature measurements by two C type thermocouples (< 1 500 °C)
- Radiative flux measured by a five wavelengths pyrometer (> 1 000 °C)

--> **Problems of pollution !**

--> High temperature blackbody



radiative flux, collected by the collimator, goes to the pyrometer

CO_2 laser heating

sapphire window

sighting hole

heat insulating

electric insulating

Inductor

aluminum chamber

ceramique

Second apparatus (work in progress)

Aerodynamic levitation with:

- CO_2 laser heating,
- IR and visible fast cameras,
- multispectral pyrometer

Glorieux *et al.*, AIP Conference Proceedings **552**, 316 (2001)

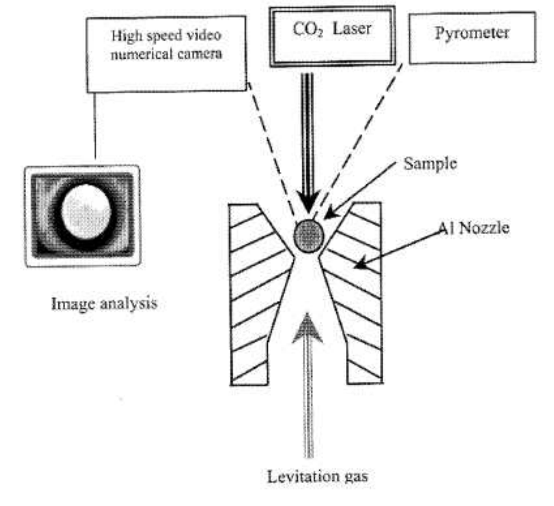
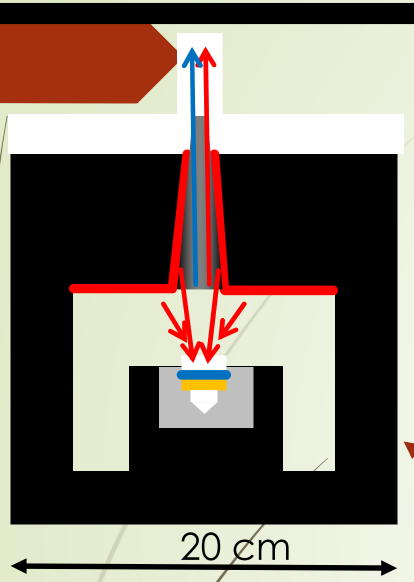
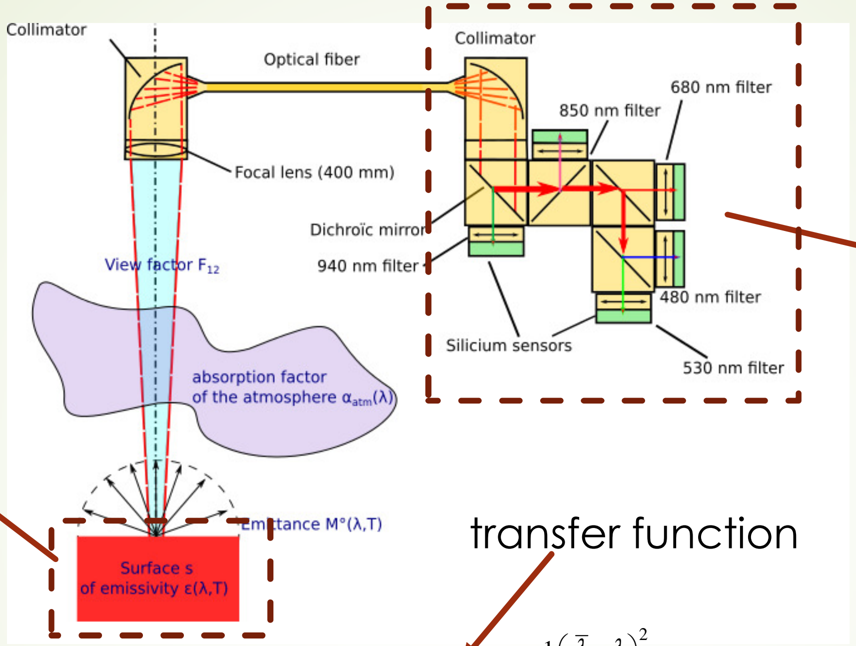
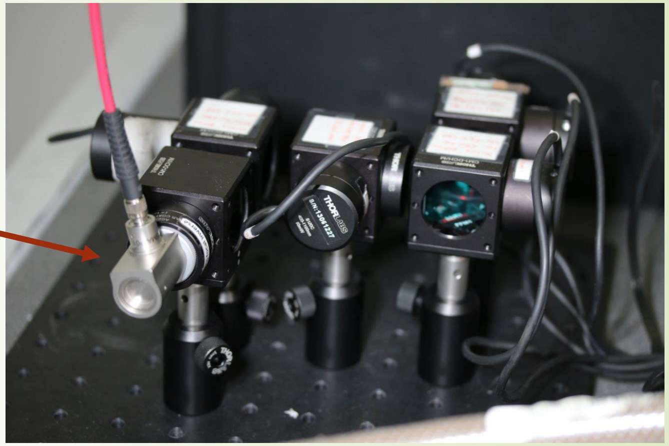


FIGURE 1. Aerodynamic Levitation Set-Up with Aluminum Nozzle.

Multispectral pyrometer presentation and theoretical flux received by each sensor







Dejaeghere, PhD thesis, 2016

- high temperature apparatus
- pyrometer
- transfer function
- calibration

transfer function

$$\Phi_i^{th} = sF_{12}\int_0^{\infty}\Big[\underbrace{\varepsilon'_{\lambda,s}M^0\left(\lambda,T_s^{th}\right)}_{\text{sample emission}}+\underbrace{\rho^{\cap'}_{\lambda;s}\varepsilon'_{\lambda,env}M^0\left(\lambda,T_{env}\right)}_{\text{environment emission reflected by the sample}}\Big]\alpha_{atm}\left(\lambda,T_{atm}\right)H_{M_i}e^{-\frac{1}{2}\left(\frac{\bar{\lambda}_i-\lambda}{\sigma_i}\right)^2}d\lambda$$

Possible observables: temperature $T_s^{\exp}$ or radiative fluxes $\Phi_i^{\exp}$.

--> Parameter estimation by front face flash method in solid and liquid phases of the sample:

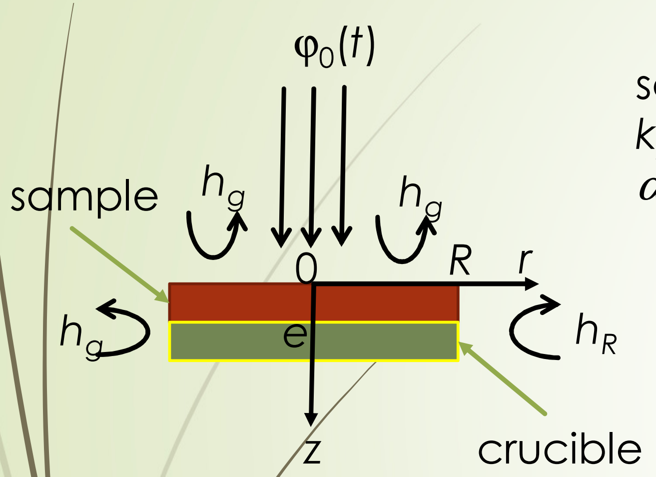
- **thermal diffusivity, thermal conductivity**

--> No fluid mechanics in the sample in its liquid phase during the heating --> thin thickness : possible lumped body condition !!! --> short time analysis (Fo < 1).

--> Temperature increase high enough (laser heating time)

Mathematical models for the front face flash method

Numerical simulation with Comsol Multiphysics, validated until 1 500 °C --> complex and time-consuming.



sample
$k_z = k_r = 50$ W·m^{-1}·K^{-1}
$\alpha_z = 1.13 \times 10^{-5}$ m^2·s^{-1}

crucible
$k_z = 1$ W·m^{-1}·K^{-1}
$\alpha_z = 7.4 \times 10^{-7}$ m^2·s^{-1}

$h_g = 40$ W·m^{-2}·K^{-1}

$\varphi_0 = 1$ MJ·m^{-2}
$\tau = 0.05$ s

--> analytical (forward) models

$$k_{jz}\frac{\partial^2 T_j}{\partial z^2} + k_{jr}\left(\frac{\partial^2 T_j}{\partial z^2} + \frac{1}{r}\frac{\partial T_j}{\partial z}\right) = \left(\rho c_p^m\right)_j \frac{\partial T_j}{\partial t}$$

j = 1 (sample) or 2 (crucible)

$$t = 0 \quad T_j\left(r, z, 0\right) = 0$$

geometry	**heating $\varphi_0(t)$**	**single layer: finite specimen**	**bilayer: finite specimen infinite crucible**	**temperature**
1D Cartesian	on all the front face	1	2	$T^{th}{}_1\left(r,0,t\right)$
2D axisymmetric		3	4	$\bar{T}_1^{th}\left(0,t\right)$
	on a reduced zone	5	6	

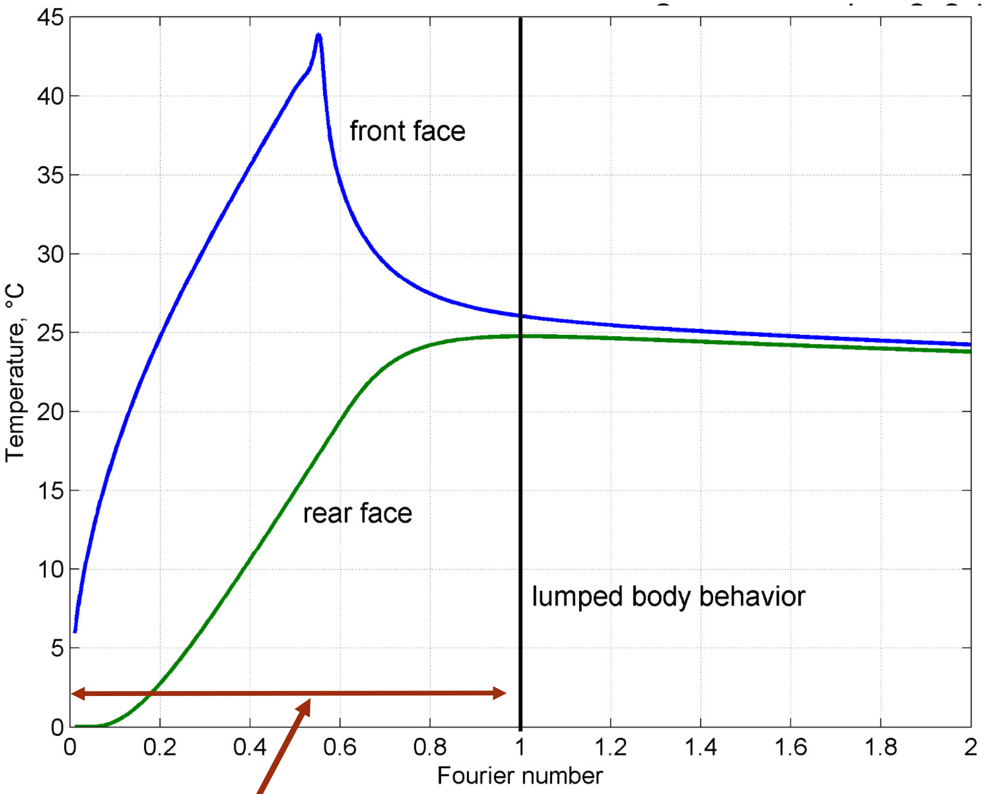
- *A priori* known parameters: e, R, φ_0, τ, h_g --> **always a bias** between experimental data and models.
- The complexity of the mathematical model is directly related to the **computational times** required for its solution.

--> Reduced mathematical models for the inverse parameter estimation problem within the Bayesian framework.

Flash method and influence of the *a priori* known parameters

classical sensitivity study with model n°6

e = 1 mm --> *t* = 0.09 s

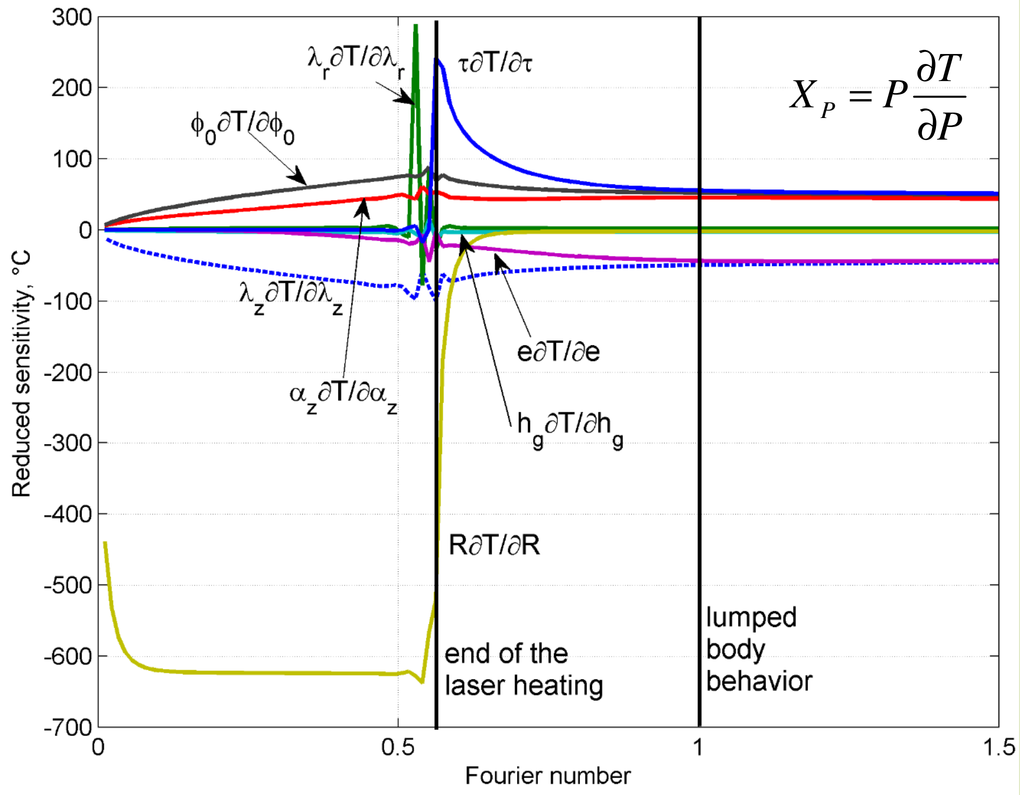


maximum estimation time $Fo_z = \frac{a_z t}{e^2}$

possible parameters to estimate: α, k_z

a priori known parameters: τ, φ_0, e, R

collateral damage: h_g



temperature few sensitive to: h_g, λ_r, and *R*.

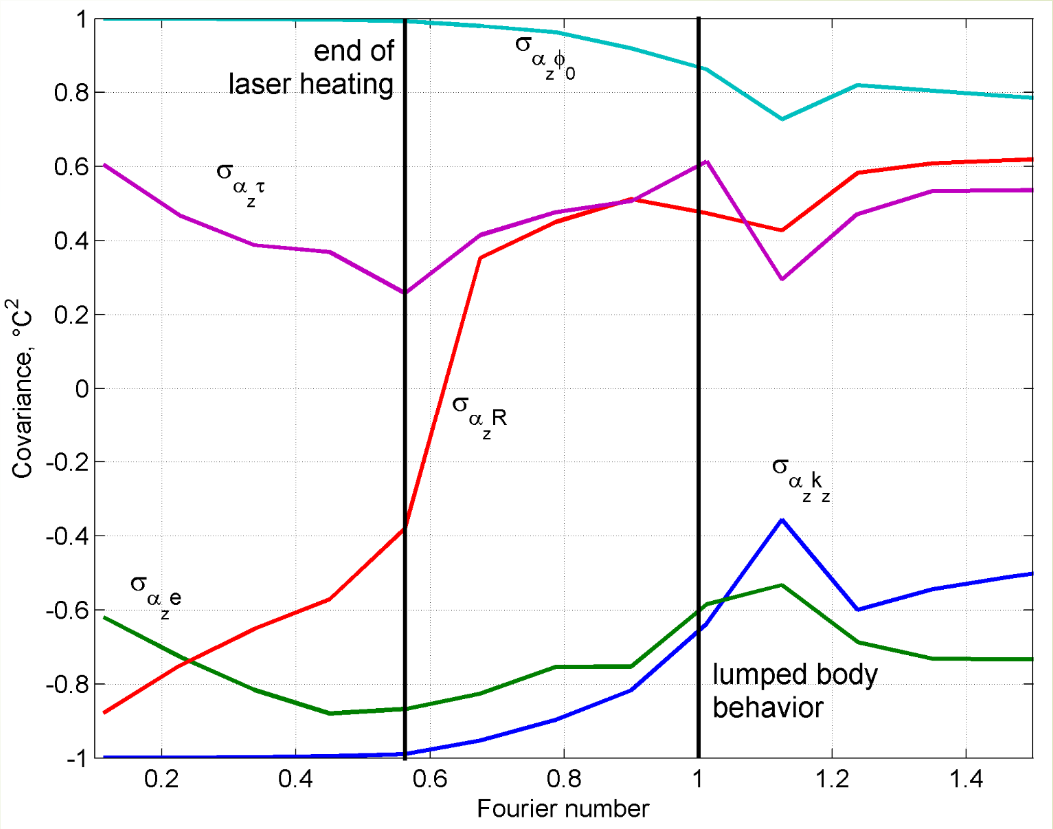
Sensitivity study and covariance matrix

example of study regarding the influence the parameters versus the thermal diffusivity α_z for e = 1 mm.

Covariance matrix

$$\begin{bmatrix}
\sigma^2_{k_z} & \sigma_{k_r k_z} & \sigma_{\alpha_z k_z} & \sigma_{h_g k_z} & \sigma_{e k_z} & \sigma_{R k_z} & \sigma_{\varphi_0 k_z} & \sigma_{\tau k_z} \\
 & \sigma^2_{k_r} & \sigma_{\alpha_z k_r} & \sigma_{h_g k_r} & \sigma_{e k_r} & \sigma_{R k_r} & \sigma_{\varphi_0 k_r} & \sigma_{\tau k_r} \\
 & & \sigma^2_{\alpha_z} & \sigma_{h_g \alpha_z} & \sigma_{e \alpha_z} & \sigma_{R \alpha_z} & \sigma_{\varphi_0 \alpha_z} & \sigma_{\tau \alpha_z} \\
 & & & \sigma^2_{h_g} & \sigma_{e h_g} & \sigma_{R h_g} & \sigma_{\varphi_0 h_g} & \sigma_{\tau h_g} \\
 & & & & \sigma^2_{e} & \sigma_{R e} & \sigma_{\varphi_0 e} & \sigma_{\tau e} \\
 & & & & & \sigma^2_{R} & \sigma_{\varphi_0 R} & \sigma_{\tau R} \\
 & & & & & & \sigma^2_{\varphi_0} & \sigma_{\tau \varphi_0} \\
 & & & & & & & \sigma^2_{\tau}
\end{bmatrix}$$

Influence of the estimation time versus the varianc



Bayesian method to study the quality of the estimation of the thermal diffusivity regarding the other para

Principle of the Bayesian method (1/4)

A Bayesian estimator is basically concerned with the analysis of the **posterior probability density** π(**P** | **Y**), which is the conditional probability of the parameters **P** given the measurements **Y.**

Bayes' formula

$$\boxed{\pi_{posterior}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi(\mathbf{Y}|\mathbf{P})\,\pi_{prior}(\mathbf{P})}{\pi(\mathbf{Y})}} \longrightarrow \boxed{posterior \propto prior \times likelihood}$$

where: $\pi(\mathbf{P}|\mathbf{Y})$ posterior probability density

$\pi(\mathbf{P})$ prior density (information about the parameters prior to the measurements)

$\pi(\mathbf{Y}|\mathbf{P})$ likelihood function (expresses the likelihood of different measurement outcomes **Y** with **P** given)

$\pi(\mathbf{Y})$ probability density of the measurements (normalizing constant)

Gaussian Prior density

$$\pi(\mathbf{P}) = (2\pi)^{-N/2}\left|\mathbf{V}\right|^{-1/2}\exp\left[-\frac{1}{2}(\mathbf{P}-\boldsymbol{\mu})^T\,\mathbf{V}^{-1}(\mathbf{P}-\boldsymbol{\mu})\right]$$

Likelihood Function

$$\pi(\mathbf{Y}|\mathbf{P}) = (2\pi)^{-M/2}\left|\tilde{\mathbf{W}}\right|^{-1/2}\exp\left\{-\frac{1}{2}[\mathbf{Y}-\mathbf{T}(\mathbf{P})]^T\,\tilde{\mathbf{W}}^{-1}[\mathbf{Y}-\mathbf{T}(\mathbf{P})]\right\}$$

P = $[k_z\ \alpha_z\ h_g\ e\ R\ \varphi_0\ \tau]$

N = 7

Y = **T**$^{\mathbf{exp}}$ = $[T_i, \ldots, T_M]$ at z = 0 and time t_i.

T = **T**$^{\mathbf{th}}$ experimental temperatures

M: number of measurements

V: covariance matrix for **P**

μ: known mean

$\tilde{\mathbf{W}}$: modified covariance matrix.

$$\tilde{\mathbf{W}} = \mathbf{W} + \mathbf{W}_\eta$$

due to the measurement errors

due to the approximation error

A reduced model implies the use of an **approximation error model** compared to a more complete one.

Principle of the Bayesian method (2/4)

The **statistical inversion approach** is based on the following principles*:

- All variables included in the formulation are modeled as random variables.
- The randomness describes the degree of information concerning their realizations.
- The degree of information concerning these values is coded in the probability distributions.
- **The solution of the inverse problem is the posterior probability distribution $\pi(\mathbf{P}\,|\,\mathbf{Y})$.**

Hypotheses:

- The errors are additive, with zero mean and normally distributed.
- The statistical parameters describing the measurement errors are known (covariance matrix $\mathbf{W}$)
- There are no errors in the independent variables.
- **P** is independent of **Y**.
- **P** is Gaussian with known mean $\boldsymbol{\mu}$ and known covariance matrix **V**.

Posterior Density

$$\ln\left[\pi(\mathbf{P}\,|\,\mathbf{Y})\right] \propto -\frac{1}{2}\left[(M+N)\ln 2\pi + \ln|\tilde{\mathbf{W}}| + \ln|\mathbf{V}| + S_{MAP}(\mathbf{P})\right]$$

Maximum a Posteriori Objective Function

$$S_{MAP}(\mathbf{P}) = \left[\mathbf{Y}-\mathbf{T}(\mathbf{P})\right]^T \tilde{\mathbf{W}}^{-1}\left[\mathbf{Y}-\mathbf{T}(\mathbf{P})\right] + (\boldsymbol{\mu}-\mathbf{P})^T \mathbf{V}^{-1}(\boldsymbol{\mu}-\mathbf{P})$$

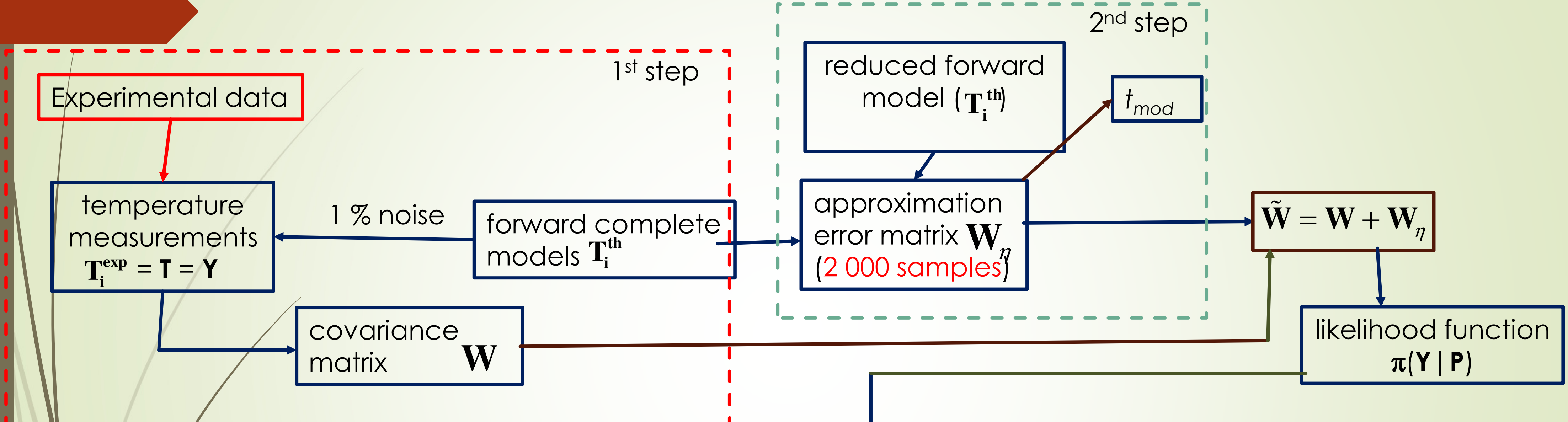
The maximizing of the posterior density is obtained by the minimizing the maximum a posteriori objective function.

Computation performed by Markov chain Monte Carlo algorithm: Metropolis-Hastings algorithm*.

---

* C. A. A. Mota et al., *Bayesian Estimation of Temperature-Dependent Thermophysical Properties and Transient Boundary Heat Flux*. Heat Transfer Engineering, 2010.

Principle of the Bayesian method (3/4)



1st step

Experimental data

temperature measurements $\mathbf{T}_i^{exp} = \mathbf{T} = \mathbf{Y}$

1 % noise

forward complete models $\mathbf{T}_i^{th}$

covariance matrix $\mathbf{W}$

2nd step

reduced forward model ($\mathbf{T}_i^{th}$)

t_{mod}

approximation error matrix $\mathbf{W}_\eta$ (2 000 samples)

$\tilde{\mathbf{W}} = \mathbf{W} + \mathbf{W}_\eta$

likelihood function $\pi(\mathbf{Y} \mid \mathbf{P})$

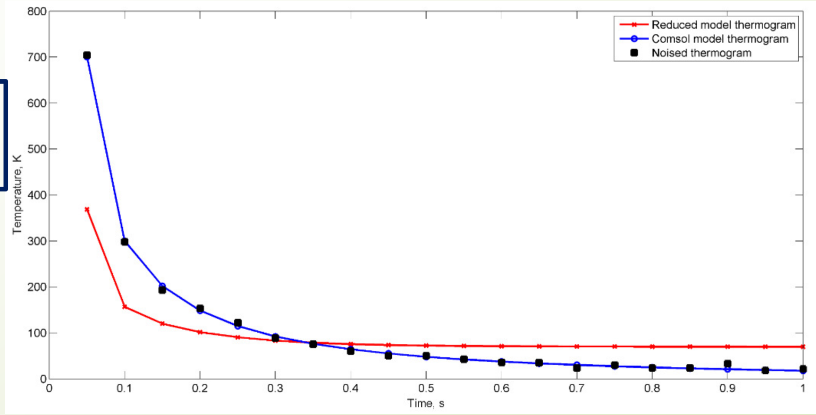
parameters $\mathbf{P}^T$

1 % to 5 % noise

V, μ

prior function $\pi(\mathbf{P})$

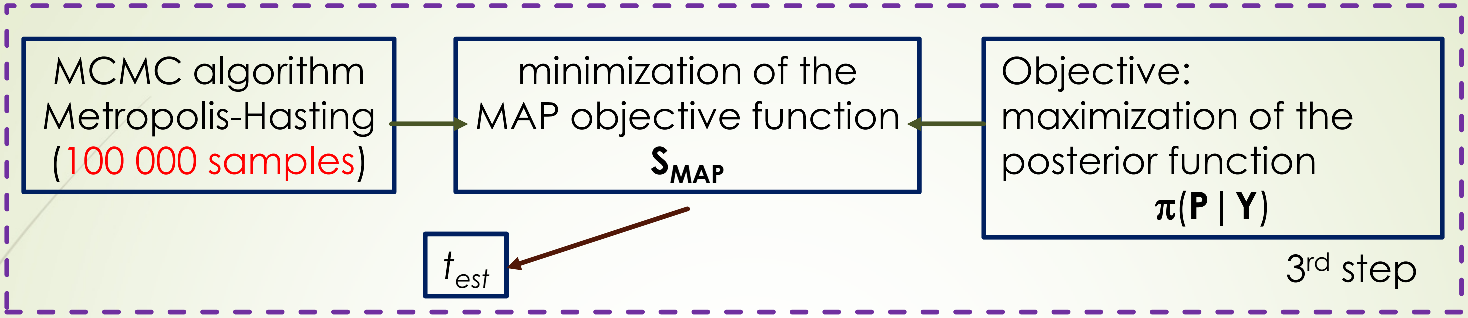
posterior function $\pi(\mathbf{P} \mid \mathbf{Y})$



$$\boxed{S_{MAP}(\mathbf{P}) = [\mathbf{Y} - \mathbf{T}(\mathbf{P})]^T \tilde{\mathbf{W}}^{-1} [\mathbf{Y} - \mathbf{T}(\mathbf{P})] + (\boldsymbol{\mu} - \mathbf{P})^T \mathbf{V}^{-1} (\boldsymbol{\mu} - \mathbf{P})}$$

--> to 3rd step

Principle of the Bayesian method (4/4)



MCMC algorithm
Metropolis-Hasting
(100 000 samples)

minimization of the
MAP objective function
$\mathbf{S_{MAP}}$

Objective:
maximization of the
posterior function
$\pi(\mathbf{P} \mid \mathbf{Y})$

t_{est}

3rd step

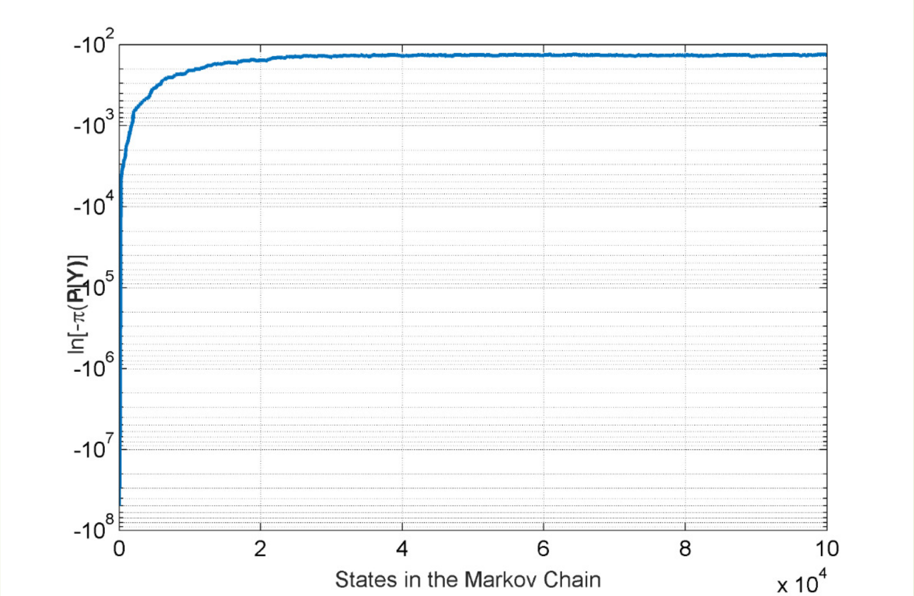
$$\boxed{\ln\left[\pi(\mathbf{P} \mid \mathbf{Y})\right] \propto -\frac{1}{2}\left[(M+N)\ln 2\pi + \ln|\tilde{\mathbf{W}}| + \ln|\mathbf{V}| + S_{MAP}(\mathbf{P})\right]}$$

METROPOLIS-HASTINGS ALGORITHM

1. Sample a *Candidate Point* $\mathbf{P}^*$ from a proposal distribution $p(\mathbf{P}^*, \mathbf{P}^{(t-1)})$
2. Calculate the acceptance factor:

$$\alpha = \min\left[1, \frac{\pi(\mathbf{P}^* \mid \mathbf{Y})\, p(\mathbf{P}^{(t-1)}, \mathbf{P}^*)}{\pi(\mathbf{P}^{(t-1)} \mid \mathbf{Y})\, p(\mathbf{P}^*, \mathbf{P}^{(t-1)})}\right]$$

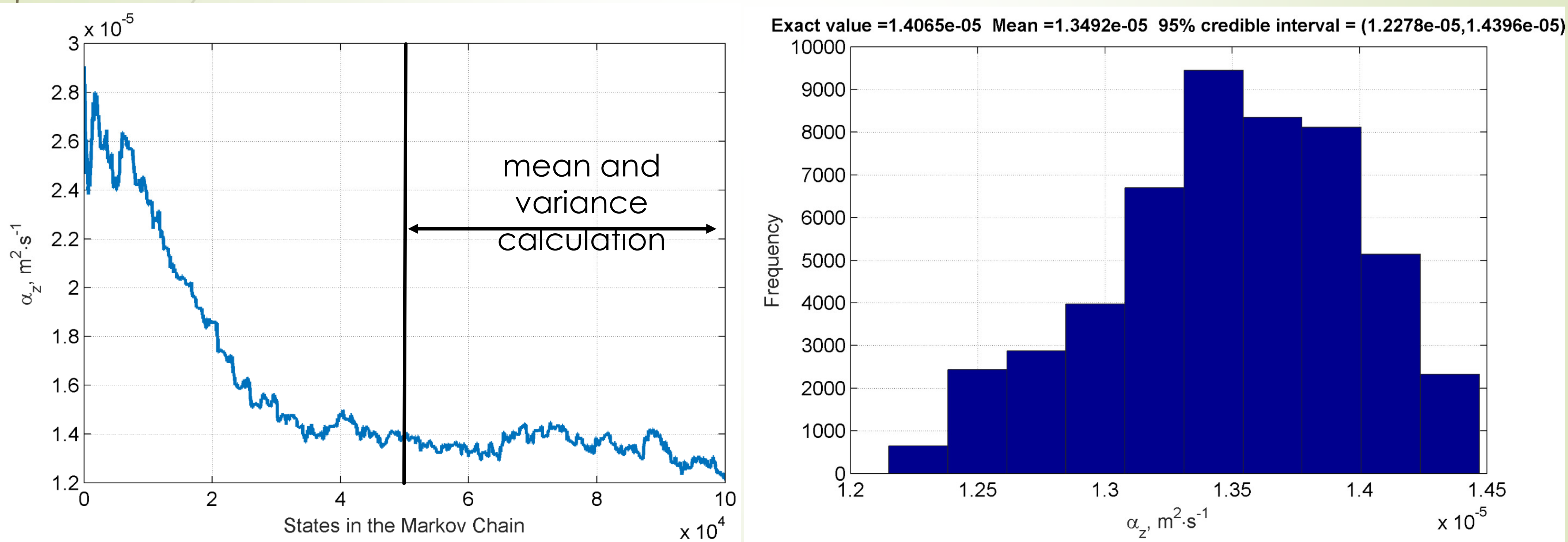
3. Generate a random value U that is uniformly distributed on (0,1).
4. If $U < \alpha$, set $\mathbf{P}^{(t)} = \mathbf{P}^*$. Otherwise, set $\mathbf{P}^{(t)} = \mathbf{P}^{(t-1)}$.
5. Return to step 1.



Tests and results

Two tests: parameter estimation from reduced model 1 compared with measurement data from:
1-/ models 2 to 6
2-/ Comsol Multiphysics model.

thermal diffusivity α_z estimation results


Exact value =1.4065e-05 Mean =1.3492e-05 95% credible interval = (1.2278e-05,1.4396e-05)

mean and variance calculation

Estimation results between different models and model 1

2^{nd} step : 2 000 samples
3rd step : 100 000 samples

Imposed parameters:

- α_z = 14.5 $mm^2 \cdot s^{-1}$
- $k_z = k_r$ = 50 $W \cdot m^{-1} \cdot K^{-1}$
- h_g = 40 $W \cdot m^{-2} \cdot K^{-1}$
- τ = 0.05 s
- φ_0 = 1 $MJ \cdot m^{-2}$
- e = 4 mm

		complete model / reduced model 1					
	n°	2	3	4	5	6	Comsol
calculation time	t_{mo}	8 s	30 min	26 min	38 min	28 min	3 h 44 min
	t_{sim} (s)	224 (for 100 000 samples)					
	parameters	estimated					
measurement	σ (K)	3.6					
diffusivity	μ_α ($mm^2 \cdot s^{-1}$)	14.0	14.7	14.1	14.8	14.5	14.6
	σ_α (10^{-8} $mm^2 \cdot s^{-1}$)	15.00	1.31	1.46	3.08	1.62	1.37
conductivity	μ_k ($W \cdot m^{-1} \cdot K^{-1}$)	50.23	50.33	50.07	50.17	50.34	50.28
	σ_k ($mW \cdot m^{-1} \cdot K^{-1}$)	74	37	43	062	39	9
heat exchange coefficient	μ_h ($W \cdot m^{-2} \cdot K^{-1}$)	39.7	40.1	39.9	40.0	39.9	40.0
	σ_h ($mW \cdot m^{-2} \cdot K^{-1}$)	7	19	6	26	16	9
flash excitation duration	μ_τ (ms)	50.8	53.1	51.4	52.1	51.4	51.8
	σ_τ (ns)	211	5.11	4.13	7.09	14.9	164
flash energy	μ_ϕ ($MJ \cdot m^{-2}$)	1.00	1.00	1.00	1.00	1.00	1.00
	σ_ϕ ($J \cdot m^{-2}$)	83	67	65	61	67	41.26
sample thickness	μ_e (mm)	5.10	4.30	3.90	4.30	3.80	4.10
	σ_e (nm)	0.68	0.47	0.45	0.42	0.44	1.02

Conclusion and outlooks

C1- The development of a high temperatures apparatus (2 500 °C):
- conditioning of samples in solid or liquid state
- front face flash method

C2- Definition of a methodology for the estimation of the thermal properties of solid and liquid metals and alloys using the Bayesian methods:
- influence of the a priori known parameters

O1- Development of the second apparatus
- no pollution problem on the sample
- simpler new analytical models (spherical coordinates)
- adaptation of the Bayesian method
- less parameters of influence (radius, flux, heating time)
- increasing of the error measurements

O2- Parameter estimation with the radiative fluxes as observables of the multispectral pyrometer for temperature above 1 000 °C

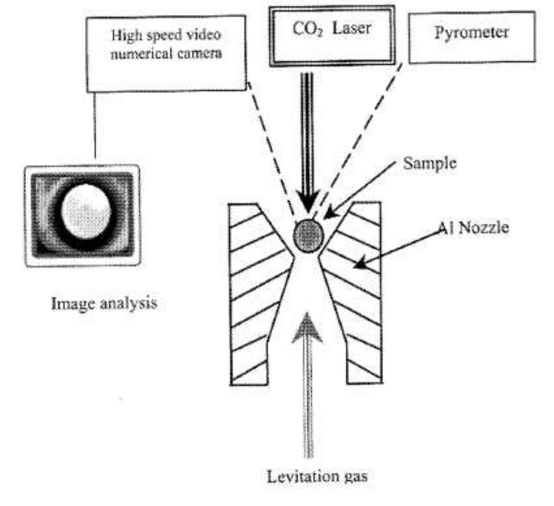
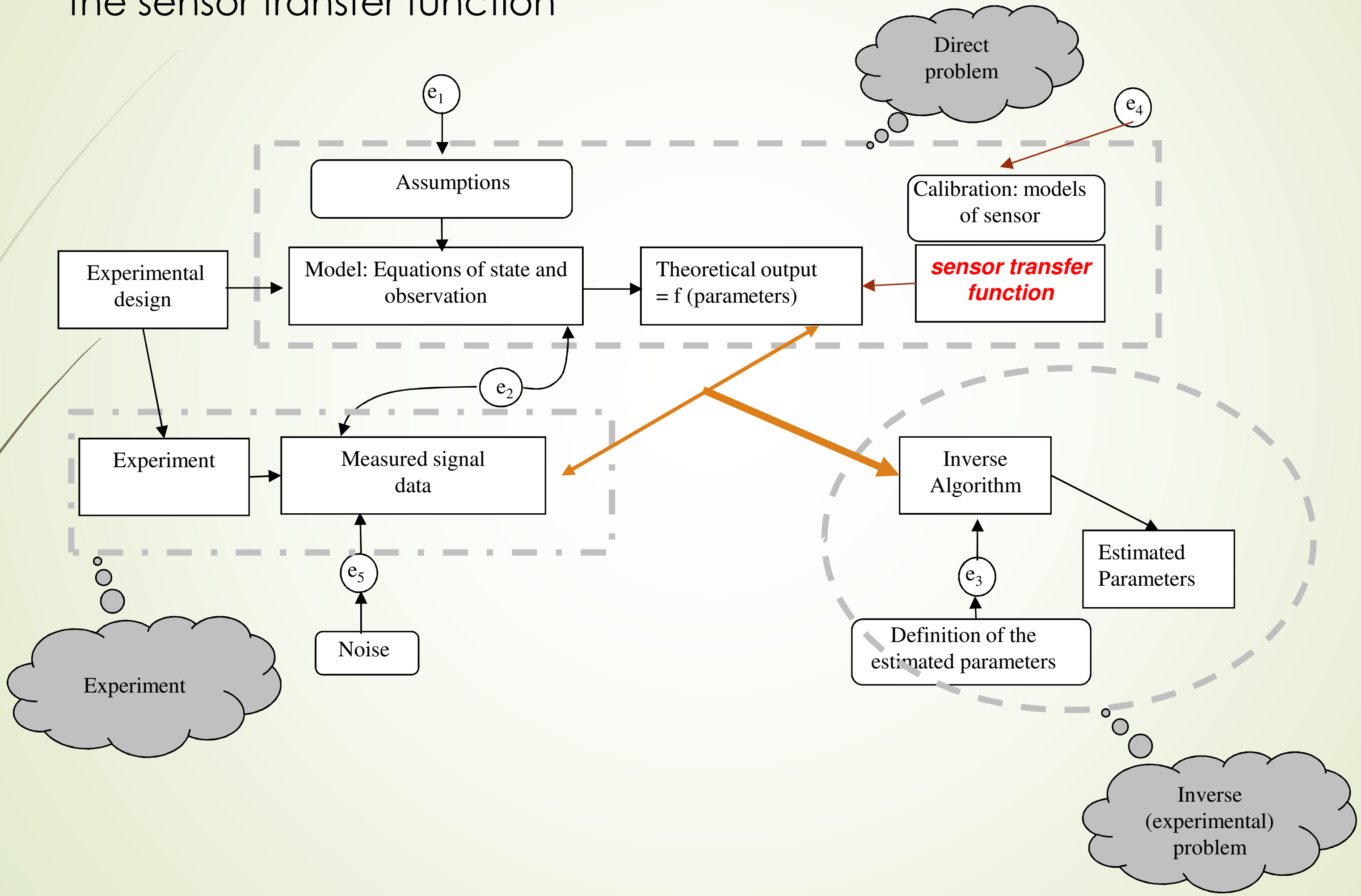


FIGURE 1. Aerodynamic Levitation Set-Up with Aluminum Nozzle.

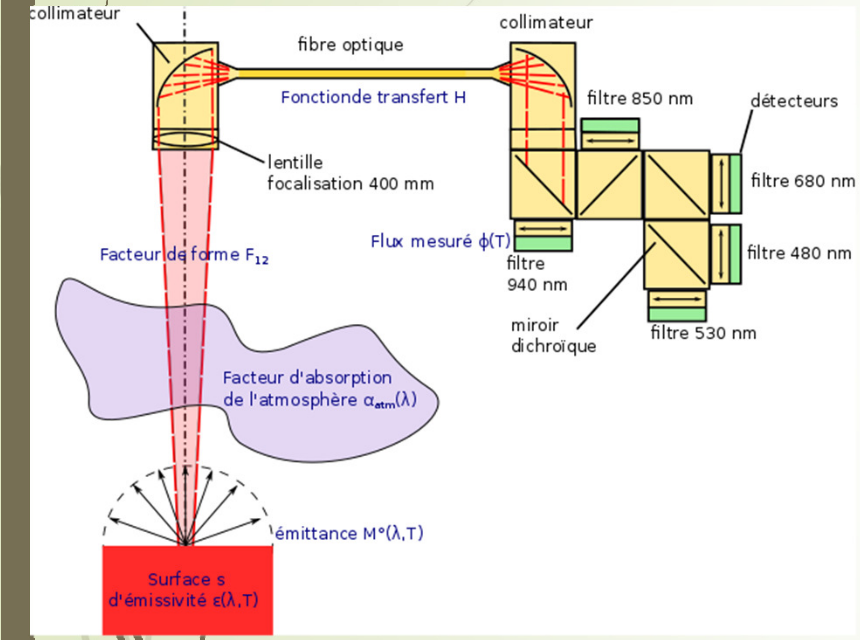
- *1. Thermal Radiation*
- 2. Radiatives Properties
- 3. Radiative measurements
- 4. Calibration
- 5. Other techniques
- 6. Applications
- 7. Inverse method

8. Prospect…

New approach for the bayesian method : the direct problem with the sensor transfer function



For our studies



- Calculation of the temperature field in the plate or in the sphere
- Integration of the flux on the surface

$$\Phi_i^{th}(\lambda, T, \varepsilon) = F_{12}\, s \int_0^{\infty} \varepsilon(\lambda, T)\alpha_{atm}(\lambda, T)M^0(\lambda, T)\, H_{M_i} e^{-\frac{1}{2}\left(\frac{\overline{\lambda_i}-\lambda}{\sigma_i}\right)^2} d\lambda$$

- Estimation of the parameters:

$$\left[\Phi_i^{th}(T_s) - \Phi_i^{exp}\right]^2 = 0$$

- Application of the bayesian method…

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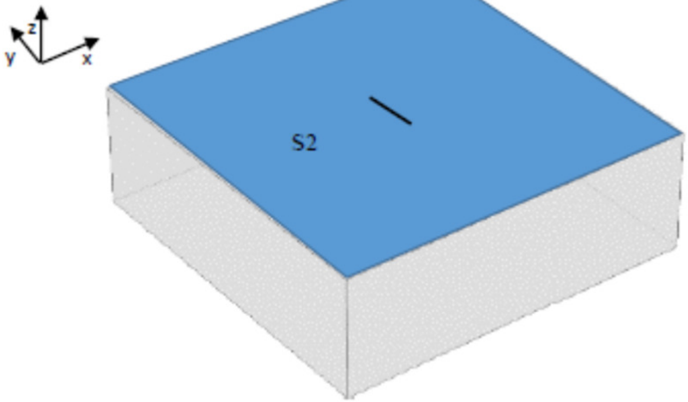

A: facteur d'absorption
P: puissance laser
h: coefficient d'échange global
σ: Rayon du faisceau laser à 1/e
T_{ext}: température extérieure
φ: Densité de flux du laser
G:fonction Gaussienne
R: fonction Rectangle
L: longueur de la ligne laser

$$\left[\begin{array}{l} \varphi = A.P.\frac{1}{\sigma_y\sqrt{2\pi}}e^{\frac{-(y-y_0)^2}{2\sigma_y^2}}.\frac{1}{\sigma_x\sqrt{2\pi}}e^{\frac{-(x-x_0-vt)^2}{2\sigma_x^2}} \\ \\ \varphi(x,y,t) = \frac{A.P.G((x-vt).cos\theta - y.sin\theta).R((x-vt).sin\theta + y.cos\theta)}{L} \\ \\ \text{Échange avec l'extérieur (convection + rayonnement)} \end{array}\right.$$

Faisceau laser gaussien

Faisceau laser rectangulaire avec rotation

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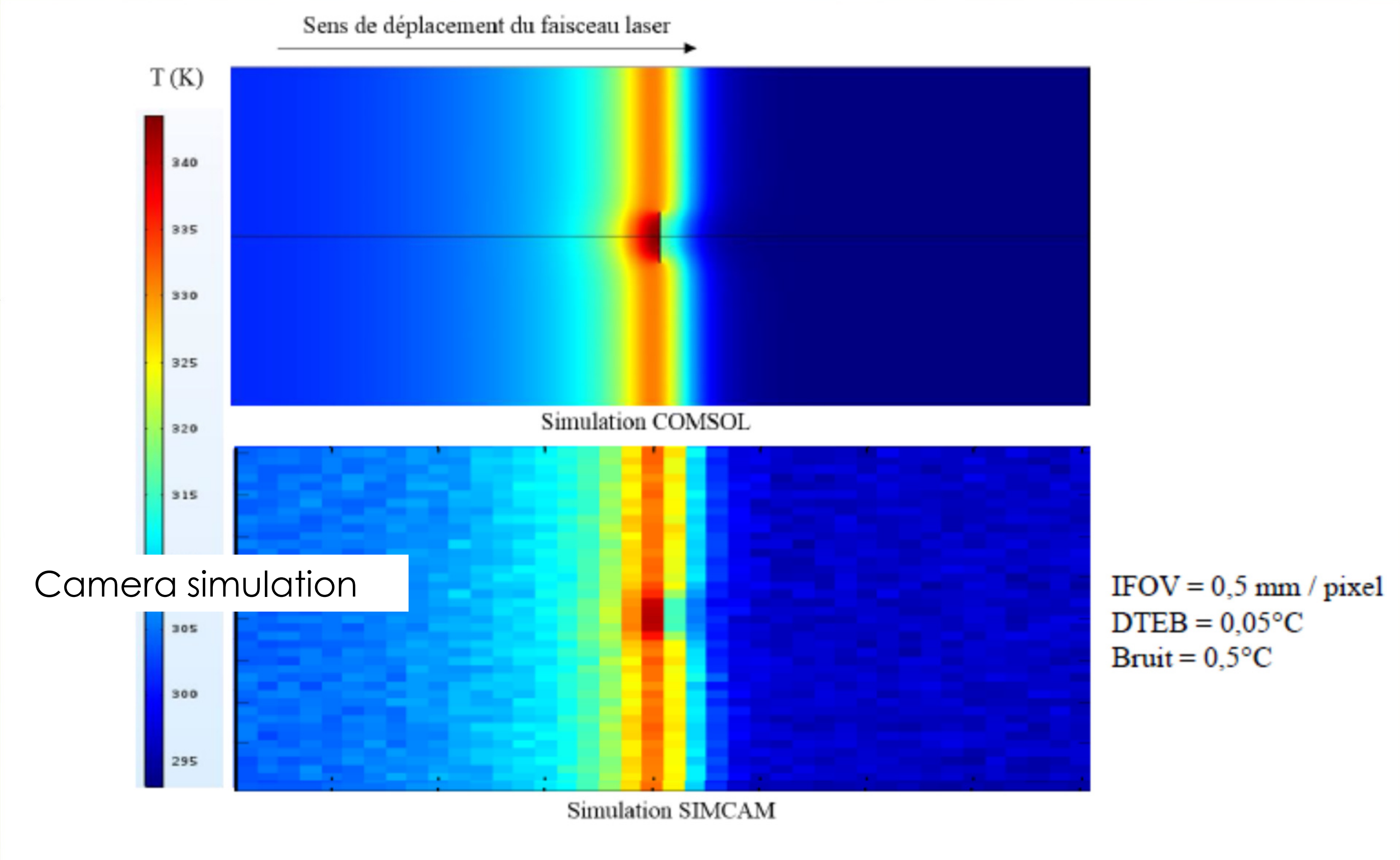

Camera simulation

IFOV = 0,5 mm / pixel
DTEB = 0,05°C
Bruit = 0,5°C

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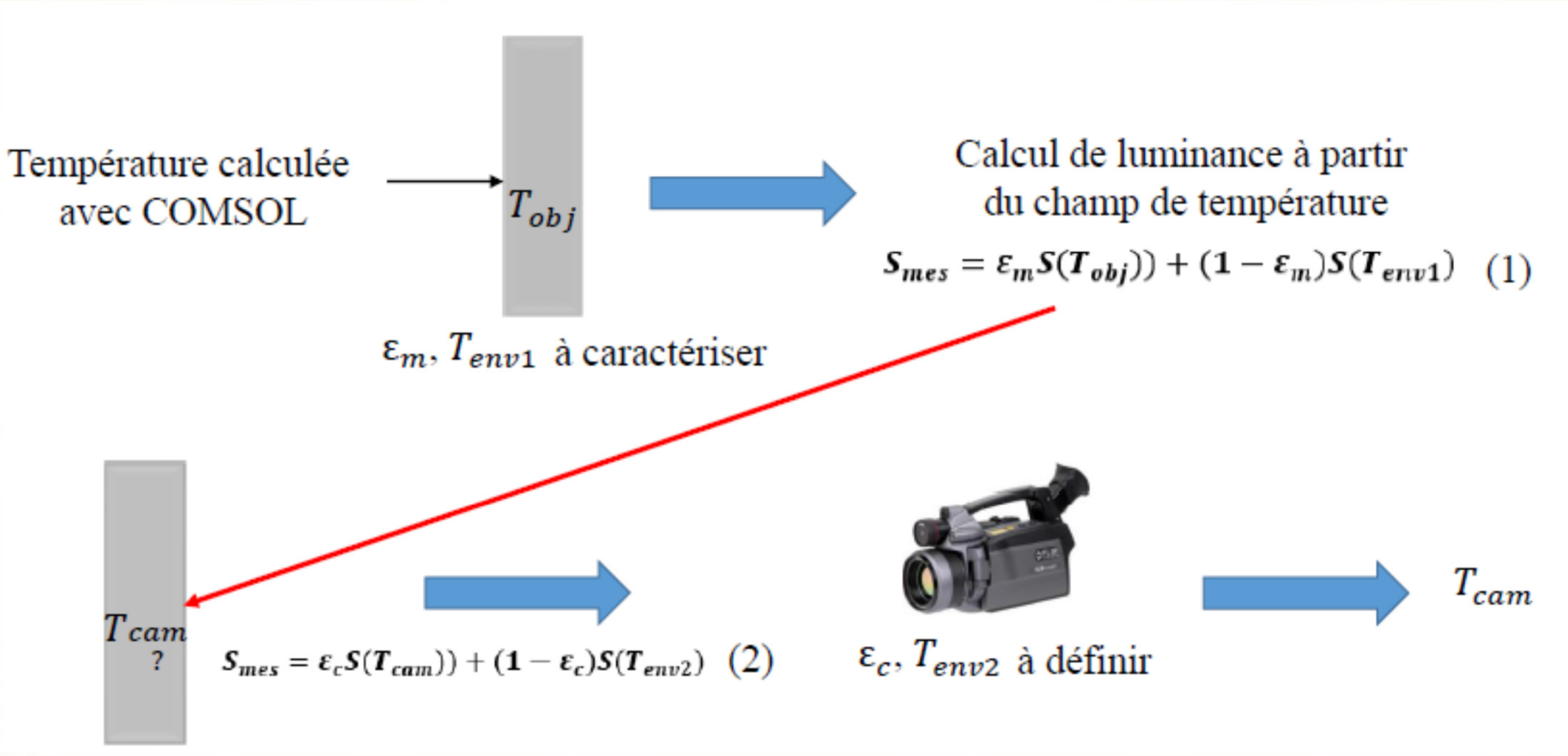

Température calculée avec COMSOL

T_{obj}

ε_m, T_{env1} à caractériser

Calcul de luminance à partir du champ de température

$S_{mes} = \varepsilon_m S(T_{obj})) + (1-\varepsilon_m)S(T_{env1})$ (1)

T_{cam} ?

$S_{mes} = \varepsilon_c S(T_{cam})) + (1-\varepsilon_c)S(T_{env2})$ (2)

ε_c, T_{env2} à définir

T_{cam}