Mini-course 07: Kalman and Particle Filters
Particle Filters
Fundamentals, algorithms and applications

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Outline

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- Why Particle Filters are important?
- Bayesian Framework
- State Estimation Problem
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History of Particle Filters

History of Particle Filters

- Sequential Monte Carlo methods for on-line learning within a Bayesian framework.
- **Known as**
  - Particle filters
  - Sequential Sampling-Importance Resampling (SIR)
  - Bootstrap filters
  - Condensation trackers
  - Interacting particle approximations
  - Survival of the fittest

History of Particle Filters

- **FIRST ATTEMPTS – SIMULATIONS OF GROWING POLYMERS**
  - M. N. Rosenbluth and A.W. Rosenbluth, “Monte Carlo calculation of the average extension of molecular chains,” *Journal of Chemical Physics*, vol. 23, no. 2, pp. 356–359, 1955.

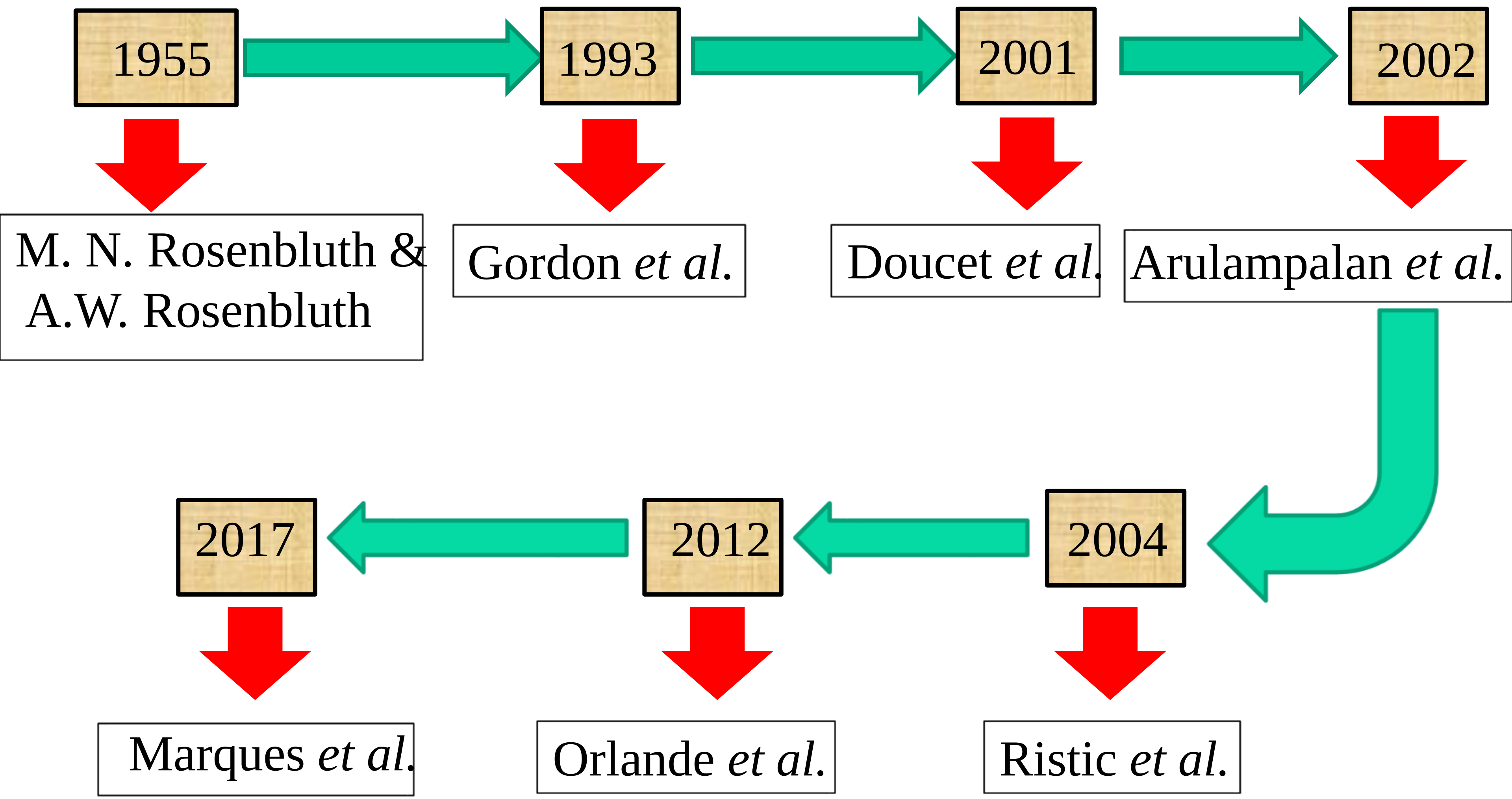
- **FIRST APPLICATION IN SIGNAL PROCESSING - 1993**
  - N. J. Gordon, D. J. Salmond, and A. F. M. Smith, “Novel approach to nonlinear/non-Gaussian Bayesian state estimation,” *IEE Proceedings-F*, vol. 140, no. 2, pp. 107–113, 1993.

History of Particle Filters

- **BOOKS**
  - A. Doucet, N. de Freitas, and N. Gordon, Eds., Sequential Monte Carlo Methods in Practice, Springer, 2001.
  - B. Ristic, S. Arulampalam, N. Gordon, Beyond the Kalman Filter: Particle Filters for Tracking Applications, Artech House Publishers, 2004.

- **TUTORIALS**
  - M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, “A tutorial on particle filters for online nonlinear/non-gaussian Bayesian tracking,” IEEE Transactions on Signal Processing, vol. 50, no. 2, pp. 174–188, 2002.

History of Particle Filters



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Why Particle Filters are important?

Why Particle Filters are important?

The elegance of the Kalman Filter provided some solace, but at the cost of pretending to live in a linear Gaussian world. Once into the nonlinear, non-Gaussian domain, we were without a universally effective approach and driven into a series of ingenious approximations, some based on flexible mixtures of tractable distributions to approximate and propagate uncertainty, or on local linearisations of nonlinear systems. (**Doucet et al, 2001**) .

Why Particle Filters are important?

1. Need to estimate an unknown variables or unobservable variables from a set of experimental data.
2. Commonly, these data or observations are generated in real-time and the estimation must be done in real time.
3. Tool for tracking the state of a dynamic system modeled by a Bayesian network.

Why Particle Filters are important?

4. Applications similar to those of the Kalman Filters, but computationally tractable for large/high dimensional problems.
5. **These methods are very flexible, easy to implement and to be applied in different areas, for nonlinear non-Gaussian problems.**
6. Key idea: Find an approximate posterior distribution.

Bayesian Framework

Bayesian Framework

The formal mechanism to combine the new information (measurements) with the previously available information (prior) is known as the **Bayes' theorem:**

$$\boxed{\pi_{posterior}(\mathbf{x}) = \pi(\mathbf{x}|\mathbf{z}) = \frac{\pi(\mathbf{x})\pi(\mathbf{z}|\mathbf{x})}{\pi(\mathbf{z})}}$$

where $\pi_{posterior}(\mathbf{x})$ is the posterior probability density, $\pi(\mathbf{x})$ is the prior density, $\pi(\mathbf{z}|\mathbf{x})$ is the likelihood function and $\pi(\mathbf{z})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant.

Bayesian Framework

The solution of the inverse problem within the Bayesian framework is recast in the form of statistical inference from the *posterior probability density*.

Prior is the model for the unknowns that reflects all the uncertainty of the parameters without the information conveyed by the measurements

Likelihood is the measurement model incorporating the related uncertainties, that is "the conditional probability of the measurements given the unknown parameters".

Posterior probability density is the model for the conditional probability distribution of the unknown parameters given the measurements.

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State Estimation Problem

State Estimation Problem

$$\begin{cases} \underline{\textbf{State Evolution Model:}} & \boxed{\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})} \\ \underline{\textbf{Observation Model:}} & \boxed{\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{n}_k)} \end{cases}$$

$\mathbf{x} \in R^{n_x}$ = state variables to be estimated

$\mathbf{u} \in R^{n_p}$ = input variable

$\mathbf{v} \in R^{n_v}$ = state noise

$\mathbf{z} \in R^{n_z}$ = measurements

$\mathbf{n} \in R^{n_n}$ = measurement noise

Subscript k = 1, 2, …, denotes an instant t_k in a dynamic problem

State Estimation Problem

<u>Definition:</u> The state estimation problem aims at obtaining information about the $\mathbf{x}_k$ based on the state evolution model and on the measurements given by the observation model.

State Estimation Problem

State Evolution Model: $\boxed{\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})}$

Observation Model: $\boxed{\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{n}_k)}$

The *evolution-observation model* is based on the following assumptions :

(i) The sequence $\mathbf{x}_k$ for $k = 1, 2, \ldots$, is a Markovian process, that is,

$$\pi(\mathbf{x}_k | \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) = \pi(\mathbf{x}_k | \mathbf{x}_{k-1})$$

(ii) The sequence $\mathbf{z}_k$ for $k = 1, 2, \ldots$, is a Markovian process with respect to the history of $\mathbf{x}_k$, that is,

$$\pi(\mathbf{z}_k | \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_k) = \pi(\mathbf{z}_k | \mathbf{x}_k)$$

(iii) The sequence $\mathbf{x}_k$ depends on the past observations only through its own history, that is,

$$\pi(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{z}_{1:k-1}) = \pi(\mathbf{x}_k | \mathbf{x}_{k-1})$$

State Evolution Problem

State Evolution Model: $\boxed{\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})}$

Observation Model: $\boxed{\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{n}_k)}$

Different problems can be considered:

1. The *prediction problem*, concerned with the determination of $\pi(\mathbf{x}_k | \mathbf{z}_{1:k-1})$;
2. The *filtering problem*, concerned with the determination of $\pi(\mathbf{x}_k | \mathbf{z}_{1:k})$;
3. The *fixed-lag smoothing problem*, concerned the determination of $\pi(\mathbf{x}_k | \mathbf{z}_{1:k+p})$, where $p \geq 1$ is the fixed lag;
4. The *whole-domain smoothing problem*, concerned with the determination of $\pi(\mathbf{x}_k | \mathbf{z}_{1:K})$, where $\mathbf{z}_{1:K} = \{\mathbf{z}_i, i = 1, \ldots, K\}$ is the complete sequence of measurements.

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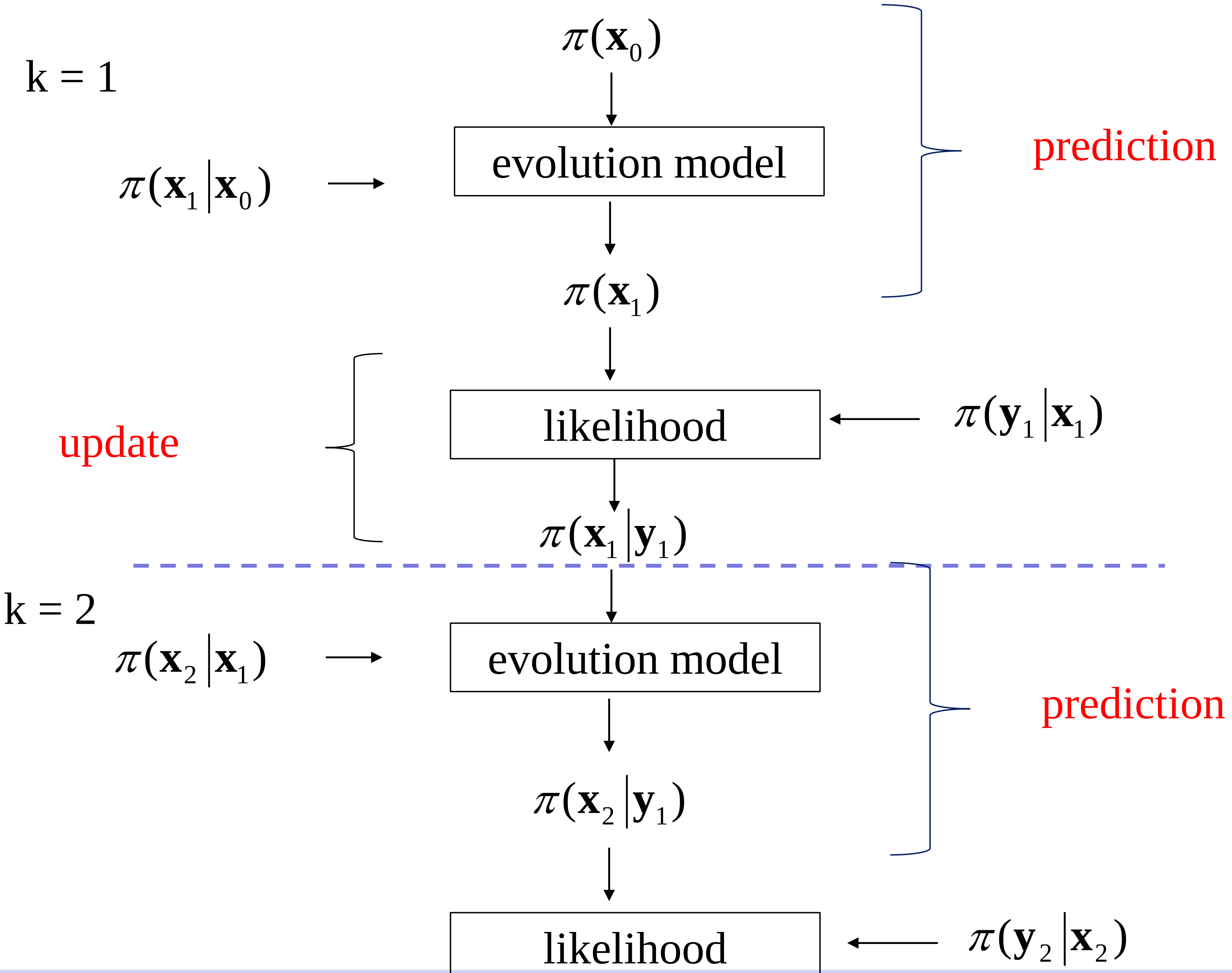
Filtering Problem

Filtering Problem

By assuming that the probability density of the initial state is available $\pi(\mathbf{x}_0 | \mathbf{y}_0) = \pi(\mathbf{x}_0)$, the Markov transition kernels $\pi(\mathbf{x}_k | \mathbf{x}_{k-1})$ and the likelihood functions $\pi(\mathbf{y}_k | \mathbf{x}_k), k = 1, 2, \ldots$ the state is estimated with Bayesian filters in two steps: **prediction and update**


k = 1
$\pi(\mathbf{x}_0)$
$\pi(\mathbf{x}_1|\mathbf{x}_0)$
evolution model
prediction
$\pi(\mathbf{x}_1)$
update
likelihood
$\pi(\mathbf{y}_1|\mathbf{x}_1)$
$\pi(\mathbf{x}_1|\mathbf{y}_1)$
k = 2
$\pi(\mathbf{x}_2|\mathbf{x}_1)$
evolution model
prediction
$\pi(\mathbf{x}_2|\mathbf{y}_1)$
likelihood
$\pi(\mathbf{y}_2|\mathbf{x}_2)$

Problem Statement

- **Goal:** Tracking the state of a system as it evolves in time
- **Information :** Sequential (noisy or ambiguous) observations
- **We want to know:** Best possible estimate of the state variables

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Particle Filtering Methods

Particle Filtering Methods

• Introduced in the 50's, but no much used until 2000's because of limited computational resources.

• Monte-Carlo techniques are the most general and robust for nonlinear and/or non-Gaussian distributions.

• The algorithms are based on Sequential Importance Sampling (SIS) .

•The sequential importance sampling (SIS) algorithm is a Monte Carlo (MC) method that forms the basis for most filters developed over the past decades.

Sequential Importance Sampling (SIS) Algorithm

Bayes' Theorem ⇒ $\pi(\mathbf{x}_k|\mathbf{z}_k) \;\; \alpha \;\; \pi(\mathbf{z}_k|\mathbf{x}_k)\, \pi(\mathbf{x}_k)$

The key idea is to represent the required posterior density function by a set of random samples (particles) with associated weights, and to compute the estimates based on these samples and weights.

$$\pi\left(\mathbf{x}_k \middle| \mathbf{z}_k\right) = \left\{\mathbf{x}_k^i, w_k^i\right\}_{i=1\ldots N}$$

$$w_k^i = \frac{\pi\left(\mathbf{x}_k \middle| \mathbf{z}_k\right)}{q\left(\mathbf{x}_k \middle| \mathbf{z}_k\right)} \Rightarrow w_k^i = w_{k-1}^i \frac{\pi\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right)\pi\left(\mathbf{x}_k \middle| \mathbf{x}_{k-1}^i\right)}{q\left(\mathbf{x}_k \middle| \mathbf{x}_{k-1}^i, \mathbf{z}_k\right)} \Rightarrow w_k^i = w_{k-1}^i \pi\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right)$$

Importance Density

Sequential Importance Sampling (SIS) Algorithm

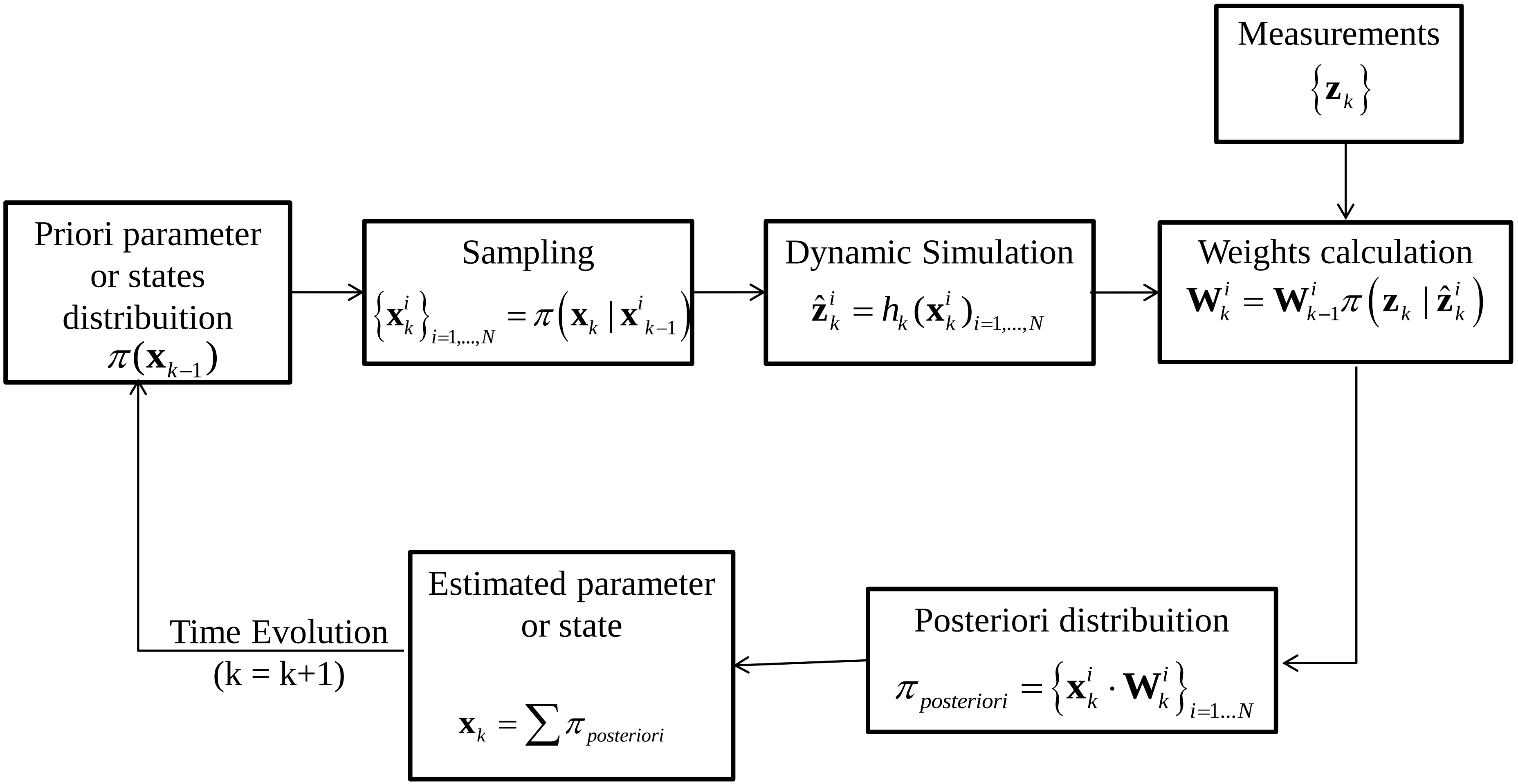
Algorithm 1: SIS Particle Filter

$[\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}] = \text{SIS}[\{\mathbf{x}_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, \mathbf{z}_k]$

- FOR $i = 1: N_s$
  - Draw $\mathbf{x}_k^i \sim q(\mathbf{x}_k | \mathbf{x}_{k-1}^i, \; \mathbf{z}_k)$
  - Assign the particle a weight, w_k^i, according to $\mathbf{w}_k^i = \mathbf{w}_{k-1}^i \pi\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right)$
- END FOR

ARULAMPALAM, M. S., MASKELL, S., GORDON, N. AND CLAPP, T. "**A tutorial on particle filters for online nonlinear/non Gaussian Bayesian tracking**," *IEEE Trans. Signal Processing*, vol. 50, no. 2, pp. 174–188, Feb. 2002.

Sequential Importance Sampling (SIS) Algorithm



Problems the SIS Filter

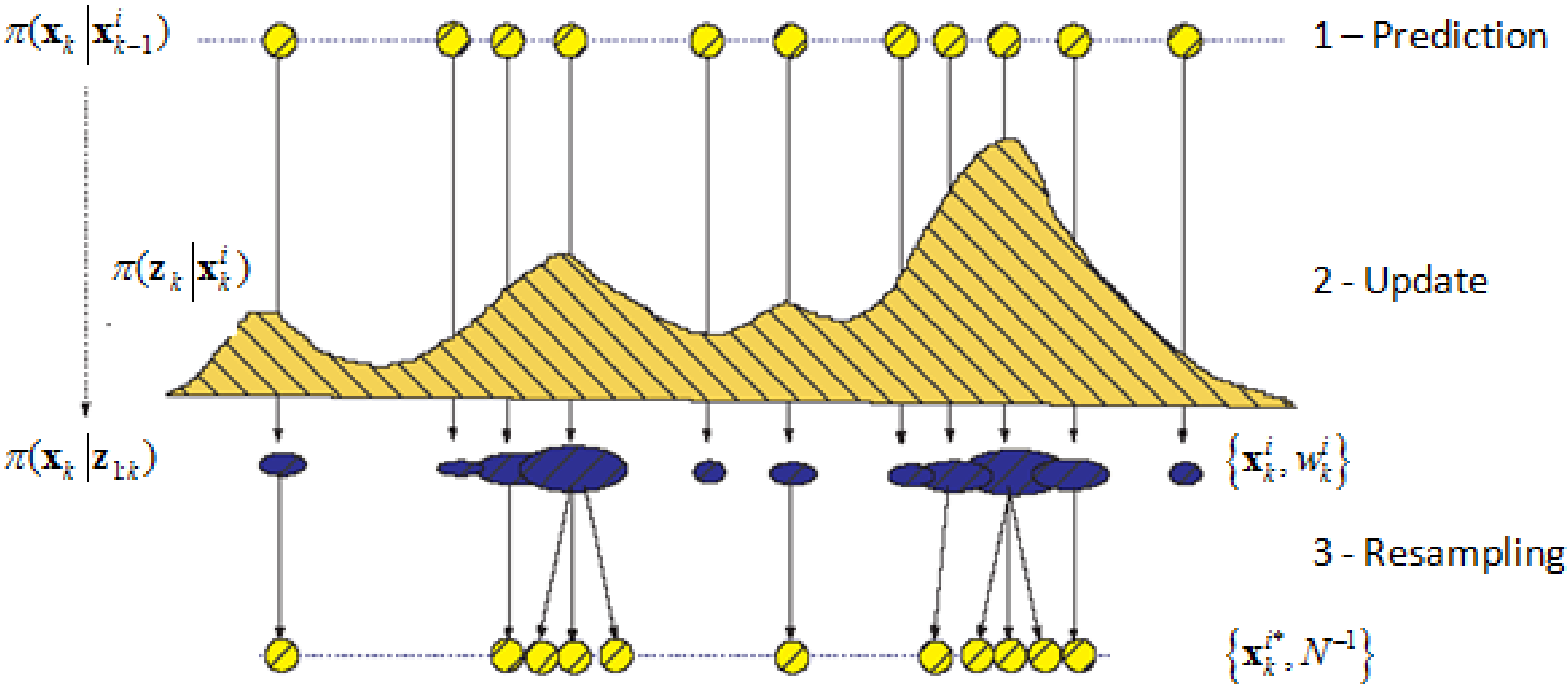
Degeneracy Problem:

- ✓ A common problem with the SIS particle filter is the degeneracy phenomenon, where after a few iterations, few particles will have negligible weight.

- ✓ This degeneracy implies that a large computational effort is devoted for updating particles whose contribution to the approximation to $\pi\left(\mathbf{x}_k \middle| \mathbf{z}_k\right)$ is almost zero.

Sampling Importance Resampling (SIR) Algorithm

(Ristic, B., Arulampalam, S., Gordon, N., 2004, *Beyond the Kalman Filter*, Artech House, Boston)



Sampling Importance Resampling (SIR) Algorithm

(Ristic, B., Arulampalam, S., Gordon, N., 2004, *Beyond the Kalman Filter*, Artech House, Boston)

Although the resampling step reduces the effects of degeneracy, it introduces other practical problems:

- Particles that have high weights are statistically selected many times: Loss of diversity, known as sample impoverishment, specially if the evolution model errors are small.

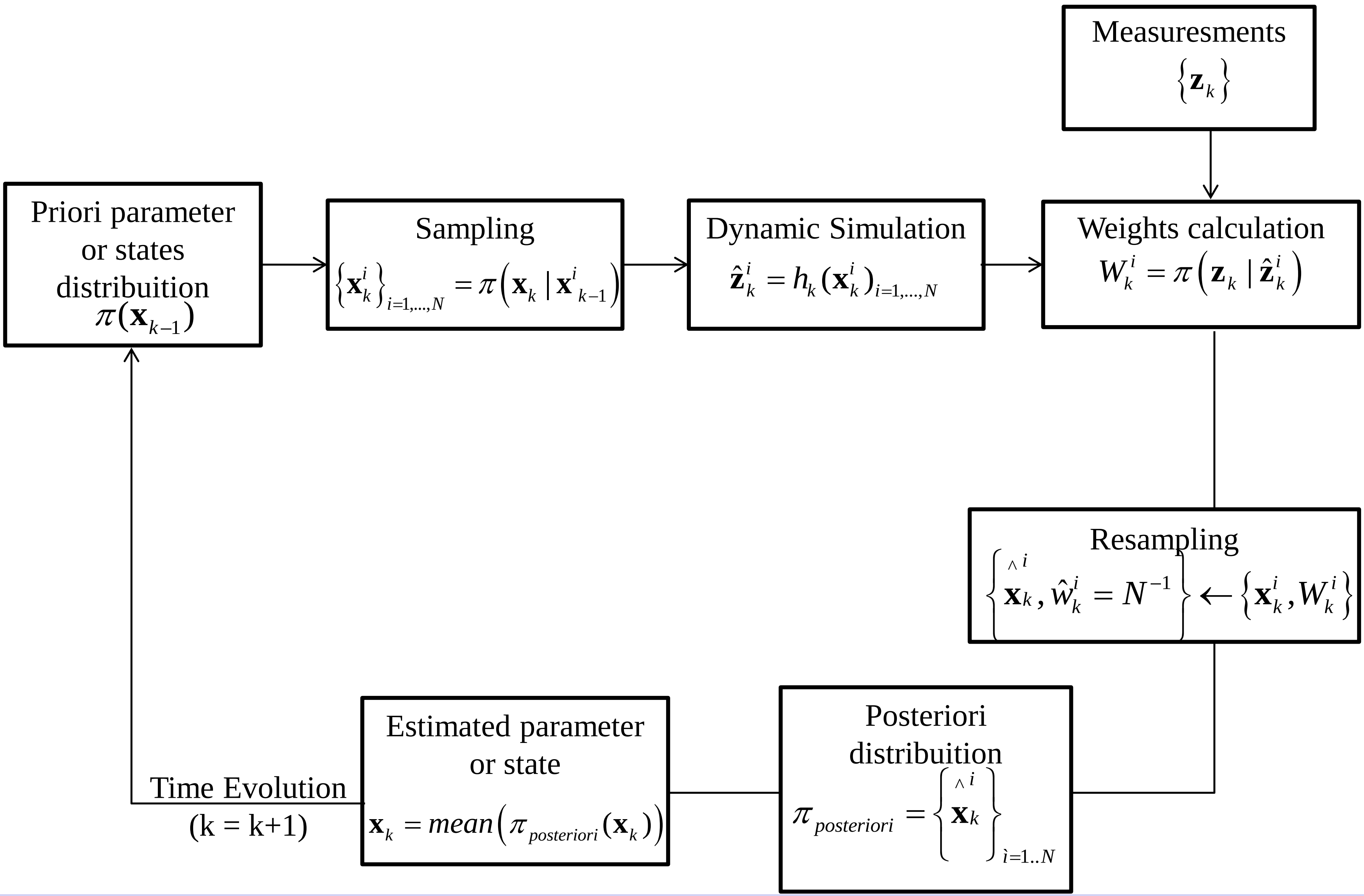
Sampling Importance Resampling (SIR) Algorithm

(Ristic, B., Arulampalam, S., Gordon, N., 2004, *Beyond the Kalman Filter*, Artech House, Boston)

Step 1
For $i=1,\cdots,N$ draw new particles $\mathbf{x}_k^i$ from the prior density $\pi\left(\mathbf{x}_k \middle| \mathbf{x}_{k-1}^i\right)$ and then use the likelihood density to calculate the correspondent weights $w_k^i = \pi\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right)$.
Step 2
Calculate the total weight $T_w = \sum_{i=1}^{N} w_k^i$ and then normalize the particle weights, that is, for $i=1,\cdots,N$ let $w_k^i = T_w^{-1} w_k^i$
Step 3
Resample the particles as follows : Construct the cumulative sum of weights (CSW) by computing $c_i = c_{i-1} + w_k^i$ for $i=1,\cdots,N$, with $c_0 = 0$. Let $i=1$ and draw a starting point u_1 from the uniform distribution $U\left[0, N^{-1}\right]$ For $j=1,\cdots,N$ Move along the CSW by making $u_j = u_1 + N^{-1}\left(j-1\right)$ While $u_j > c_i$ make $i=i+1$. Assign sample $x_k^j = x_k^i$ Assign sample $w_k^j = N^{-1}$

- Weights are easily evaluated and importance density easily sampled.
- Importance of sampling density is independent of the measurements at current time. The filter can be sensitive to outliers.
- Resampling is applied in every iteration, which can result in fast loss of diversity of the particles.

Sampling Importance Resampling (SIR) Algorithm



Improvements to SIR

- Variety of resampling schemes with different performance in terms of the variance of the particles $\mathrm{var}(N_i)$:
  - **The cumulative sum of weights (CSW) (Arulampalan et al, 2002*)***
  - Residual sampling (Liu & Chen, 1998).
  - Systematic sampling (Carpenter *et al.*, 1999).
  - Mixture of SIS and SIR, only resample when necessary (Liu & Chen, 1995; Doucet *et al.*, 1999).
- Degeneracy and Impoverishment may still be a problem:
  - During resampling a sample with high importance weight may be duplicated many times.
  - Samples may eventually collapse to a single point.

Improvements to SIR

- Local Monte Carlo methods for alleviating sample impoverishment:
  - Local linearization - using an EKF (Doucet, 1999; Pitt & Shephard, 1999) or **UKF** (Doucet *et al*, 2000) to estimate the importance distribution.
  - Rejection methods (Müller, 1991; Doucet, 1999; Pitt & Shephard, 1999).
  - **Auxiliary particle filters** (Pitt & Shephard, 1999)
  - Kernel smoothing (Gordon, 1994; Hürzeler & Künsch, 1998; Liu & West, 2000; Musso *et al.*, 2000).
  - MCMC methods (Müller, 1992; Gordon & Whitby, 1995; Berzuini *et al.*, 1997; Gilks & Berzuini, 1998; Andrieu *et al.*, 1999).
  - **Move-Reweighting** (Marques *et al.*, 2017)

Ingredients for SMC

- Importance sampling function
  - Gordon *et al* → $p(x_k \mid x^i_{0:k-1}, D_k)$
  - Optimal → $p(x_k \mid x^i_{k-1})$
  - UKF → pdf from UKF at x^i_{k-1}
- Redistribution scheme (Resampling)
  - Gordon *et al* → SIR
  - Liu & Chen → Residual
  - Carpenter *et al* → Systematic
  - Liu & Chen, Doucet *et al* → Resample when necessary
- Careful initialization procedure (for efficiency)

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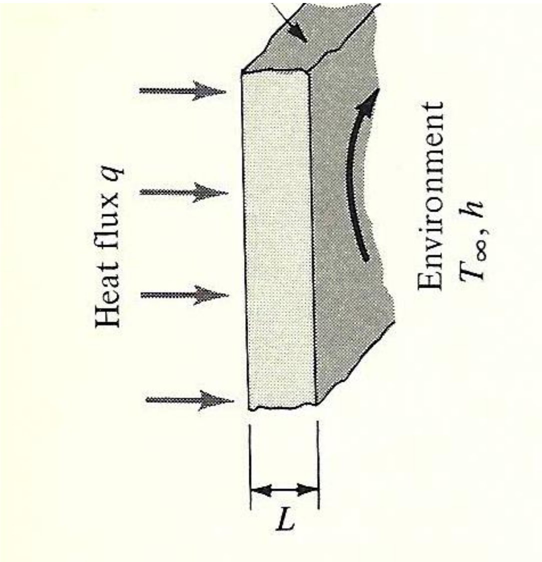

$$\frac{d\theta(t)}{dt} + m\theta(t) = \frac{mq(t)}{h} \qquad \text{for } t > 0$$

$$\theta = \theta_0 \qquad \text{for } t = 0$$

where

$$\theta(t) = T(t) - T_\infty$$

$$\theta_0 = T_0 - T_\infty$$

$$m = \frac{h}{\rho c L}$$

Consider a slab of thickness L, initially at the uniform temperature T_0, which is subjected to a uniform heat flux $q(t)$ over one of its surfaces, while the other exchanges heat by convection and linearized radiation with a medium at a temperature.

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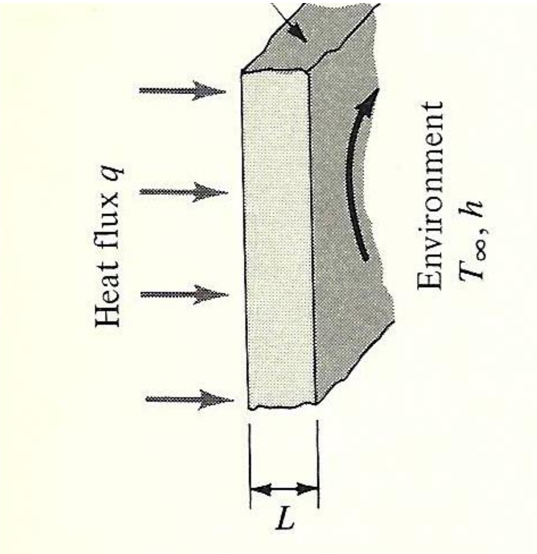

$$\frac{d\theta(t)}{dt} + m\theta(t) = \frac{mq(t)}{h} \qquad \text{for } t > 0$$

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where

$$\theta(t) = T(t) - T_\infty$$

$$\theta_0 = T_0 - T_\infty$$

$$m = \frac{h}{\rho c L}$$

Two illustrative cases are now examined, namely: (i) Heat Flux $q(t) = q_0$ constant and deterministically known; and (ii) Heat Flux $q(t) = q_0 f(t)$ with unknown time variation. Results are obtained for these two cases, by assuming that the plate is made of aluminum (r = 2707 kgm^{-3}, c = 896 Jkg^{-1}K^{-1}), with thickness L = 0.03 m, q_0 = 8000 Wm^{-2}, T_∞=20 ^{o}C, h = 50 Wm^{-2}K^{-1} and T_0 = 50 ^{o}C ·

ORLANDE, H. R. B. ; COLACO, M. J. ; DULIKRAVICH, G. S ; VIANNA, F. L. V. ; SILVA. W. B. ; FONSECA, H. M. ; FUDYM, O. . Kalman and Particle filters. In: METTI V - Thermal Measurements and Inverse Techniques, 2011, Roscoff. METTI V - Thermal Measurements and Inverse Techniques, 2011.

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Heat Flux $q(t) = q_0$ constant and deterministically known

The analytical solution for this problem is given by:

$$\theta(t) = \theta_0 e^{-mt} + \frac{q_0}{h}(1 - e^{-mt})$$

The only state variable in this case is the temperature $\theta(t_k) = \theta_k$ since the applied heat flux q_0 is constant and deterministically known, as the other parameters appearing in the formulation. By using a forward finite-differences approximation for the time derivative in equation , we obtain:

$$\theta_k = (1 - m\Delta t)\theta_{k-1} + \frac{mq_0}{h}\Delta t$$

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Heat Flux $q(t) = q_0 f(t)$ with unknown time variation

The analytical solution for this problem is given by:

$$\theta(t) = e^{-mt}\left\{\theta_0 + \frac{mq_0}{h}\int_{t'=0}^{t} e^{mt'} f(t')\,dt'\right\}$$

In this case, the state variables are given by the temperature $\theta(t_k) = \theta_k$ and the function that gives the time variation of the applied heat flux, that is, $f(t_k) = f_k$ .As in the case examined above, the applied heat flux q_0 is constant and deterministically known, as the other parameters appearing in the formulation.

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TOY EXAMPLE

Heat Flux $q(t) = q_0 f(t)$ with unknown time variation

The analytical solution for this problem is given by:

$$\theta(t) = e^{-mt}\left\{\theta_0 + \frac{mq_0}{h}\int_{t'=0}^{t} e^{mt'} f(t')\,dt'\right\}$$

By using a forward finite-differences approximation for the time derivative in equation, we obtain the equation for the evolution of the state variable $\theta(t_k) = \theta_k$:

$$\theta_k = (1 - m\Delta t)\theta_{k-1} + \left(\frac{mq_0}{h}\Delta t\right) f_{k-1}$$

A random walk model is used for the state variable $f(t_k) = f_k$, which is given in the form: $f_k = f_{k-1} + \varepsilon_{k-1}$

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For the cases studied here, two different functions were examined for the time variation of the applied heat flux, specifically, a step function in the form:

$$f(t) = \begin{cases} 1, & 0 < t \leq \dfrac{t_{final}}{2} \\ 0, & \dfrac{t_{final}}{2} < t < t_{final} \end{cases}$$

and a ramp function in the form

$$f(t) = t \big/ t_{final}$$

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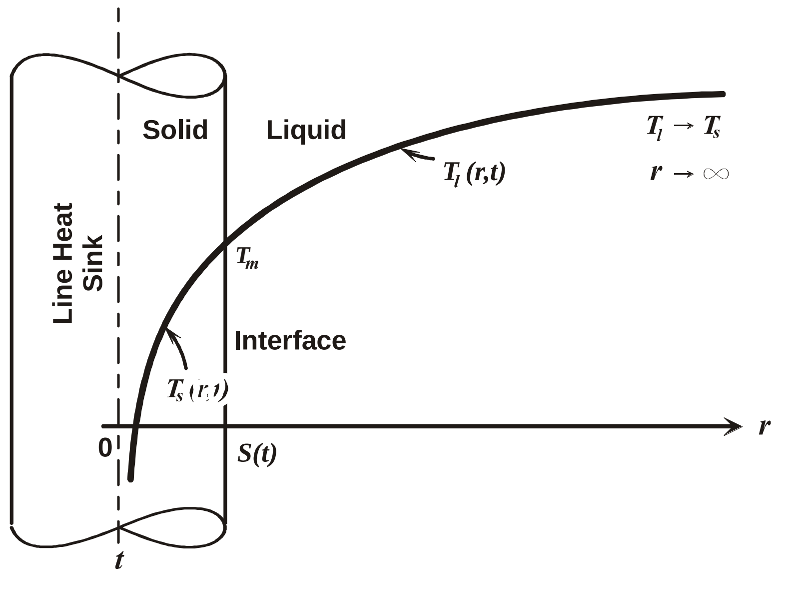
APPLICATIONS

APPLICATIONS

Estimation of the location of the solidification front and the intensity of a line heat sink



Silva, W. B. Orlande, H. R. B.; Colaço, M. J., Fudym, O., 2011, Application Of Bayesian Filters To A One-Dimensional Solidification Problem, *21st Brazilian Congress of Mechanical Engineering*, Natal, RN, Brazil 45

APPLICATIONS

The mathematical formulation for the solid phase is given as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s(r,t)}{\partial r}\right)=\frac{1}{\alpha_s}\frac{\partial T_s(r,t)}{\partial t} \quad in \quad 0<r<S(t) \quad and \quad t>0 \tag{1.a}$$

while the liquid phase is described as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_l(r,t)}{\partial r}\right)=\frac{1}{\alpha_l}\frac{\partial T_l(r,t)}{\partial t} \quad in \quad S(t)<r<\infty \quad and \quad t>0 \tag{1.b}$$

$$T_l(r,t)\to T_i \quad in \quad r\to\infty \quad and \quad t>0 \tag{1.c}$$

$$T_l(r,t)=T_i \quad in \quad t=0 \quad and \quad r>0 \tag{1.d}$$

At the interface between liquid and solid phases, the following conditions must be satisfied

$$T_s(r,t)=T_l(r,t)=T_m \quad in \quad r=S(t) \quad and \quad t>0 \tag{1.e}$$

$$k_s\frac{\partial T_s(r,t)}{\partial r}-k_l\frac{\partial T_l(r,t)}{\partial r}=\rho L\frac{\partial S(t)}{\partial t} \quad in \quad r=S(t) \quad and \quad t>0 \tag{1.f}$$

APPLICATIONS

An analytical solution of this problem can be obtained for this physical problem and it is given by [8]:

$$T_s\left(r,t\right)=T_m+\frac{Q_s}{4\pi k_s}\left[E_i\left(\frac{-r^2}{4\alpha_s t}\right)-E_i\left(-\lambda^2\right)\right]\quad 0<r<S(t) \tag{2.a}$$

$$T_l\left(r,t\right)=T_i-\frac{\left(T_i-T_m\right)}{E_i\left(\frac{-\lambda^2\alpha_s}{\alpha_l}\right)}\left[E_i\left(\frac{-r^2}{4\alpha_s t}\right)\right]\quad S(t)<r<\infty \tag{2.b}$$

where the eigenvalues λ and the solidification front S (t) are given by

$$\frac{Q}{4\pi k_s}e^{-\lambda^2}+\frac{k_l\left(T_i-T_m\right)}{E_i\left(\frac{-\lambda^2\alpha_s}{\alpha_l}\right)}e^{\frac{-\lambda^2\alpha_s}{\alpha_l}}=-\lambda^2\alpha_s\rho L \tag{3.a}$$

$$S(t)=2\lambda\sqrt{\alpha_s t} \tag{3.b}$$

In the above equations T_i is the uniform initial temperature, T_m is the melting temperature of the material, L is the latent heat of solidification of the material, ρ is the density, k_s and k_l are the thermal conductivities of the solid and liquid phases, respectively, α_s and α_l are the thermal diffusivities of the solid and liquid phases, respectively, and T_s and T_l are temperatures of the solid and liquid phases, respectively.

APPLICATIONS

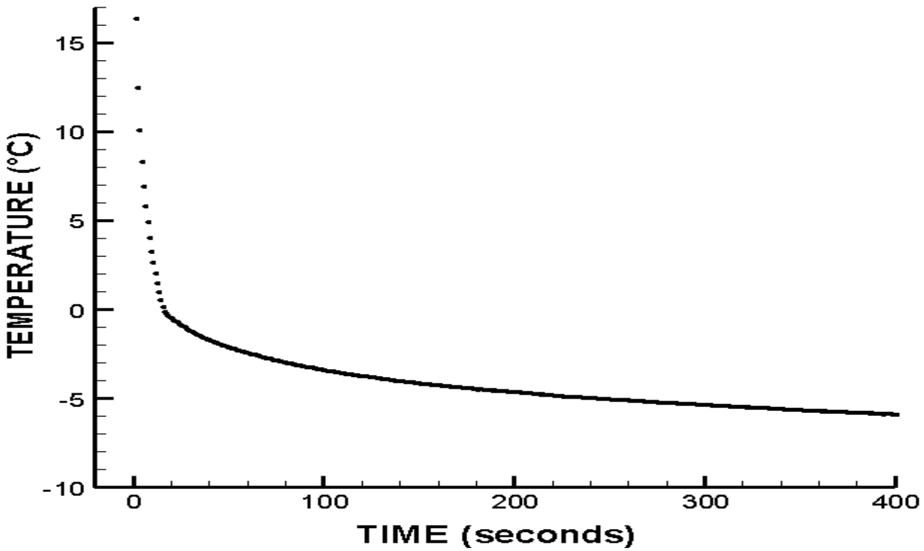
Estimation of the location of the solidification front and the intensity of a line heat sink

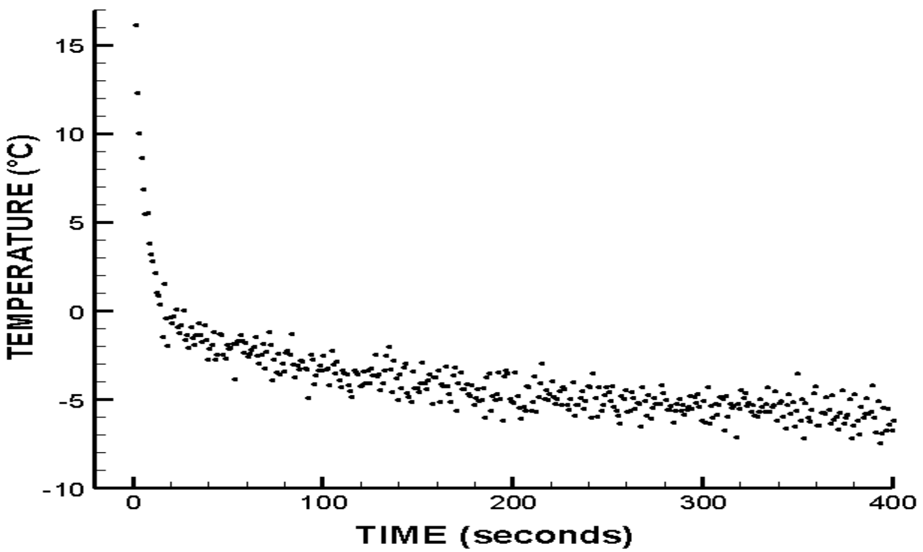
The physical problem defined by Eqs. (1.a-f) was solved analytically, where we used the following data, corresponding to solidifying water: $T_i = 25°C$, $T_m = 0°C$, $\alpha_s = 0.00118\frac{m^2}{s}$, $\alpha_l = 0.000146\frac{m^2}{s}$, $k_s = 2.22\frac{w}{m°c}$, $k_l = 0.61\frac{w}{m°c}$, $\rho = 997.1\frac{kg}{m^3}$, $L = 80\ \frac{\mathrm{J}}{\mathrm{kg}}$. The line heat sink was supposed to have a constant value equals to $\mathrm{Q} = 50\frac{\mathrm{W}}{\mathrm{m}}$. In this work, the measurements (for the observation model) were obtained at *r*=0.01 m. The simulated noisy measurements were uncorrelated, additive, Gaussian, with zero mean and constant standard deviation equal to 5% of the maximum temperature. Figures 3.a,b show the transient measurements obtained after applying such constant line heat sink**,** with and without errors, respectively.

Silva, W. B. Orlande, H. R. B.; Colaço, M. J., Fudym, O., 2011, Application Of Bayesian Filters To A One-Dimensional Solidification Problem, *21st Brazilian Congress of Mechanical Engineering*, Natal, RN, Brazil.

APPLICATIONS

Estimation of the location of the solidification front and the intensity of a line heat sink





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Estimation of the location of the solidification front and the intensity of a line heat sink

Bayesian filter	**Number of Particles (NP)**	**Time**	**RMS error for the solidification front (m)**	**RMS error for the line heat sink intensity (W/m)**
SIR	100	0.008 min.	$9x10^{-3}$	1.55
SIR	1000	0.997 min.	$2x10^{-3}$	1.78
SIR	5000	11.047 min.	$1x10^{-4}$	0.34
ASIR	100	0.161 min.	$7.9x10^{-5}$	0.15

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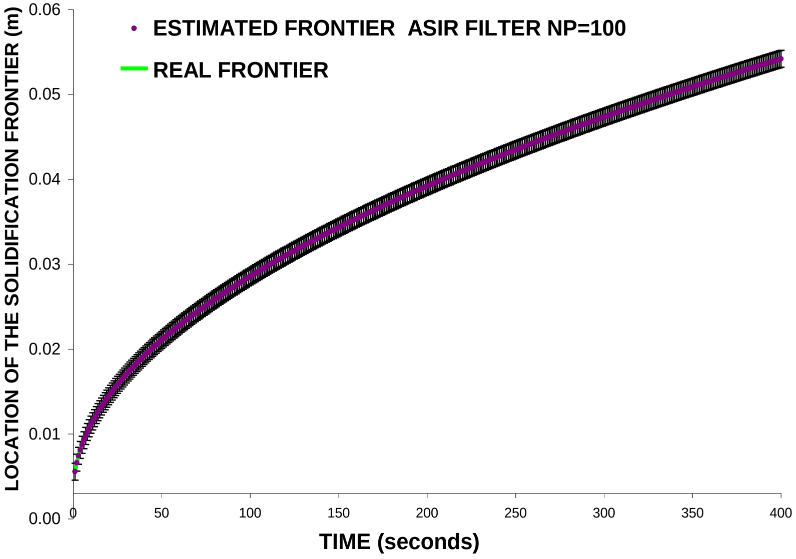
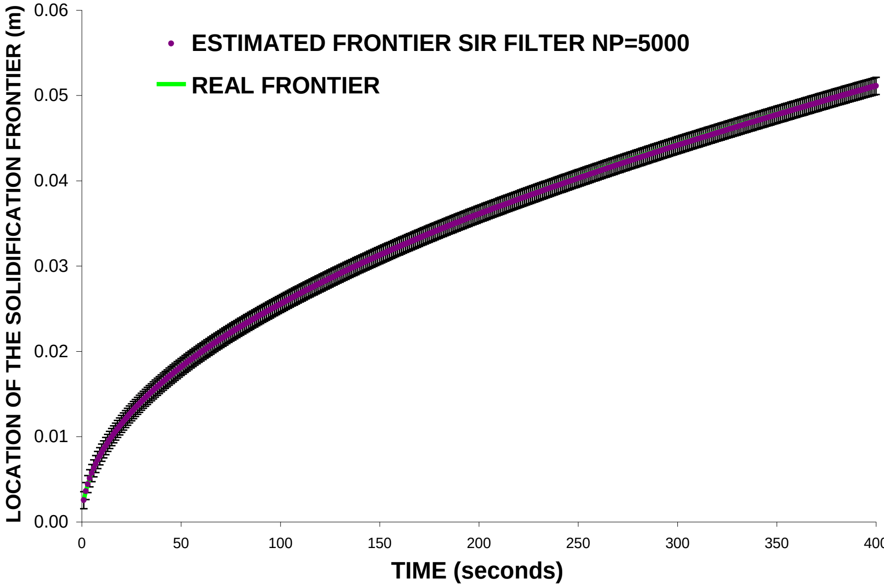
APPLICATIONS

Estimation of the location of the solidification front





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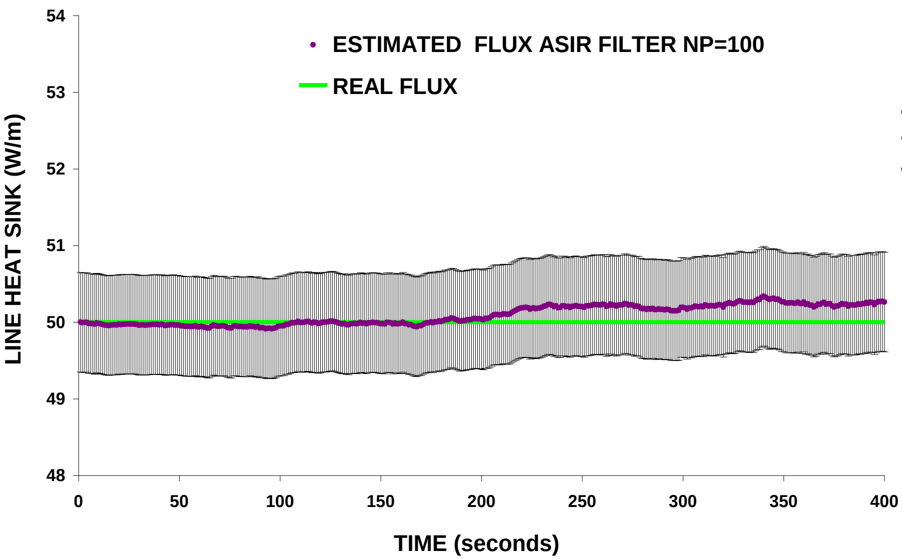
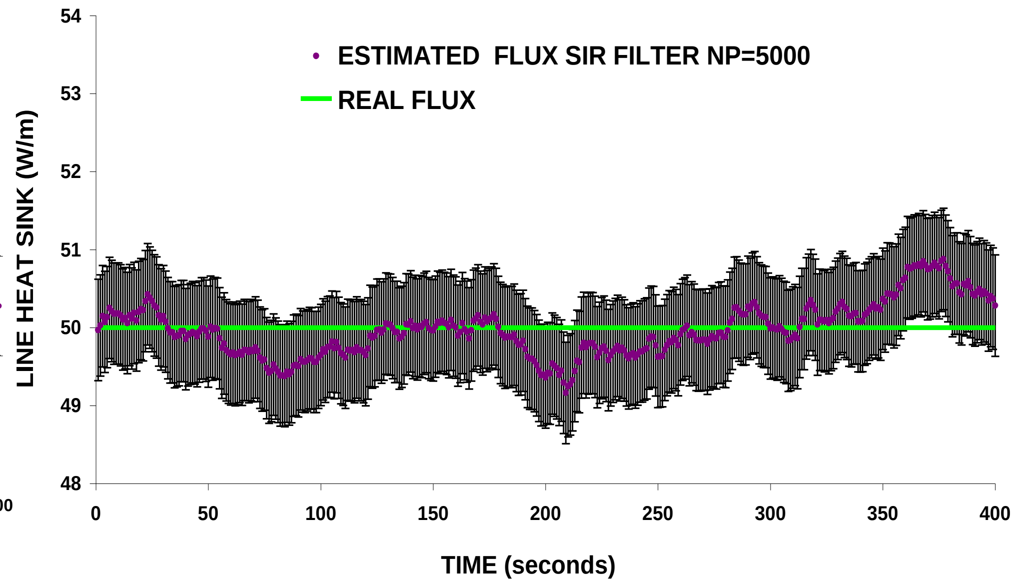
APPLICATIONS

Estimation of of a line heat sink





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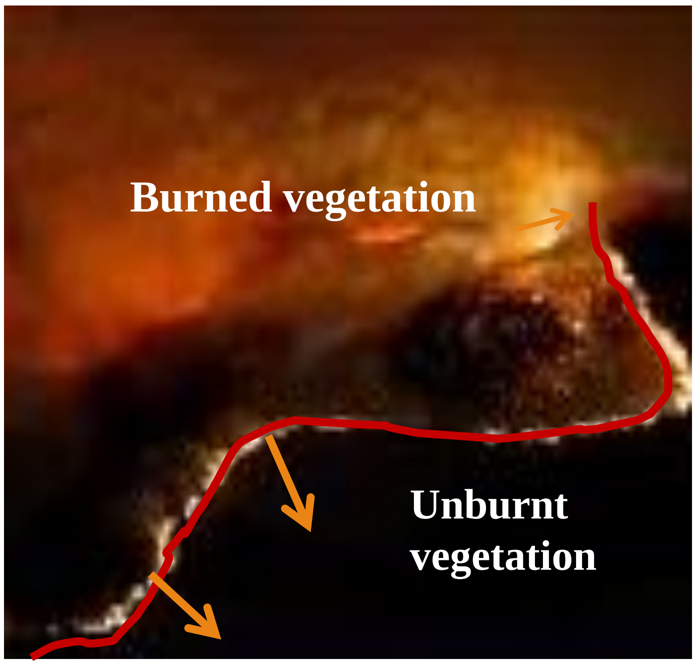
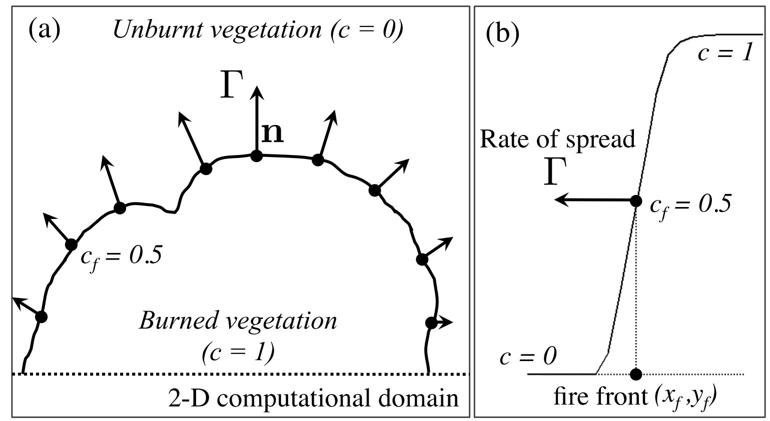
APPLICATIONS

PARTICLE FILTERS TO REGIONAL-SCALE WILDFIRE SPREAD





$$\frac{\partial c}{\partial t} + \vec{R}.\vec{\nabla} c = 0, \qquad \forall x, y \in \Omega, t \in \left[0, t_f\right]$$

$$c(x, y, 0) = c_0(x, y) \quad , \qquad \forall x, y \in \Omega$$

$$\nabla c(x, y, t) \cdot \bar{n}_{\partial\Omega}(x, y) = 0 \quad , \forall x, y \in \partial\Omega$$

$$R = R\left(\beta, \Sigma, M_f, \delta(x, y), U(x, y, t), \tan(\varphi)\right)$$

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PARTICLE FILTERS TO REGIONAL-SCALE WILDFIRE SPREAD

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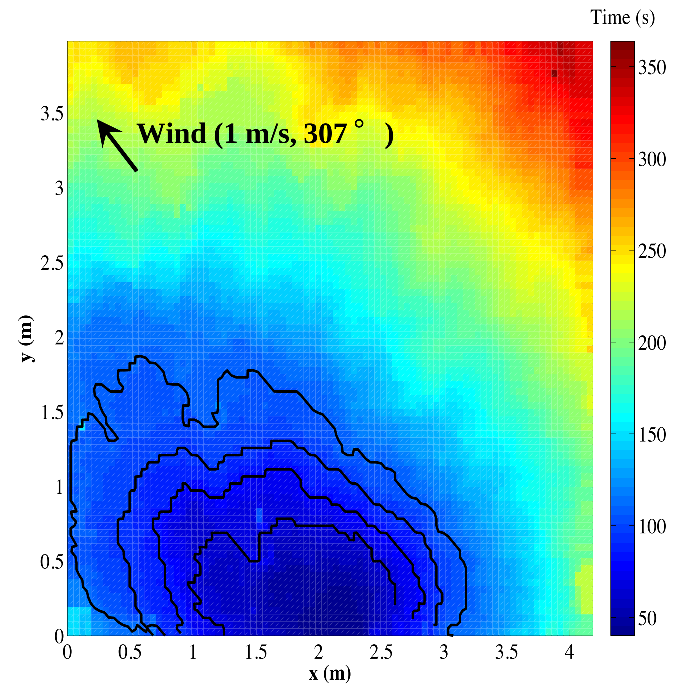

Arrival time of the fire front (in color), and observed fire fronts separated by *14 s* (at *t = 64 s, 78 s, 92 s, 106 s*) in black solid lines.

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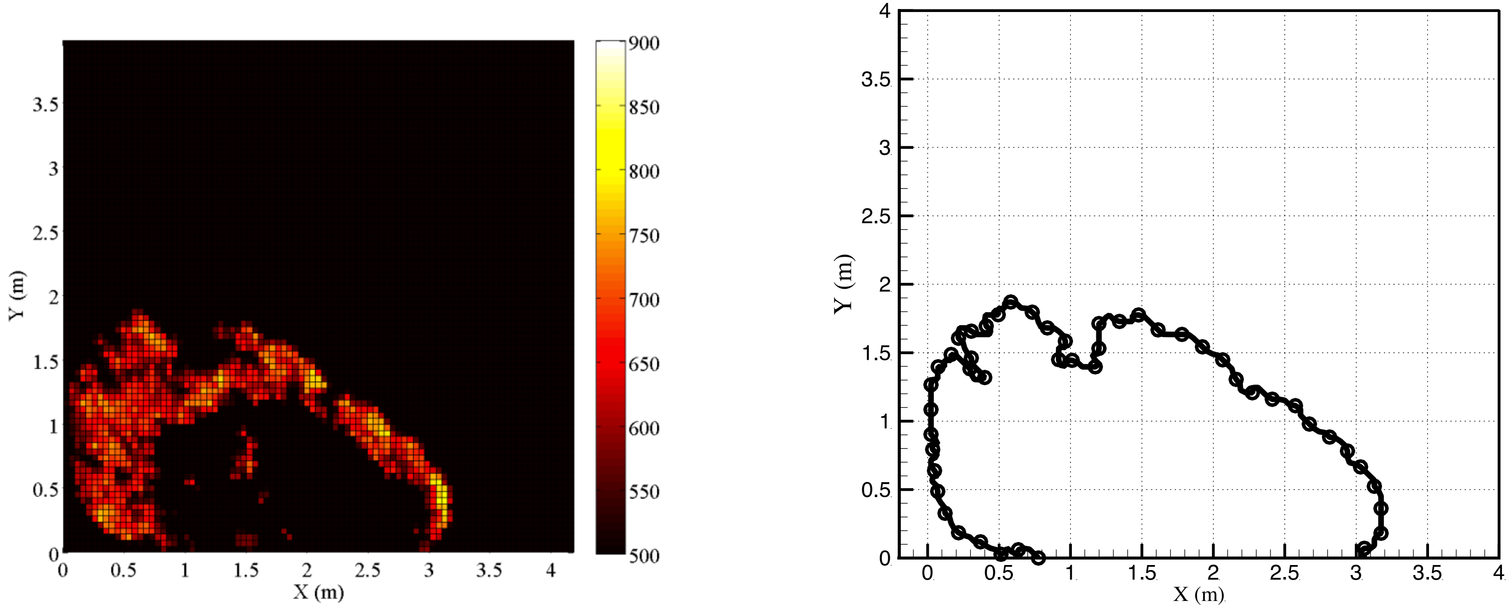

Extraction of the fire front location (right) from the thermal infrared image (left) at *t* = *106 s*; the fire front is identified as the isocontour where the temperature reaches *600 K*.

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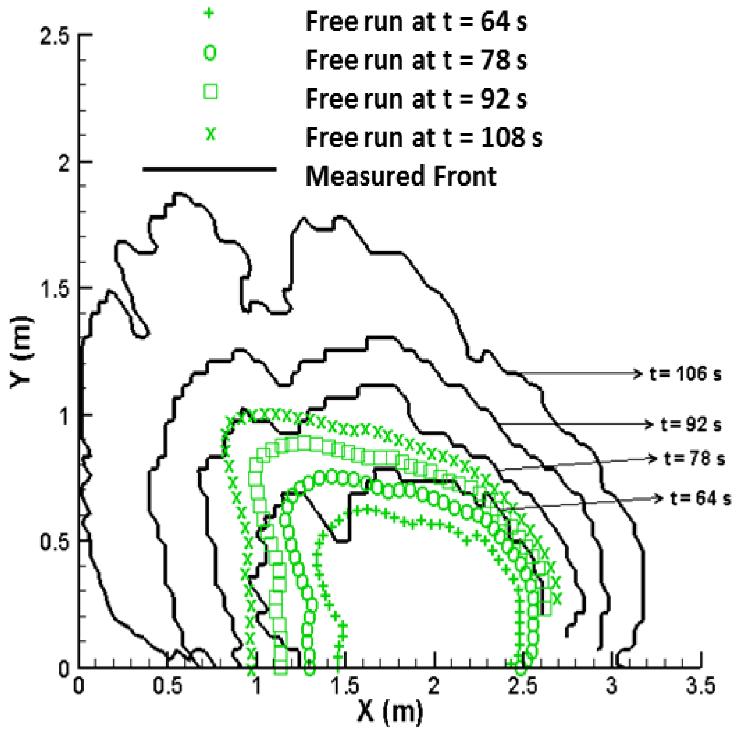

Comparison between the direct simulation (free run) and the measured fire front positions from t = 64 s to t = 106 s. Observations are represented in black solid lines, simulated fire fronts associated with the prior PDF of the control vector at t = 50 s are represented in green symbols.

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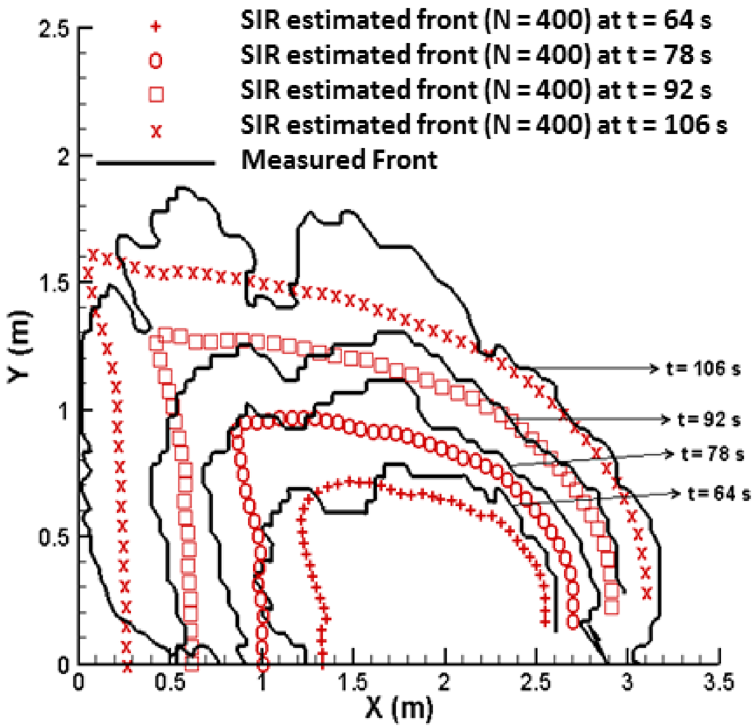

Comparison between simulated and measured fire front positions from t = 64 s to t = 106 s using the SIR filter 400 particles. Observations are represented in black solid lines; simulated fire fronts associated with the posterior PDF of the control vector are represented in red symbols.

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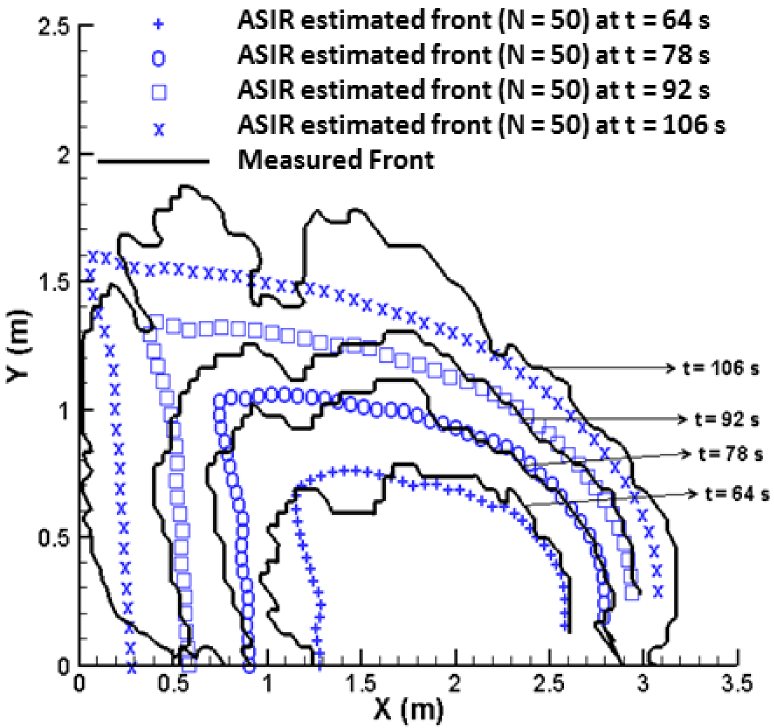

Comparison between simulated and measured fire front positions from t = 64 s to t = 106 s using the ASIR filter 50 particles. Observations are represented in black solid lines; simulated fire fronts associated with the posterior PDF of the control vector are represented in blue symbols.

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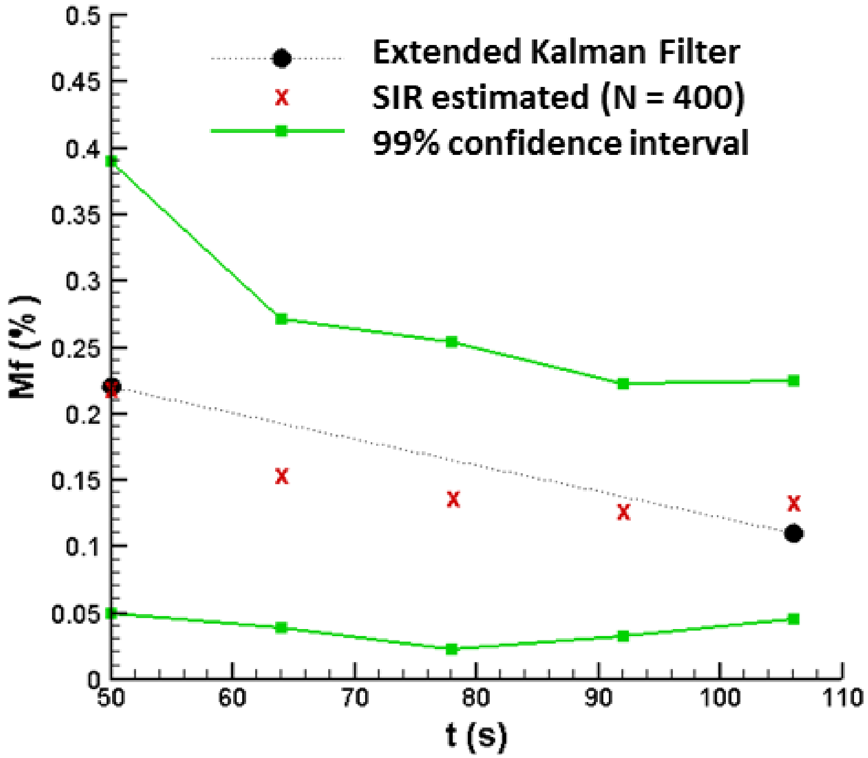

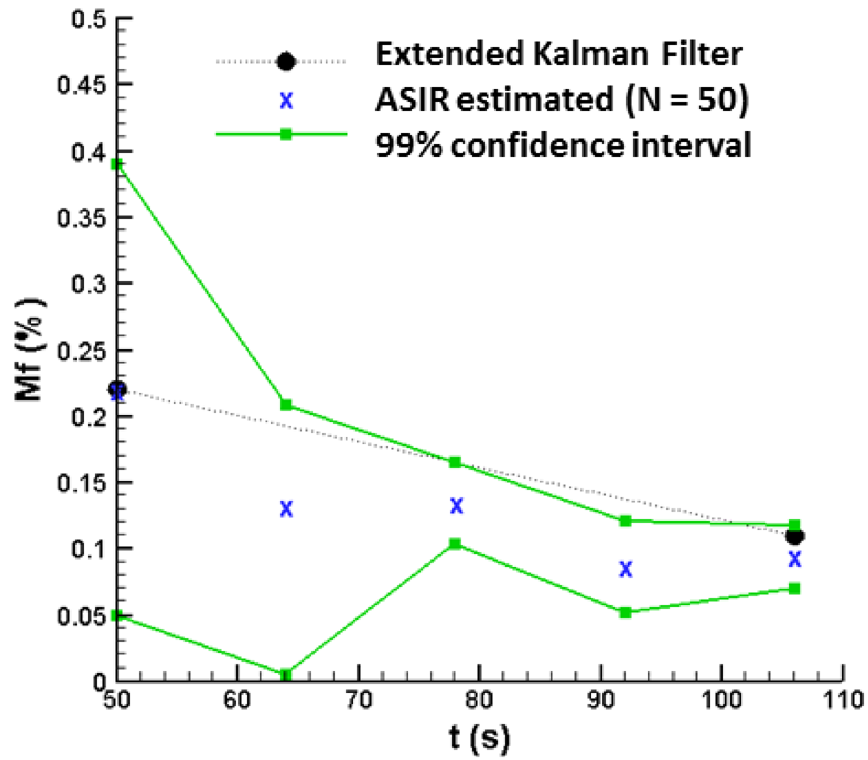

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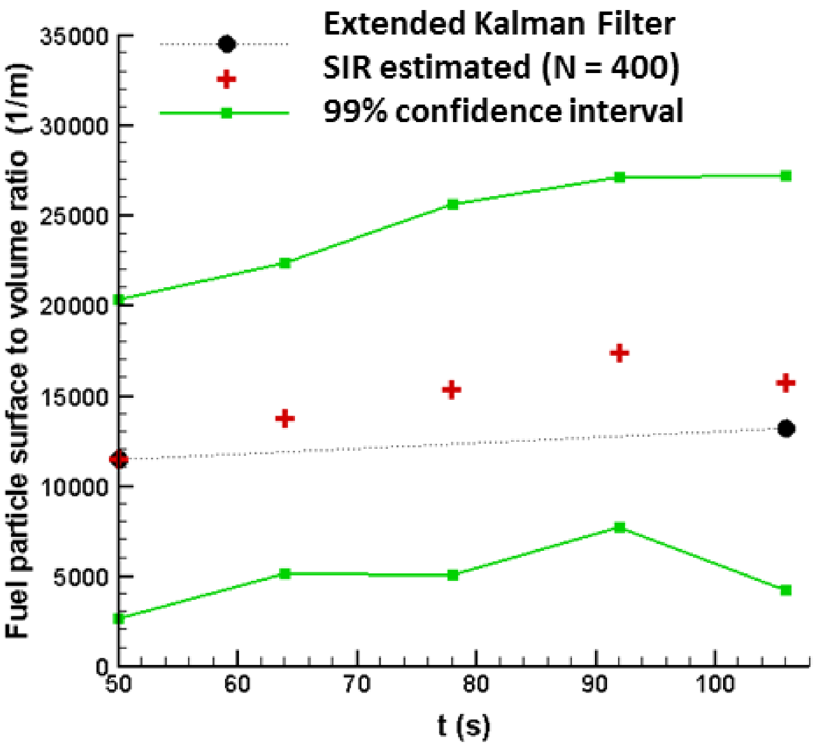

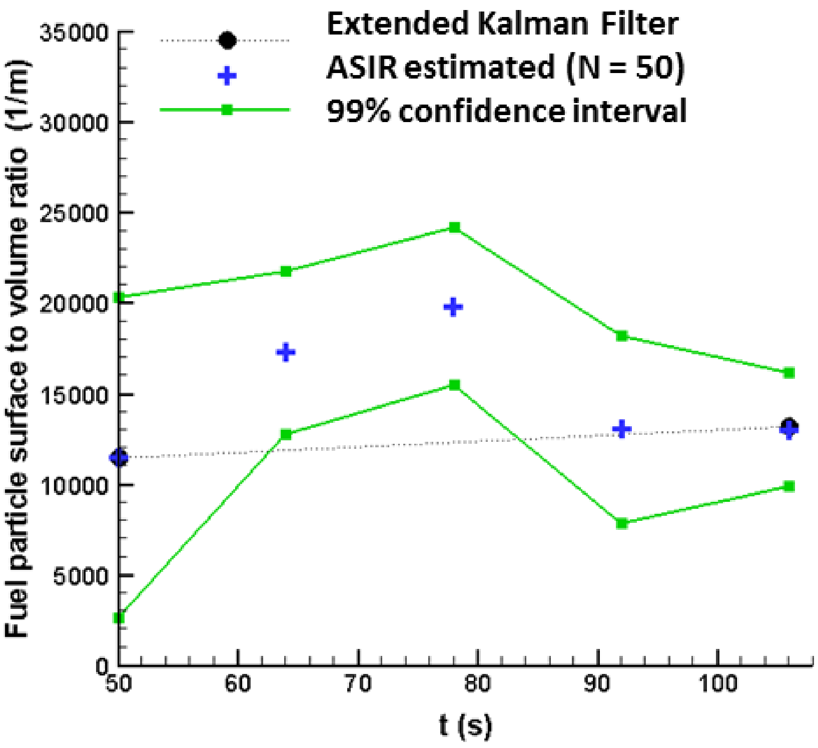

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APPLICATIONS

PARTICLE FILTERS TO REGIONAL-SCALE WILDFIRE SPREAD

$$R = R_0\left(1 + \phi_W + \phi_S\right)$$

Rate of spread

$$\phi_S = 5.275 * \beta^{-0.3}\left(\tan\varphi\right)^2$$

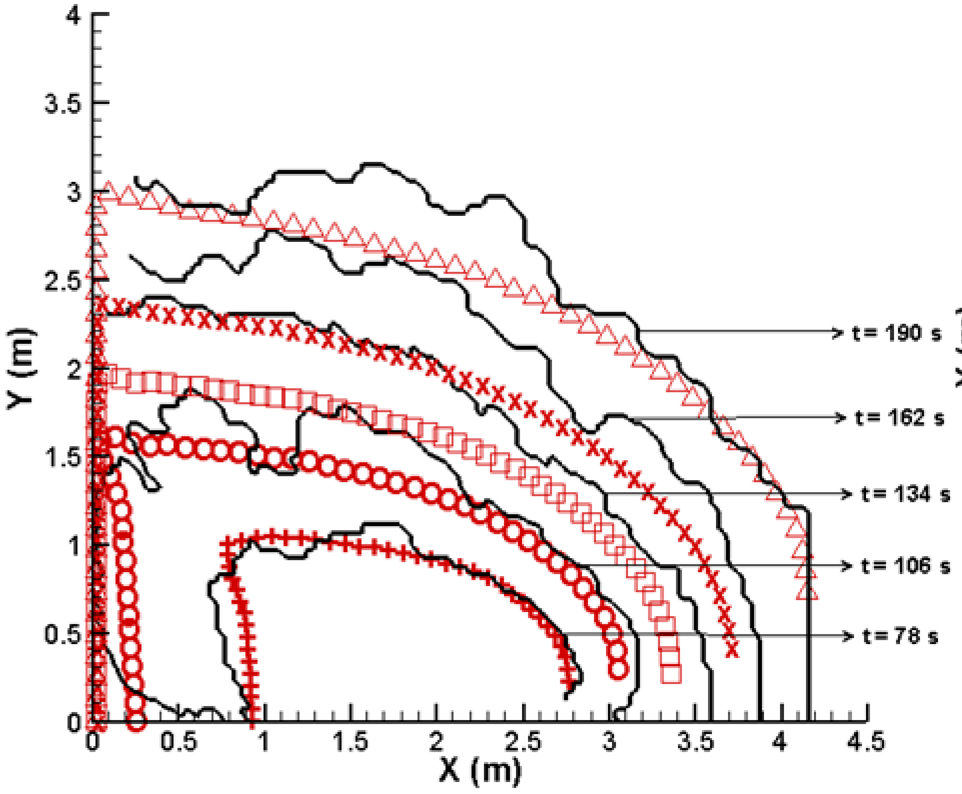
Elevation factor

$$\left.\begin{aligned}
\phi_W &= C U^B \left(\frac{\beta}{\beta_{op}}\right)^{-E} \\
C &= 7.47 * \exp\left(-0.133\Sigma^{0.55}\right) \\
B &= 0.02526 * \Sigma^{0.54} \\
E &= 0.715 * \exp\left(-3.59 * 10^{-4}\Sigma\right)
\end{aligned}\right\}$$

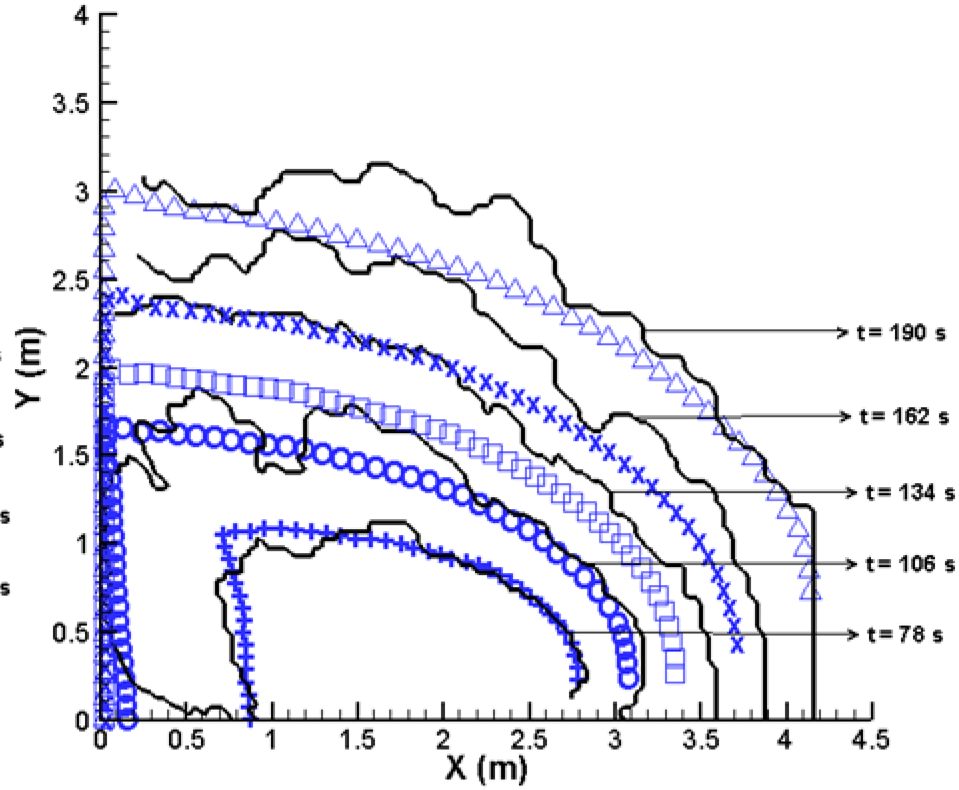
Wind factor

APPLICATIONS

PARTICLE FILTERS TO REGIONAL-SCALE WILDFIRE SPREAD



SIR filter 200 particles.



ASIR filter 50 particles.

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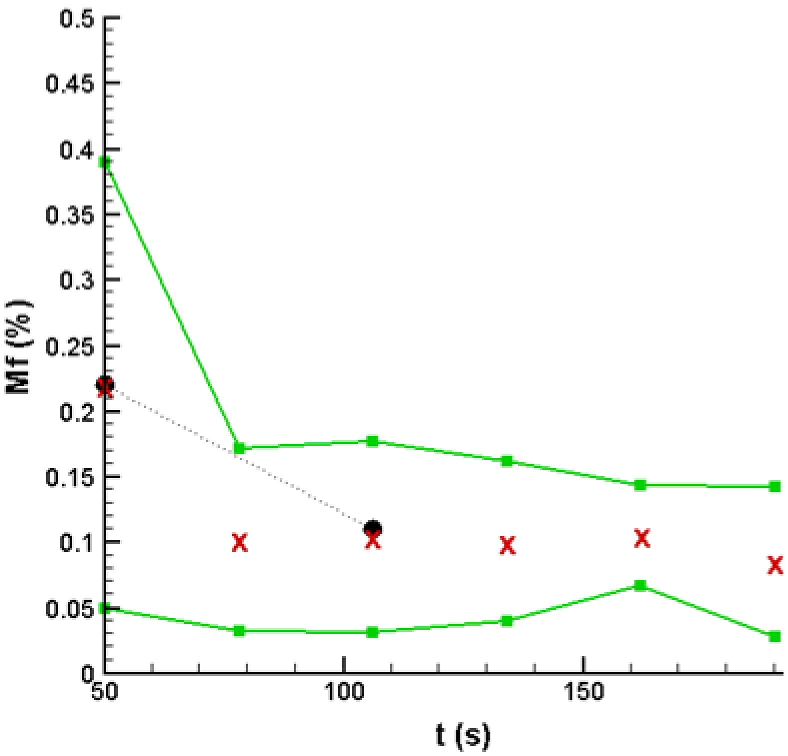
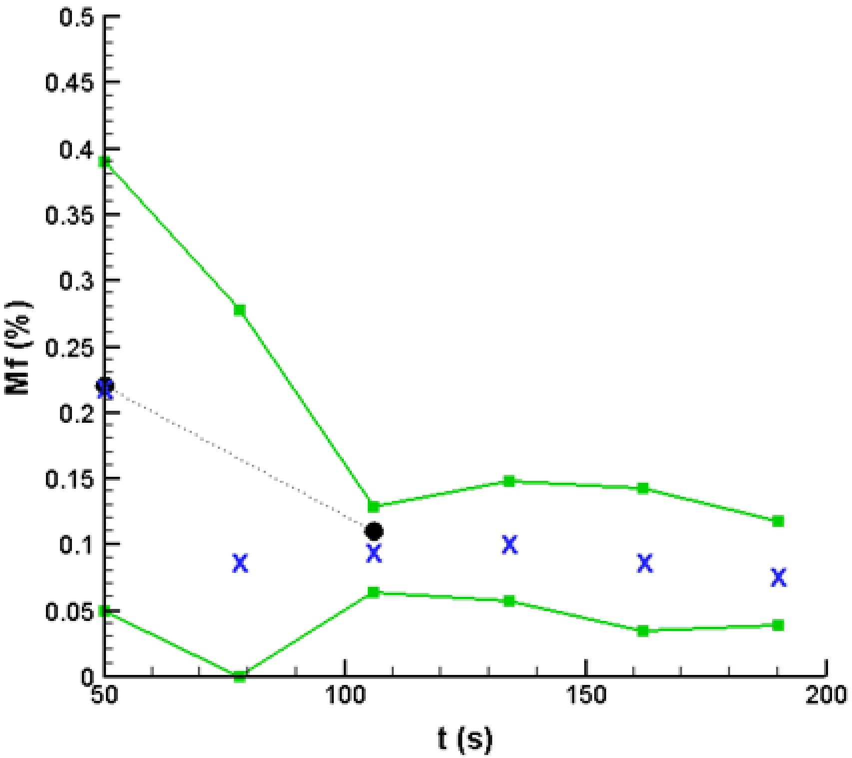


SIR filter 200 particles.



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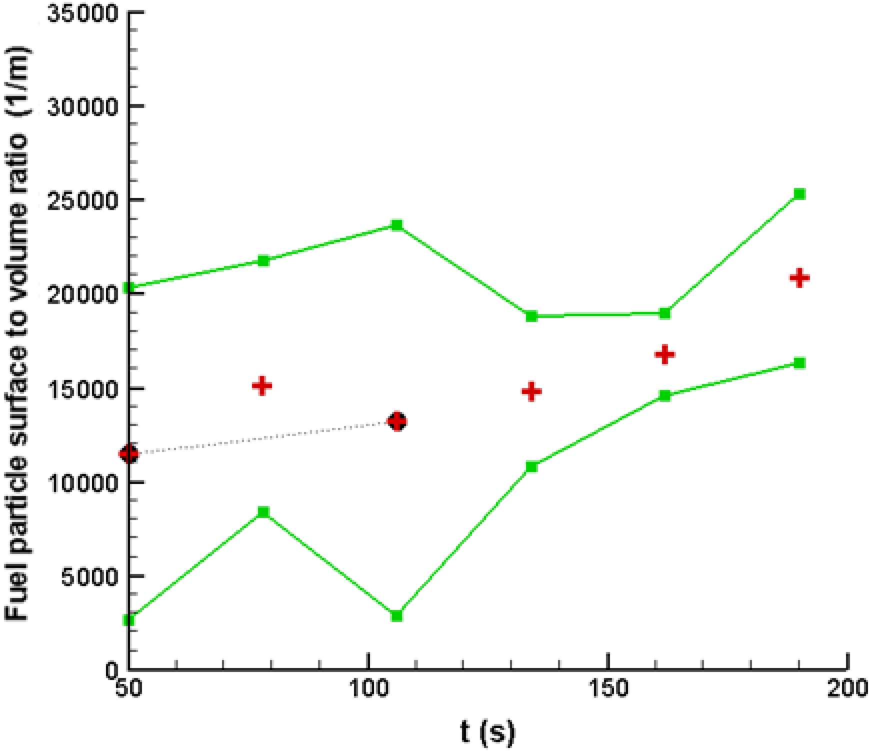
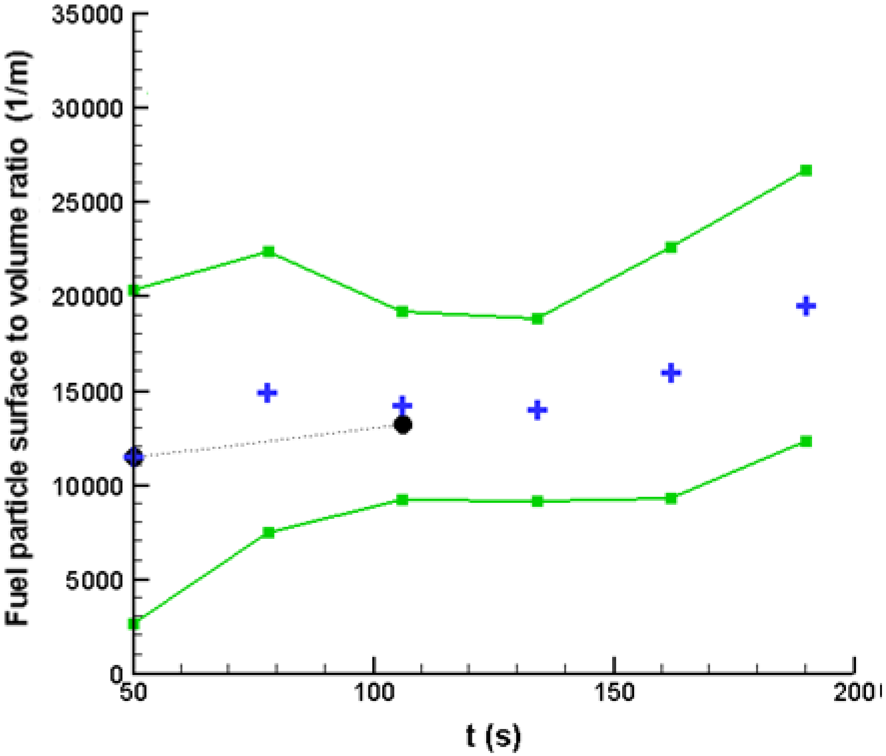


SIR filter 200 particles.



ASIR filter 50 particles.

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- EMAC: **O. Fudym and M. El Hafi**.
- COPPE/UFRJ: **Helcio Orlande and Marcelo Colaço.**