Kalman and Particle Filters

PART B: Particle Filters
Applications to control of dynamic systems

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Outline

- Introduction to Process Control
- Measurements, Uncertainties and Estimations tools
- Particle Filter as an observer for a PID control loop
- MPC using Particle Filters
- References

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Introduction to Process Control: Definitions, Design and Strategies

Operation of dynamic systems

Dynamic systems, such as industrial processes, are very diversified and integrate different equipment that demand accurate control of variables to meet performance criteria.

Industrial processes

petroleum, chemicals, food, steel, pulp and paper, power generation, ...

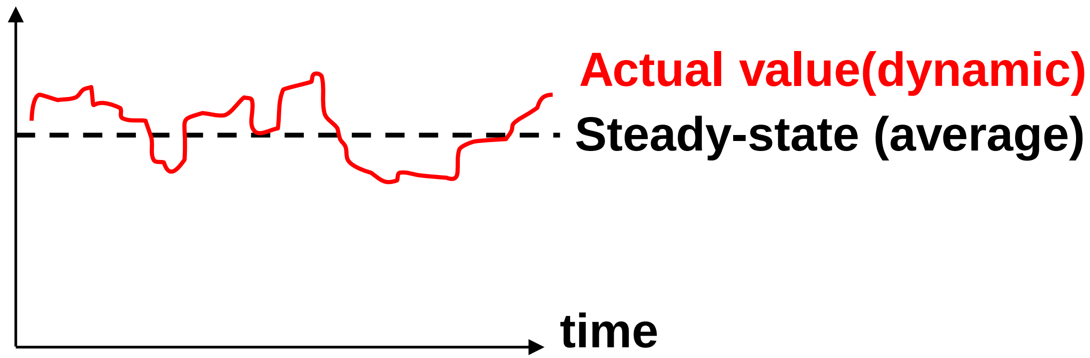
Equipment

heat exchangers, tanks, reactors, distillation columns, mills, pumps, ...

Variables

temperature, level, flow, pressure, humidity, viscosity, pH, stiffness, ...

Operation of dynamic systems



In practice, there's no steady-state:

- Feed changes
- Startup operation
- Desired changes
- Failures

External phenomena (**disturbances**) that cause the system to bounce around the desired equilibrium point, since they affect internal variables.

In addition, we can include: nonlinear dynamic behavior, uncertain and time varying parameters.

Countermeasures

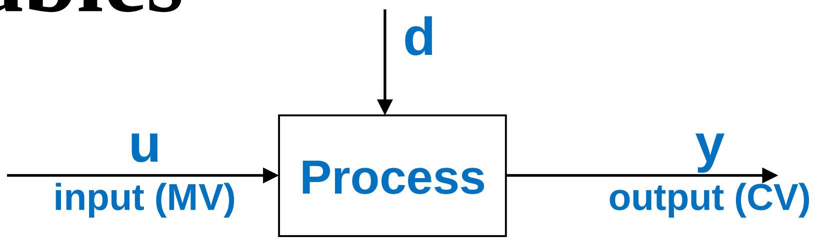
Process design

- Design system insensitive to disturbances, such way that all undesired input signal are dampened.
- Detect and remove source of disturbances using, for example, statistical tools.

Process control

- Specify electronic devices (sensors, transmitters, valves…)
- Design control structure to act dynamically on the system (usually manipulate valves) to counteract the effect of disturbances.

Classification of variables
for Process Control



Independent variables (“the cause”):

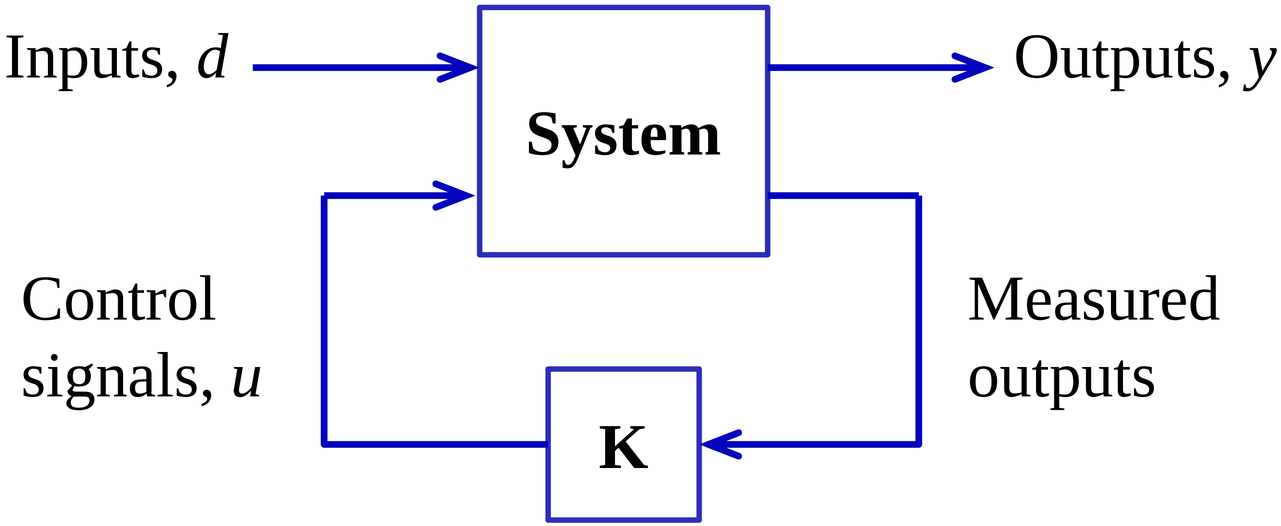
- Manipulated inputs (MV, u): Variables we can adjust (*DOF*)
- Disturbances (DV, d): Variables outside our control (it’s the *nature*)

Dependent variables (“the effect or result”):

- Primary outputs (CV, y_1): Variables we want to keep at a given setpoint
- Secondary outputs (y_2): Extra measurements that we may use to improve control

General view of control

Use inputs (u) to counteract the effect of the disturbances (d) such that the outputs (y) are kept close to their setpoints (y_s).



SKOGESTAD, S., POSTLETHWAITE, L., 2005, "Multivariable Feedback Control", New York, John Wiley and Sons.

Why control?

Control is needed to **reduce** the effect of disturbances and to **regulate** the system around a desired point (*setpoint*).

The following objectives are also included:

- Ensure **safety** (low risk of accidents)
- Ensure **stability** of the process
- **Optimization** of performance
  - Increase productivity
  - Reduce variability of product quality
  - Minimize production cost

Control hierarchy

Most of engineering systems are controlled using hierarchies of quite simple controllers:

on-off + PID-control + nonlinear fixes + some feedforward

Stabilization: regulatory layer

- *It doesn't use up any degrees of freedom.*
- *Reference value (setpoint) available for layer above.*

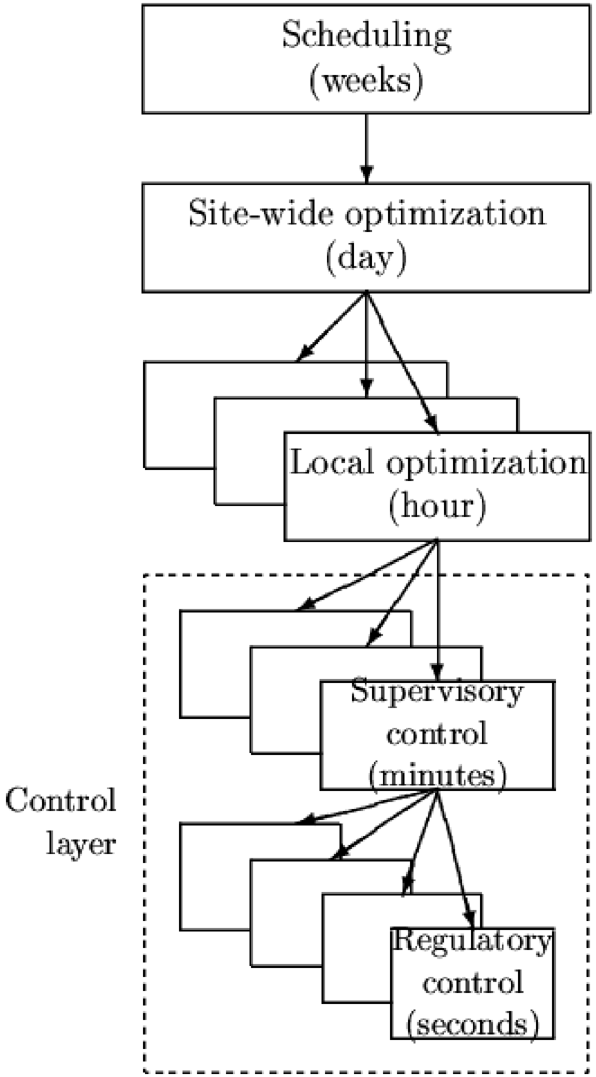
Productivity: supervisory layer

- *To meet economics (control of production rate and product quality)*

Optimization: local, site-wide or scheduling optimization

- *Reference values are determined for the lower layers in order to take into account global control objectives and market information.*

Control hierarchy



Manager

Process engineer

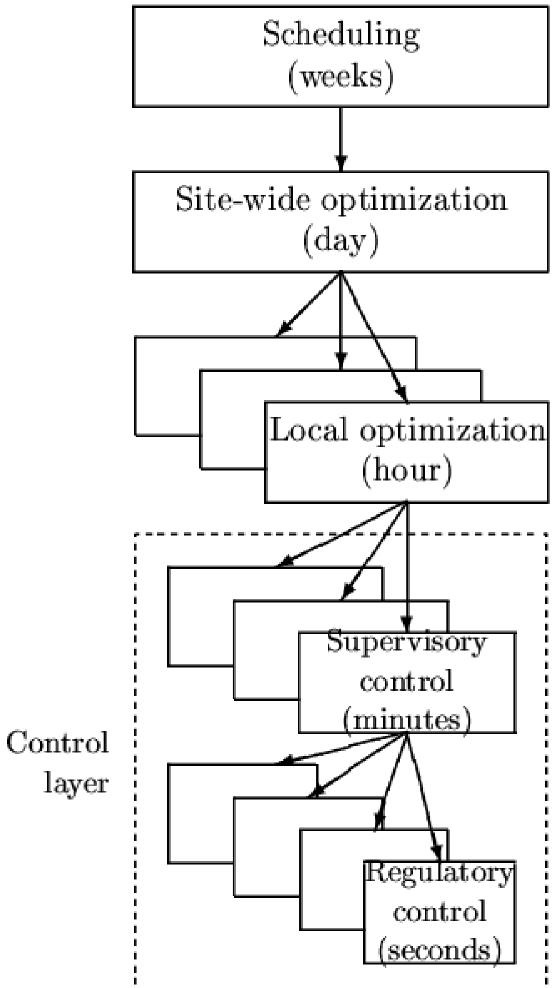
Operator, RTO

Operator, "Advanced control" / MPC

PID-control

SKOGESTAD, S., 2004, "Control structure design for complete chemical plants". *Computers and Chemical Engineering*, v. 28, pp. 219-234

Control structure design



Most (if not all) available control theories assume that a control structure is given at the outset.

Translate the operation into simple control objectives:

What should we control?

CV_1 = c ? (economics)

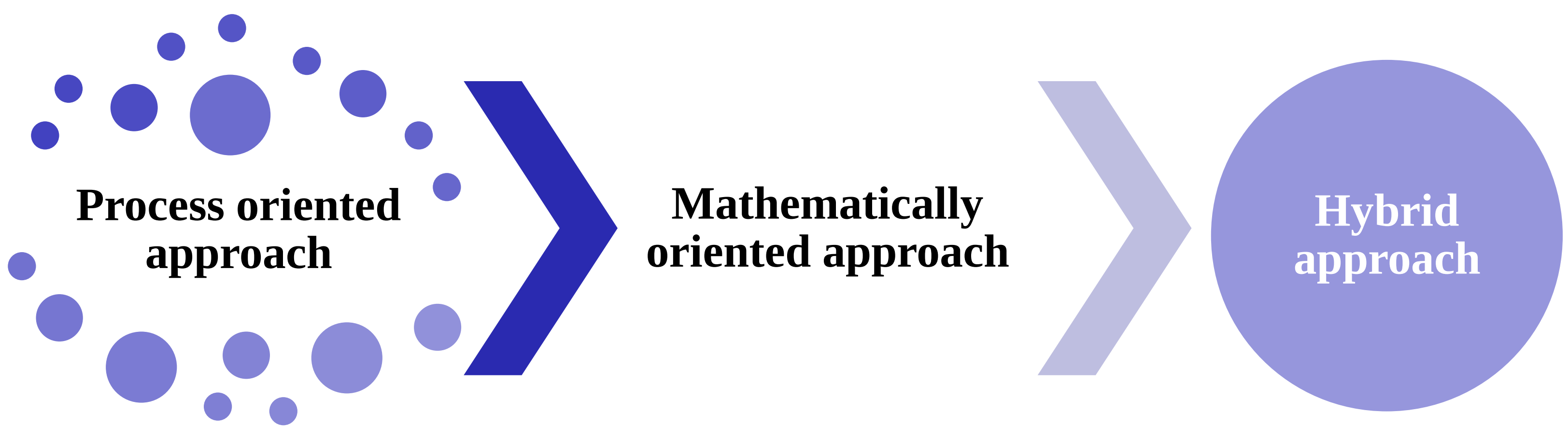
CV_2 = ? (stabilization)

Control structure design

Basic questions (Foss, 1973; Morari, 1982; Skogestad, 2004)

Which variables should be controlled,
which inputs should be manipulated,
which variables should be measured,
and which links should be made between them?

Literature proposals



Control structure design

Skogestad (2004): “self-optimizing control” procedure.

I Top-Down analysis (*structural questions*)

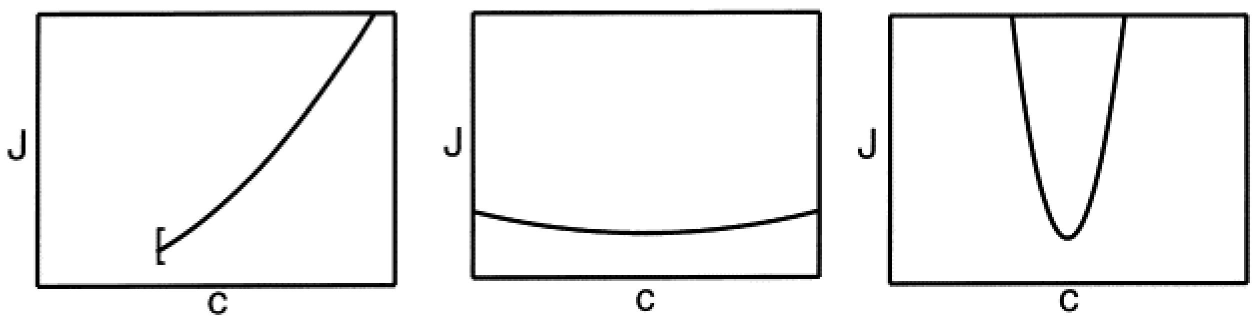
Step S1: Define operational objective (cost) and constraints

Step S2: Identify degrees of freedom and optimize operation for disturbances

Step S3: Implementation of optimal operation

- *What to control ? (primary CV’s in the sense of self-optimizing control)*

Step S4: Where set the production rate? (Inventory control)



(a) Implementation easy: Active constraint control

(b) Implementation easy: Insensitive to error in c

(c) Implementation difficult: Look for another controlled variable

Implementation of the controlled variable

Control structure design

Skogestad (2004): “self-optimizing control” procedure.

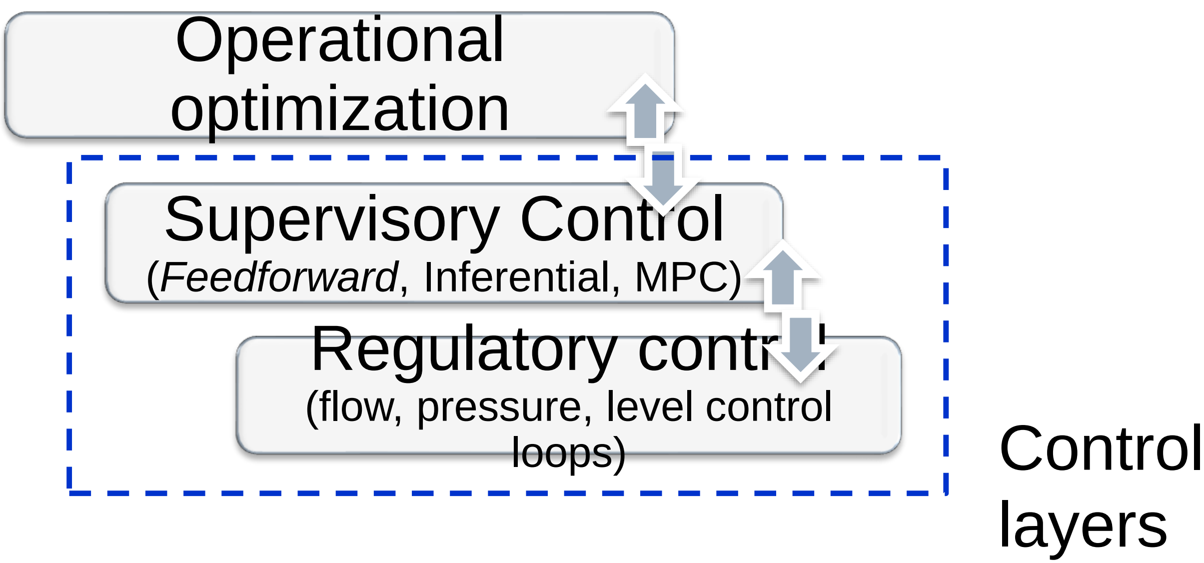
II Bottom-Up analysis (*control questions*)

Step S5: Regulatory control *(for stabilization and local disturbance rejection)*

- *What more to control? (secondary CV’s)*

Step S6: Supervisory control *(to keep outputs at optimal setpoints)*

Step S7: Real-time optimization *(if any DOF unused)*

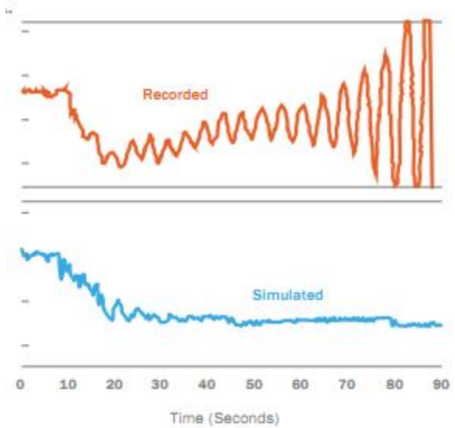


Control strategies

Regulatory control should be of "low complexity".

- It consists of single-input/single-output (SISO**)**: **PID control loops**.
- Manipulated variables that may saturate must be avoided, because otherwise control is lost and reconfiguration of loops is required.
- For stabilization: unstable modes should be detected "quickly" by the measurement.
- For local disturbance rejection: the variable is located "close" downstream of an important disturbance.

The challenge of regulatory control is to master the dynamics, even in the face of disturbances and operational changes.

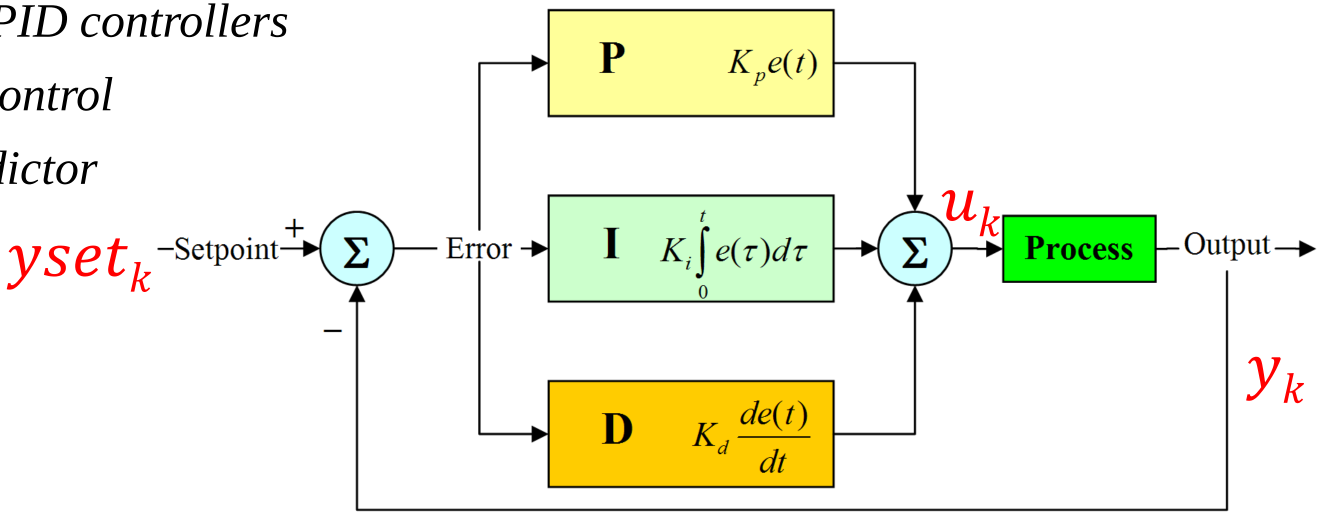


Dynamical responses.

Control strategies

PID control loops

1. <u>The most widely used algorithm in industrial control systems</u>
   - *Robust performance over a wide range of operating conditions.*
   - *Functional simplicity, allowing straightforward use.*
   - *Three parameters must tuned.*
2. <u>Some schemes are possible</u>
   - *P, PI and PID controllers*
   - *Cascade control*
   - *Smith predictor*



PID control loop

Control strategies

Supervisory control: two main approaches.

1. Decentralized single-loop control: usually for non-interacting process and for constant active constraints applying PID controllers.
   - *Advantage*: no or minimal model requirements.
   - *Disadvantage*: need to determine pairing (RGA-analysis) and logic required for reconfiguration when active constraints move.

2. Multivariable control: usually Model Predictive Control is applied.
   - *Advantage*:
     - coordinated control for interacting systems
     - easy handling of feedforward control
     - no logic required to handle changing constraints
   - *Disadvantage*:
     - requires multivariable dynamic model
     - controller tuning may be difficult
     - may have a reliability problem

Control strategies

Model Predictive Control - MPC

It reflects human behavior whereby we select control actions which we think will lead to the best predicted outcome (or output) over some limited time horizon.

Rossiter (2004)

It is the advanced control alternative the most present in the industry:

- Multivariable control problems are naturally handled while taking into account the actuator limitations and outputs constraints;
- An optimization routine is provided for system operation;
- Control update rates are relatively low in the chemical process industry applications and hence there is plenty of online computation time available;
- Unlike the most popular PID controllers, it takes into account simultaneously the effects of all manipulated variables to all controlled variables.

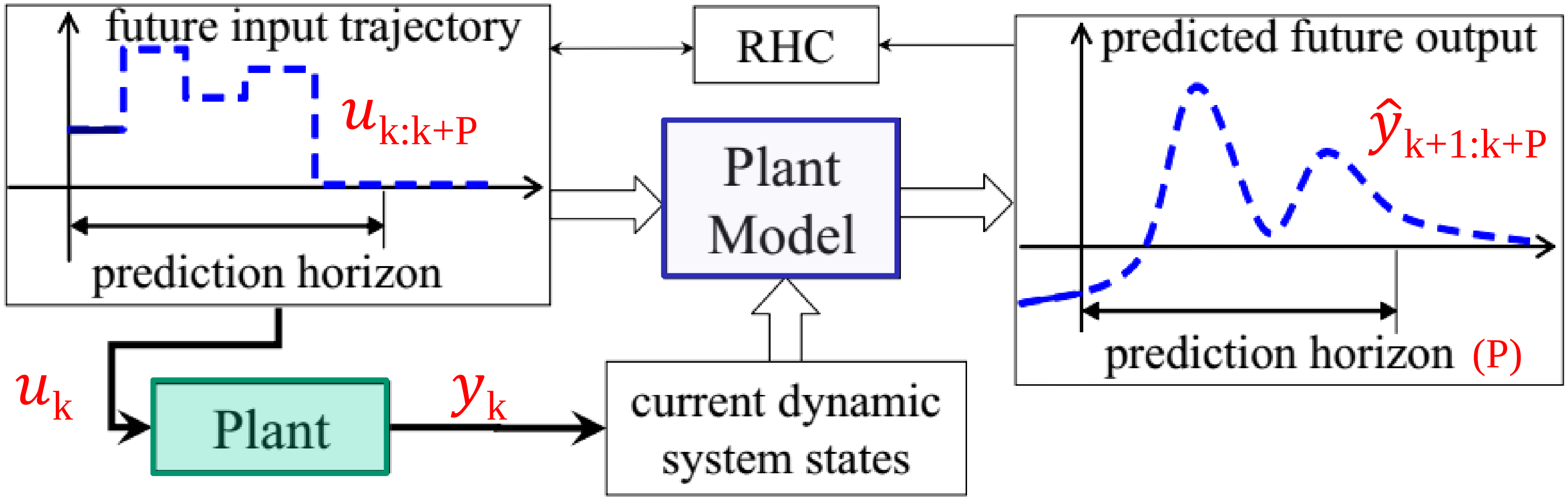
Control strategies

Rawlings (2000),
Maciejowski (2002),
Qin & Badgwell (2003),
Rawlings & Mayne (2009).

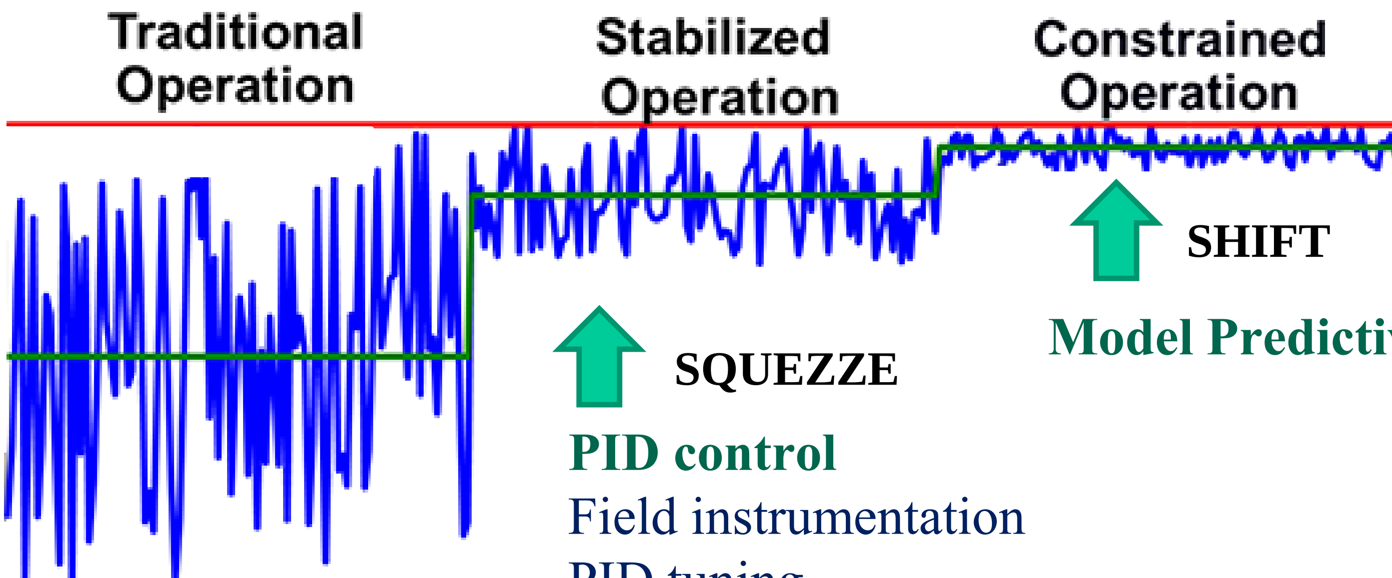
Model Predictive Control - MPC

The control move selection is based on an internal model of the system, regarding that the decisions is updated as new observations are available.

- The control law depends on predicted behavior;
- The current MV is determined by optimizing a performance index.
- The receding horizon control (RHC): the input is updated every sampling instant.



Control strategies



Model Predictive Control

PID control
Field instrumentation
PID tuning

About 90% of industrial cases still apply only PID control and rely on manual control in difficult situations.
(Rewagad & Kiss, 2012).

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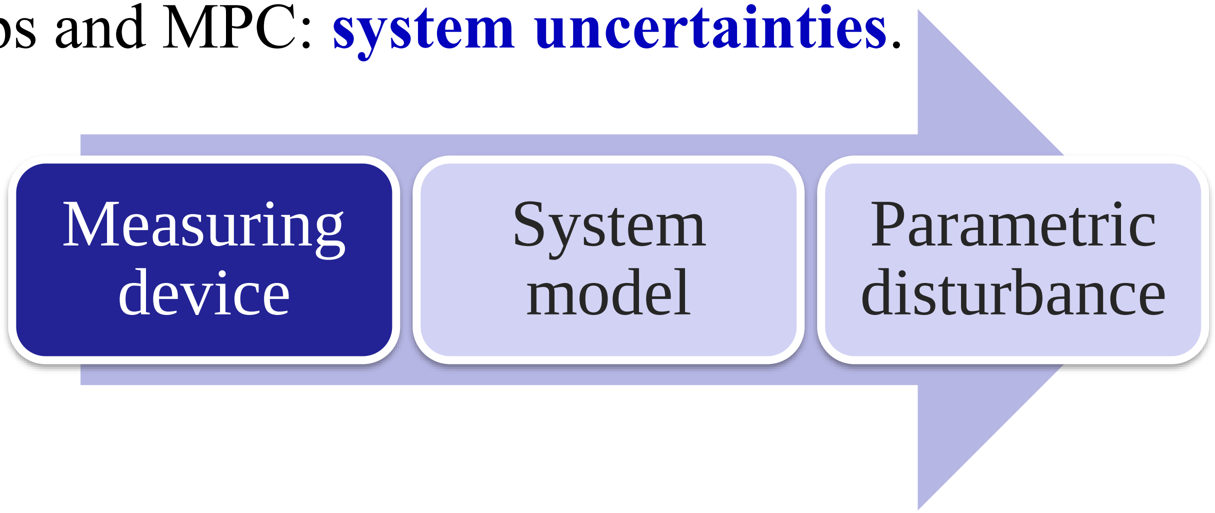
Measurements, Uncertainties and Estimations tools

Control limitations

PID loops and MPC: **system uncertainties**.



Uncertainties come from measurement errors due to the finite accuracy of measuring devices, systematic bias or gross errors, as well as from limited knowledge about the physical system and varying parameters.

In a real systems, online measuring devices are not always available.

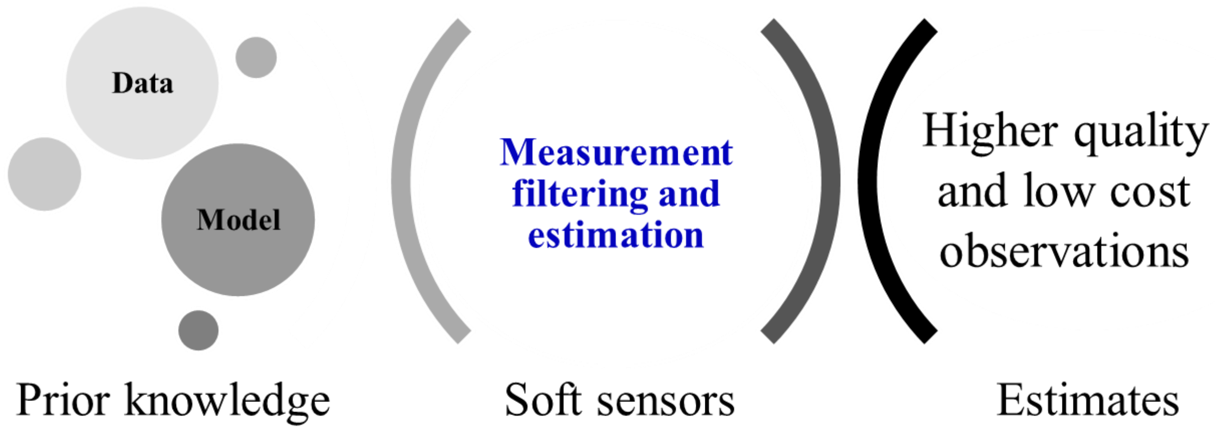
<u>Common procedure</u> : to resort to offline analysis. But, this takes time and can impair control actions, leading efficiency loss.

Soft sensors

Prata et al. (2009),
Khatibisepehr et al. (2013),
Shenoy et al. (2013).

As performance relies on online system observation, it is necessary to enhance the study on **SYSTEM MODELING** and on **SOFT SENSORS** (or virtual sensors) from observed data to infer variables and parameters.

In the last two decades, the use of such estimation tools has received much attention, since it is possible to complement or replace physical sensors minimizing the effects of uncertainties and lack of instruments.



Soft sensors

Solving an inverse problem is to determine unknown causes from desired or observed effects (Engl et al., 1996).

Producing sequential estimates for hidden variables of dynamic systems is an **INVERSE PROBLEM**, which has many practical applications.

Optimization Approach

- Data reconciliation/retification
- Moving horizon estimation (MHE)
- Artificial neural network (ANN)

Bayesian Approach

- Kalman filters family (KF, EKF, UKF, …)
- Particle filters (SIS, SIR, ASIR, …)
- Monte Carlo Markov Chain

The Bayesian approach

The available data must be combined with prior knowledge about the physical phenomena and measurement devices in order to statistically minimize the residue (Arulampalam, 2002).

Dynamical System

$$\begin{cases} \text{State Evolution Model:} & \boxed{\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})} \\ \text{Observation Model:} & \boxed{\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{n}_k)} \end{cases}$$

$$\pi_{posterior}(\mathbf{x}_k) = \pi(\mathbf{x}_k | \mathbf{z}_k) = \frac{\pi(\mathbf{x}_k)\pi(\mathbf{z}_k | \mathbf{x}_k)}{\pi(\mathbf{z}_k)} = \frac{Prior \times Likelihood}{Marginal}$$

The Bayesian approach

Doucet (2001), Arulampalam (2002), Ristic (2004), Kaipio & Somersalo (2004).

Particle Filter: Sample Importance Resampling (SIR)

SIR filter is a Sequential Monte Carlo technique for solution of the estimation problem, regarding nonlinear and non-Gaussian systems.

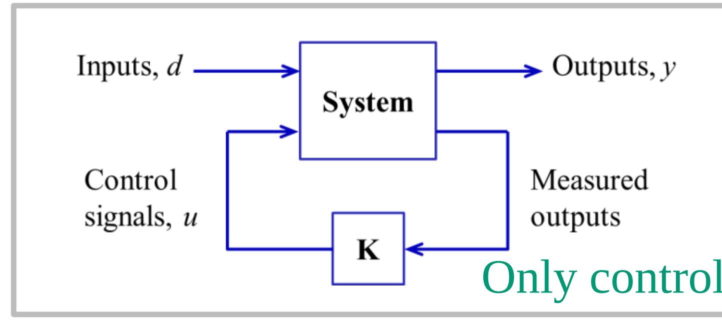
The main ideas are:

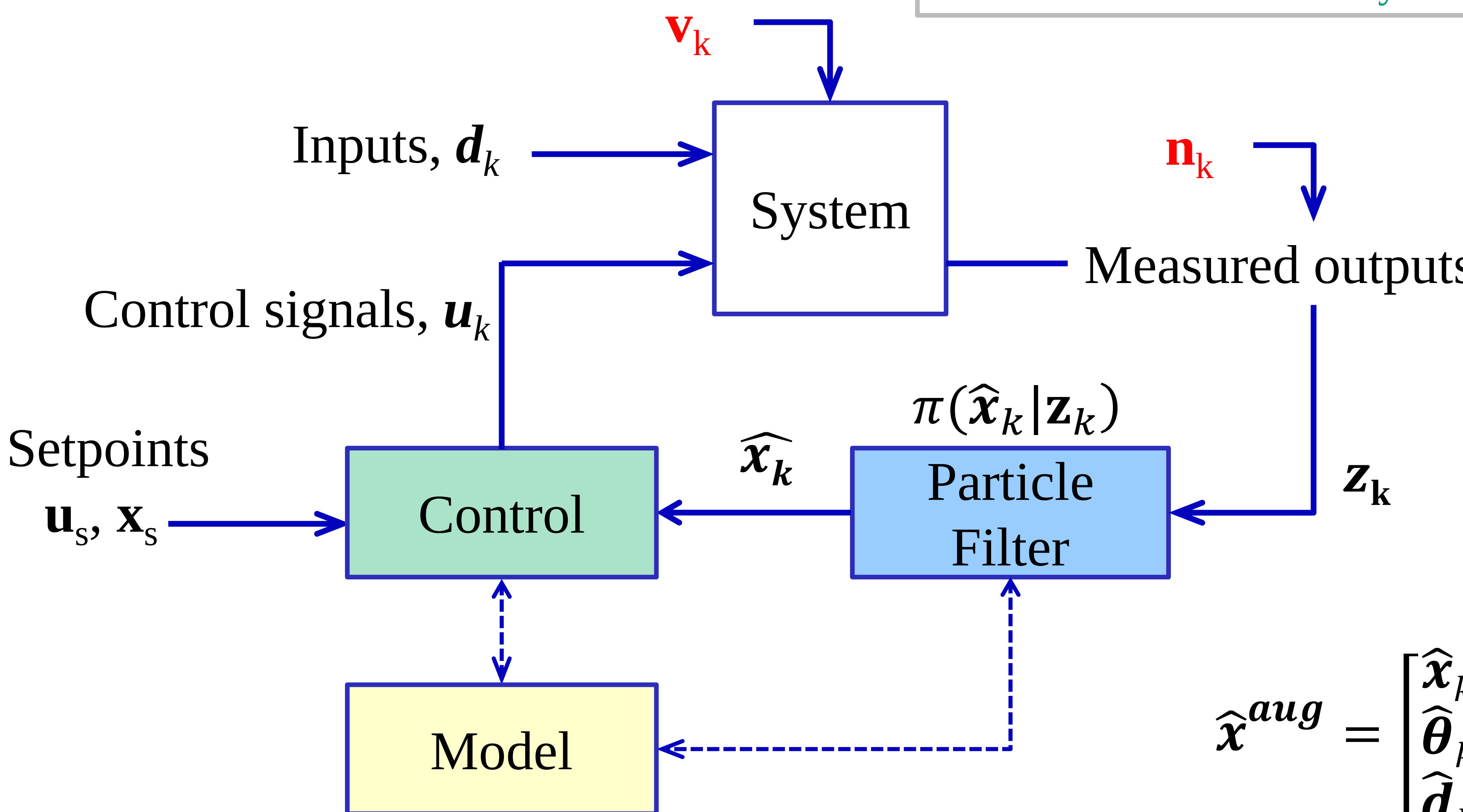
- to represent the required posterior density function by a set of random samples called particles with associated weights;
- to resample particles based on importance weights to avoid degeneracy;
- to compute the estimates based on these samples and weight.

$$\pi_{posterior}(\mathbf{x}_k) = \left\{ \mathbf{x}_k^i \cdot \mathbf{w}_k^i \right\}_{i=1\ldots N}$$

$$\mathbf{w}_k^i = \pi\left(\mathbf{z}_k \left| \mathbf{x}_k^i \right.\right) = (2\pi)^{-D/2} \left|\mathbf{W}\right|^{-1/2} \exp\left\{ -\frac{1}{2}[\mathbf{z}_k - h(x_k^i)]^T \mathbf{W}^{-1}[\mathbf{z}_k - h(x_k^i)] \right\}$$

Estimation and Control





$$\widehat{\boldsymbol{x}}^{\boldsymbol{aug}} = \begin{bmatrix} \widehat{\boldsymbol{x}}_k \\ \widehat{\boldsymbol{\theta}}_k \\ \widehat{\boldsymbol{d}}_k \end{bmatrix}$$

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SIR filter as a state observer for a PID control loop

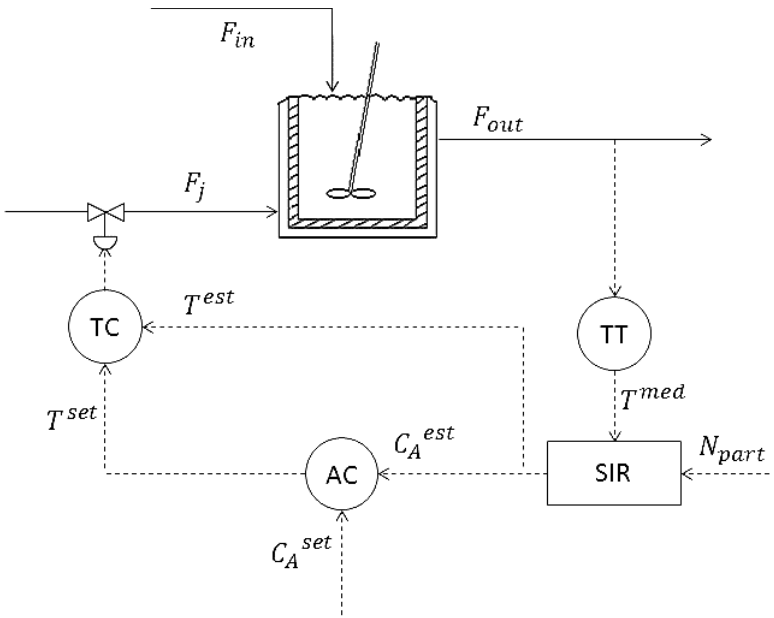
CARVALHO et al., 2016, "Filtro de partículas como observador online em um esquema de controle cascata para um reator contínuo". XIX ENMC – João Pessoa/PB, Brazil.

DIAS et al., 2016, "Online state estimation through particle filter for feedback temperature control". XXI COBEQ – Fortaleza/CE, Brazil.

DIAS et al., 2017, "Propylene Polymerization Reactor Control and Estimation Particle Filter and Neural Network". Macromolecular Reaction Engineering DOI: 10.1002/mren.201700010.

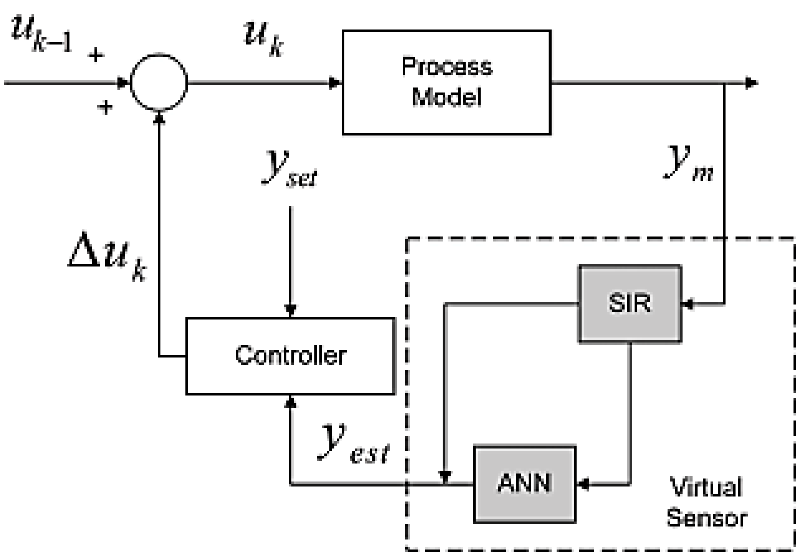
SIR + control loop

1. Non-isothermal continuous stirred reactor tank (CSTR)



Carvalho et al. (2016)

2. Propylene polymerization (PP) reactor control



Dias et al. (2017)

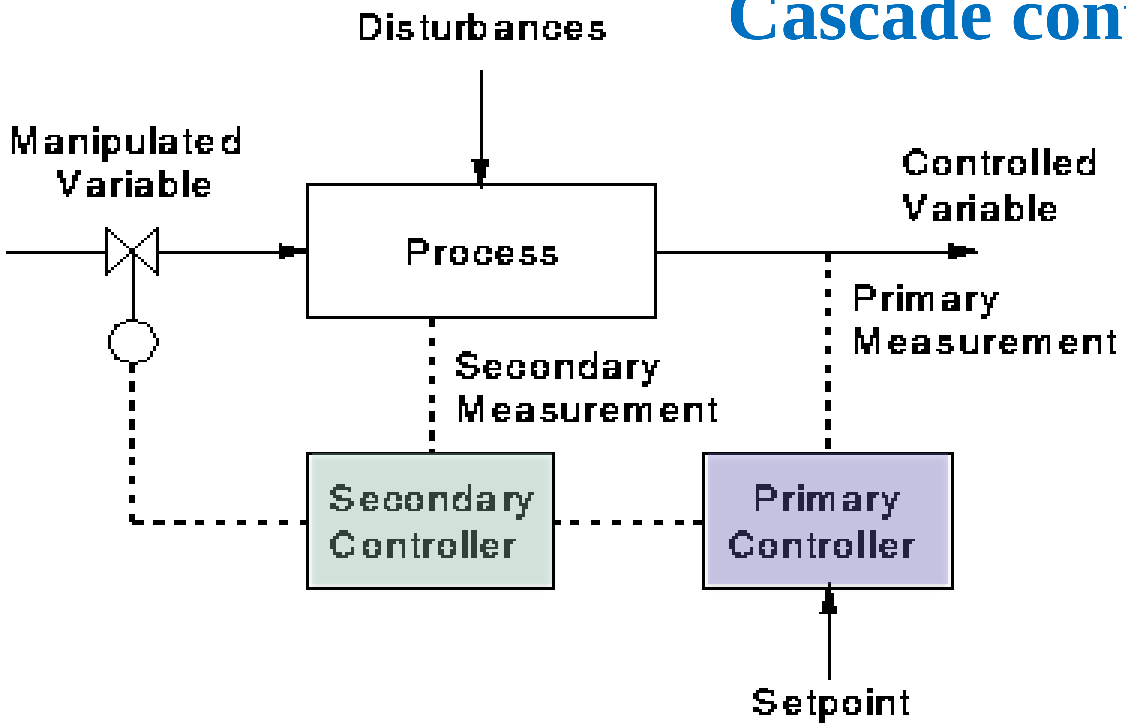
SIR + control loop

To study and perform the implementation of particle filter as an observer for a control loop, one needs:

1. Set the control objectives (regulation, supervision, optimization, …)
2. Design the control structure (CV, MV, PV, control configuration, …)
3. Specify dynamic model for the system and numerical/analytic approach
4. Set SIR filter structure
   - Variables (state and parameters) to estimate
   - Model for inverse problem
   - Measurements from the system
   - Number of particles
   - Uncertainties of the model, measuring device and variables

SIR + control loop

Cascade control



Nested loops for:

Fast dynamics: regulatory control (primary or slave controller)

Slow dynamics: supervision (secondary or master controller)

OBS: inner loop is at least three or four times faster than outer loop.

SIR + control loop

Cascade control

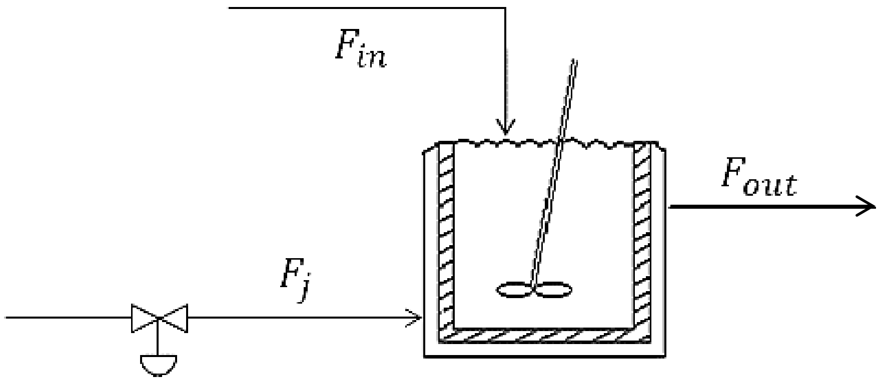
It is an advanced application of PID, in which a slave controller compensate for disturbances before they can affect the primary system variable.

Main drawbacks

- Cascade control systems require twice as much tuning (tune the slave controller first, then the master controller).
- The extra sensor tends to increase the overall costs and uncertainty sources. But, this can be softened by the use of particle filters.

1 - Control of CSTR

Carvalho et al. (2016)
Kittisupakorn & Hussain (2000)



Control objective

Keep concentration at desired point.

System model

$$\frac{dCa}{dt} = \frac{F}{Vr}(Cao - Ca) - ko\ Ca\ e^{\frac{-E}{RTr}}$$

$$\frac{dTr}{dt} = \frac{F}{Vr}(Tf - Tr) - \frac{\Delta H}{\rho Cp} ko\ Ca\ e^{\frac{-E}{RTr}} - \frac{UA}{Vr\rho Cp}(Tr - Tj)$$

1 - Control of CSTR

Control structure and configuration

- CV1: concentration (master loop) using MV= T_{set}
- CV2: temperature (slave loop) using MV=Tj

PI velocity algorithm

$$u(k) = u(k-1) + K_P \left[e(k) - e(k-1) + \frac{Ts}{\tau_I} e(k) \right]$$

<u>Tuning methods</u>

- Ziegler-Nichols
- Direct synthesis
- Internal model control
- Frequency analysis
- Integral criteria

	Controller tuning		
Loop	K_P	τ_I	Ts
master	7,0	4,0	1 min
slave	0,05	0,02	5 min

1 - Control of CSTR

SIR filter structure

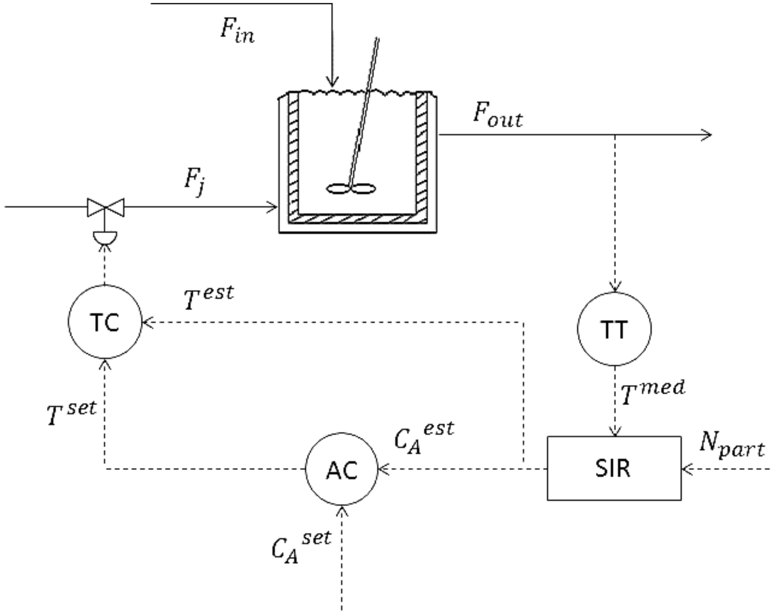
Objective: Estimate concentration from temperature

Noise: **1%** and **5%** of initial condition for measurement, $y_m = y_{exact} + n$

Npart: **10, 100 and 500**

Performance: **RMSE**

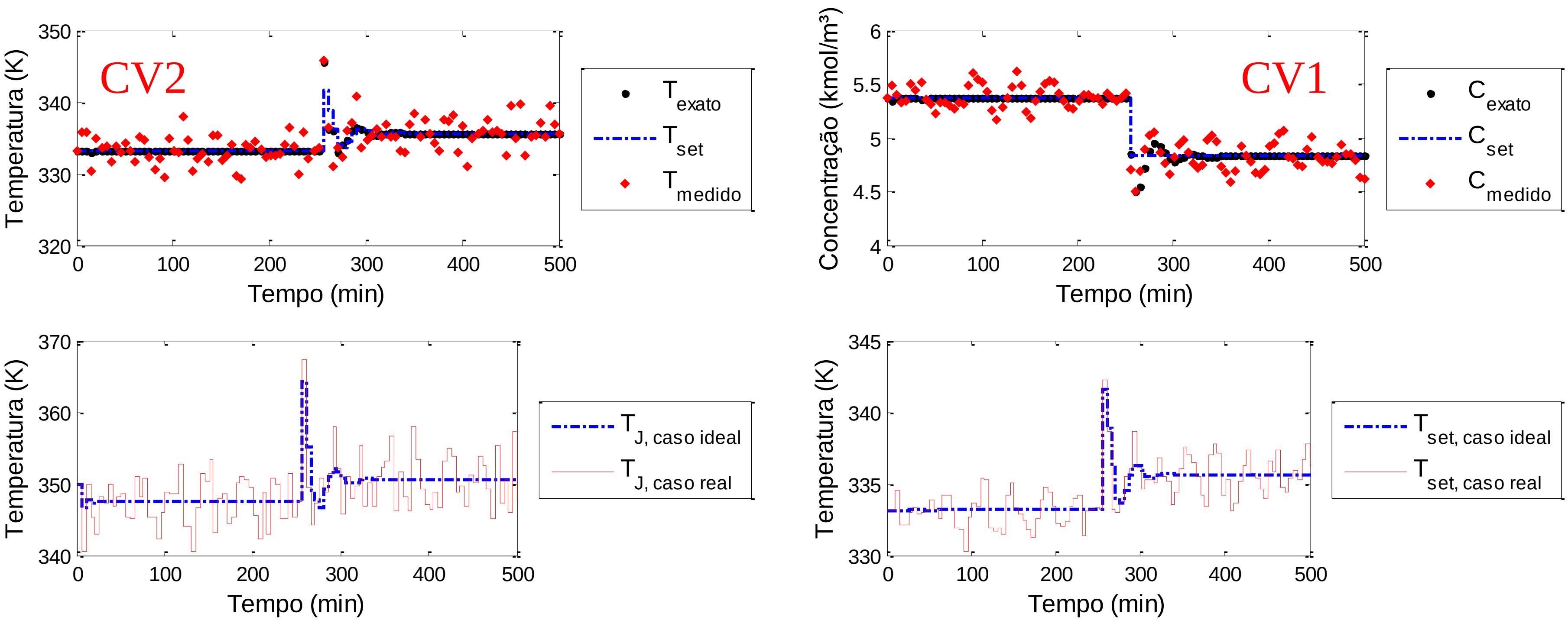
$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2}$$



1 - Control of CSTR

Numerical results

Noise effect



In the presence of noise, there are significant deviations between the desired value and the actual value during the simulation. This effect is also noticed in the manipulated variables, which oscillate during all simulation time

1 - Control of CSTR

Numerical results

Npart selection

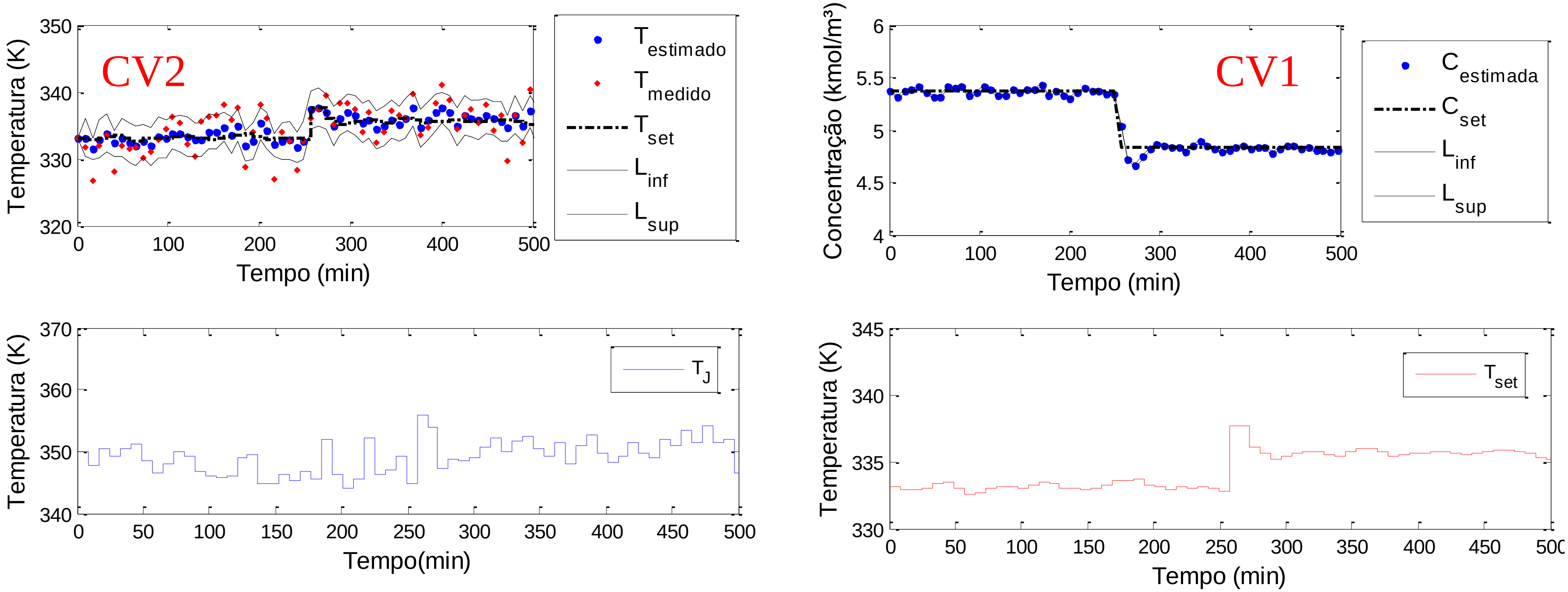
Npart	RMSE	ACT (s)
10	0.7865	0.0247
100	0.7389	0.1785
500	**0.7073**	**0.8708**

- Increasing the number of particles improves filter estimation performance.
- ACT is the average computing time required for the estimation, such way that the higher the number of particles, the higher is elapsed time.
- Anyway, it was much shorter the sampling time of the process, not being a limiting factor for the use of the tool.

1 - Control of CSTR

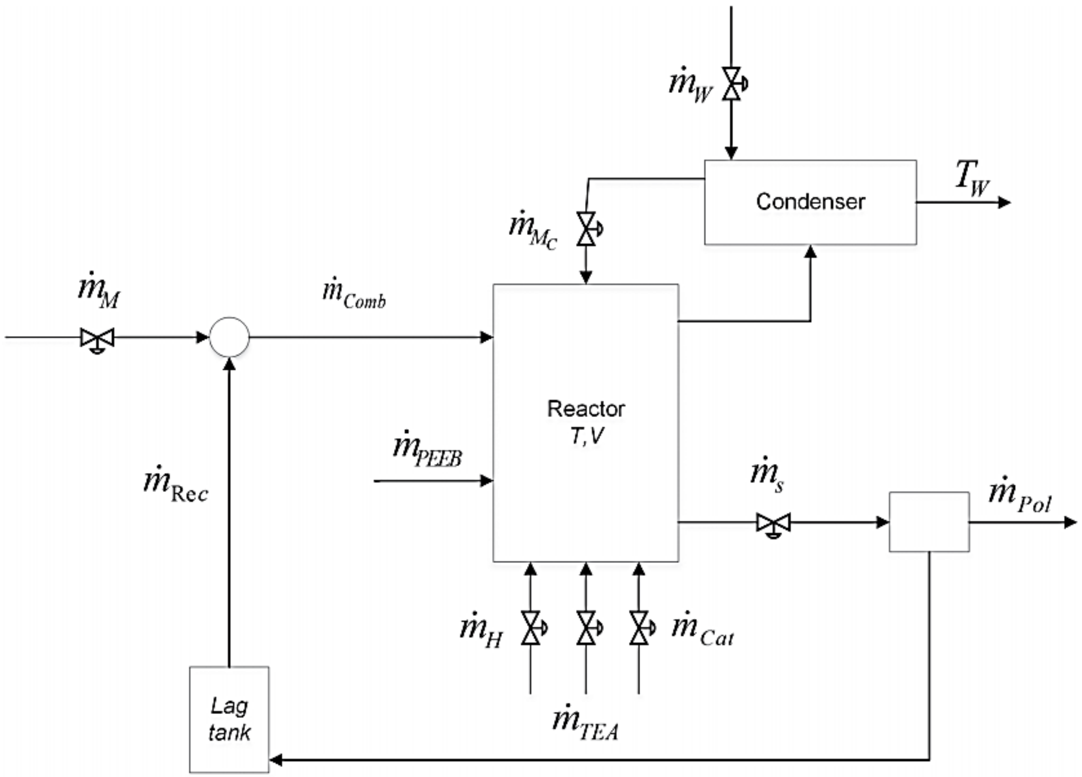
Numerical results

-10% step-like servo test



Dynamic behavior considering the control scheme with SIR filter.

2 – PP Reactor



Dias et al. (2017)
Dutra et al. (2014)

Control objective

Keep process stability
Meet product final quality:
MI and XS

System model (rigorous)

Mass and energy balances + Constitutive equations

17 ODE + 48 algebraic equations

2 – PP Reactor

Control structure and configuration

	Regulatory layer	Supervisory layer
CV	Volume Temperature Cooling fluid temperature Production rate Monomer feed flow	Melt index (MI) Xylene xtractable fraction (XS)
MV	Reactor slurry out flow Cooled monomer flow Cooling fluid flow Catalyst feed flow Fresh monomer feed flow	Hydrogen feed flow Cocatalyst feed flow ratio

2 – PP Reactor

SIR filter structure

Objective: filter observed data

Model: **simplified direct model**

Noise: **5%** of exact value

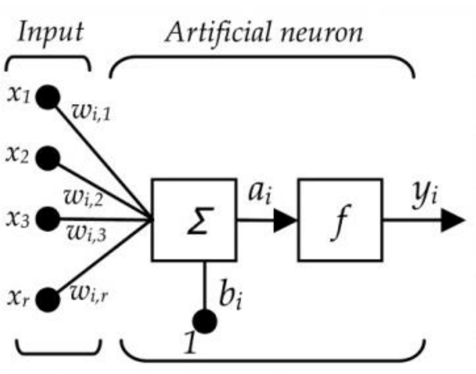
Npart: **10, 50, 100** and **200**

Performance: **MWCI, N_{eff}**

$$y_m = y_{exact} + N(0, 0.05 \cdot y_{exact}) \qquad N_{eff} = \frac{N}{1 + Var(w_k^i)}$$

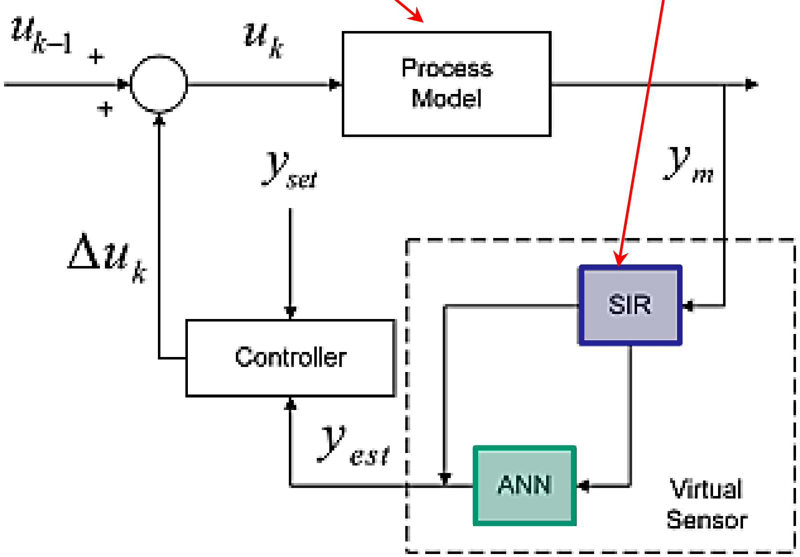
Artificial neural network (ANN)

Objective: predict product quality



Direct problem: Rigorous model

Inverse problem: Simplified model



Proposed virtual sensor

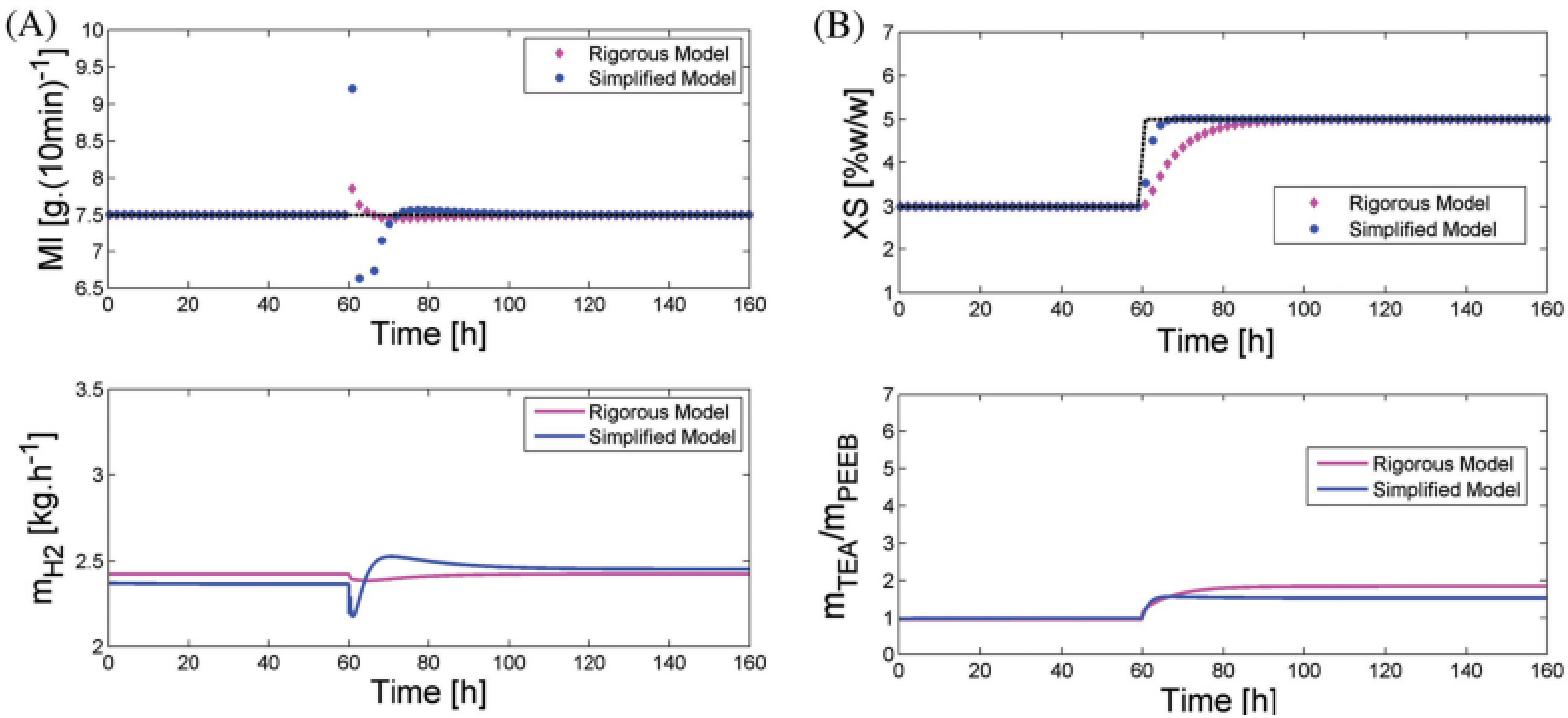
2 – PP Reactor

Numerical results

Model simplification

CV

MV



Comparison of the dynamic behavior for the rigorous and simplified model.

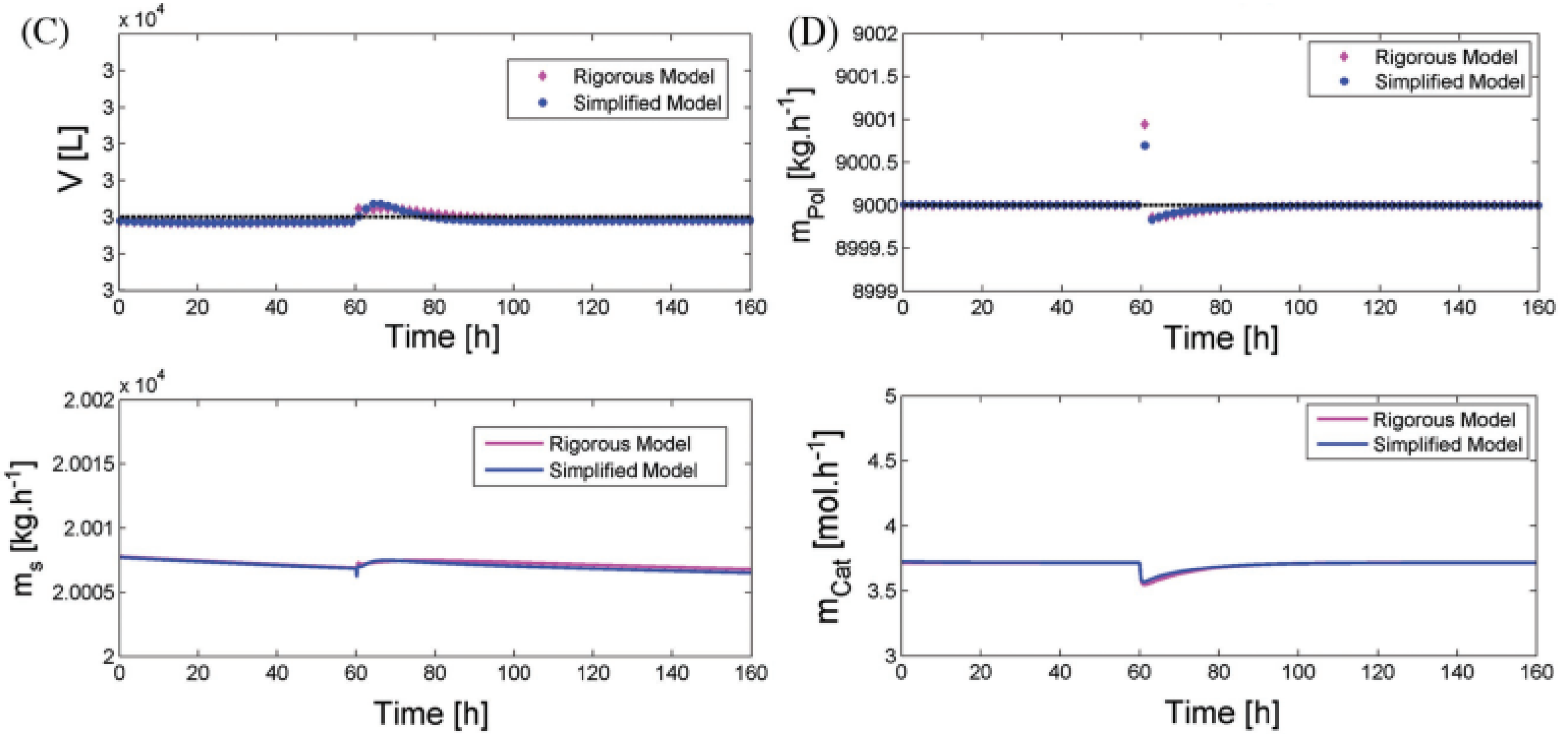
2 – PP Reactor

Numerical results

Model simplification

CV

MV



Comparison of the dynamic behavior for the rigorous and simplified model.

2 – PP Reactor

Numerical results

Virtual sensor

6 input variables

operation

Multilayer Perceptron

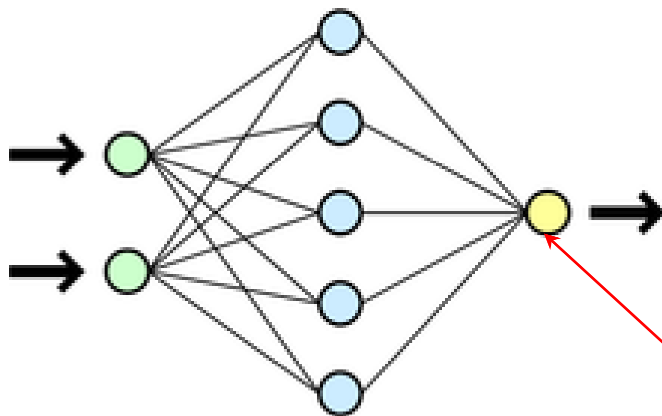
(6-10-2)

2 output variables

quality

Reactor temperature
Reactor holdup:

- Monomer
- Hydrogen
- Cocatalysts
- Polymer



MI
XS

Sigmoid function
(output layer)

Hyperbolic function
(hidden layer)

2 – PP Reactor

Numerical results

Virtual sensor

Performance of the best tested networks

ANN	R^2	R^2	R^2	R^2	SD_{RATIO}
	Training	Validation	Test	Total	
MLP 6-2-2	0.8419	0.8738	0.7791	0.8409	0.2882
MLP 6-2-2	0.8468	0.8463	0.7812	0.8356	0.2989
MLP 6-5-2	0.9769	0.9281	0.8153	0.9479	0.1660
MLP 6-8-2	0.9561	0.9529	0.8259	0.9369	0.1859
MLP 6-8-2	0.9096	0.9273	0.8733	0.9073	0.2261
MLP 6-10-2	0.9765	0.9671	0.8969	0.9631	0.1518
MLP 6-10-2	**0.9804**	**0.9673**	**0.9718**	**0.9772**	**0.1363**
MLP 6-10-2	0.9658	0.9382	0.9642	0.8093	0.1673
MLP 6-15-2	0.9748	0.9439	0.7820	0.9396	0.2013
MPL 6-15-2	0.9742	0.9344	0.8356	0.9464	0.1493

SD_{RATIO} is the ratio of the prediction error standard deviation and original data set standard deviation.
SD_{RATIO} values below 0.2 are considered to be good.

2 – PP Reactor

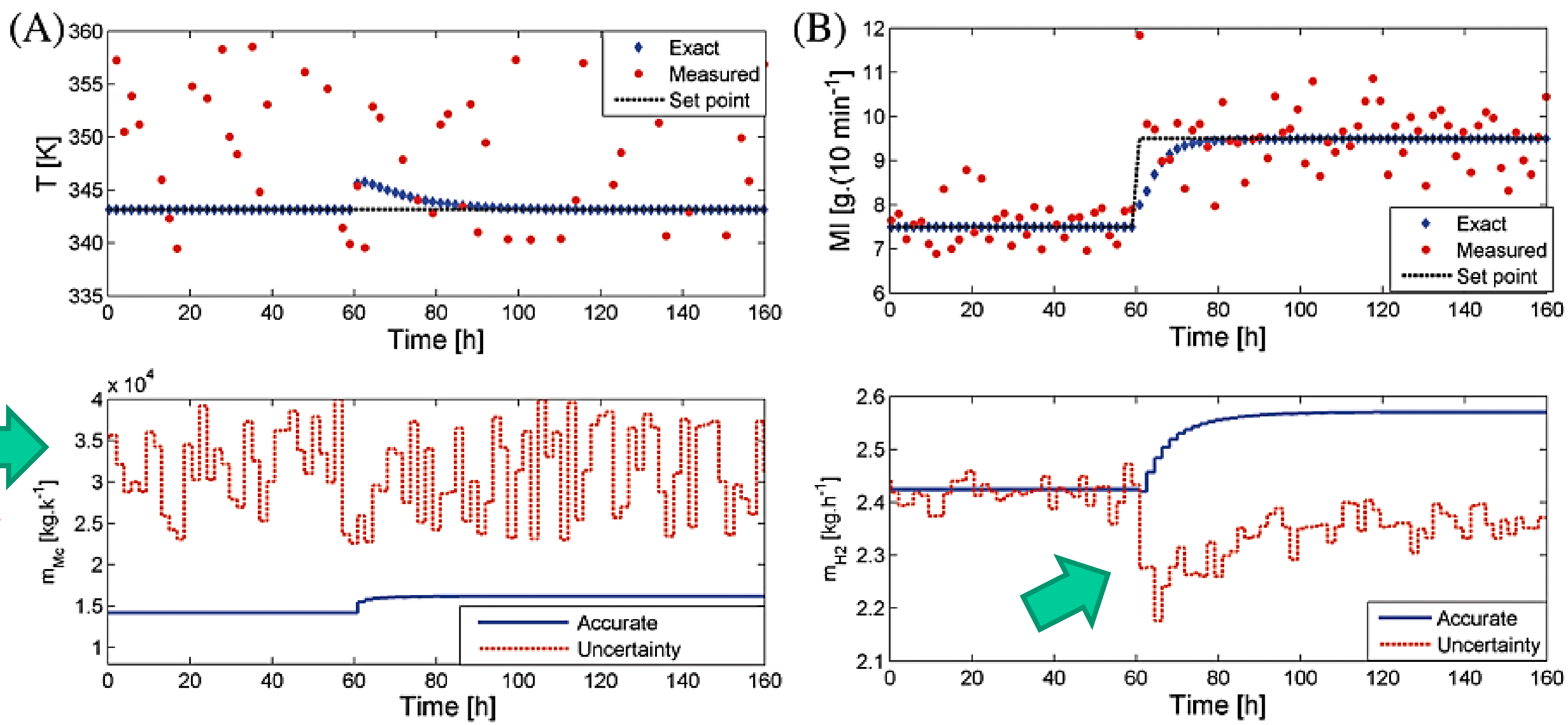
Numerical results

Uncertainties effect

$$y_m = y_{exact} + N(0, 0.05 \cdot y_{exact})$$

CV

MV



Dynamic behavior considering uncertain measurements.

2 – PP Reactor

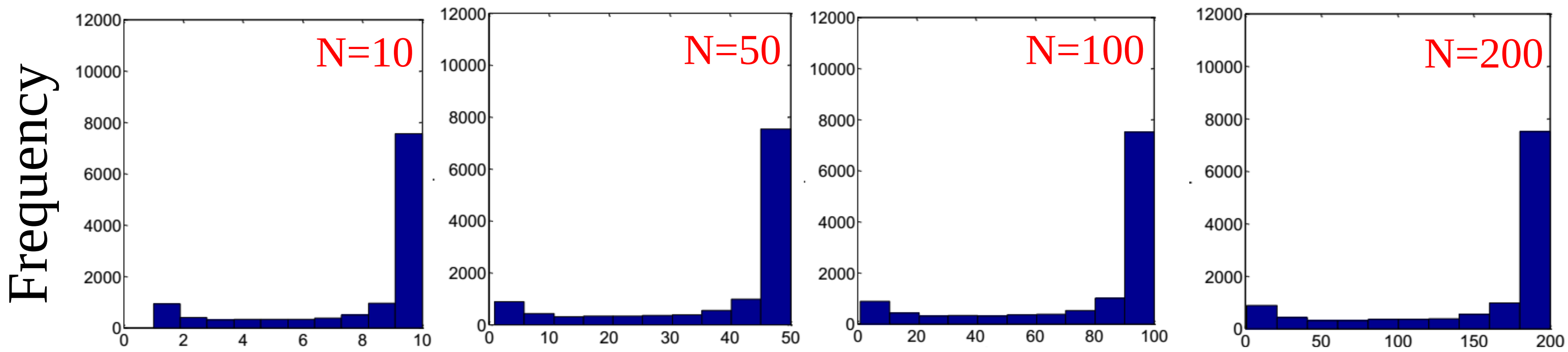
Numerical results

Particle filter performance

Maximum width of the credibility interval (MWCI)

Number of particles	V [L]	T [K]	T_W [K]	$\dot{m}_{comb}$ [kg h^{-1}]	N_{eff} [%]	Average computation time [s]
10	16.2883	1.8826	1.5128	52.3151	90.26	0.10
50	14.7388	1.7835	1.4197	50.1588	89.16	0.35
100	14.5765	1.6602	1.3122	47.2783	88.99	0.64
200	12.9021	1.2287	0.9976	34.3042	88.91	1.28

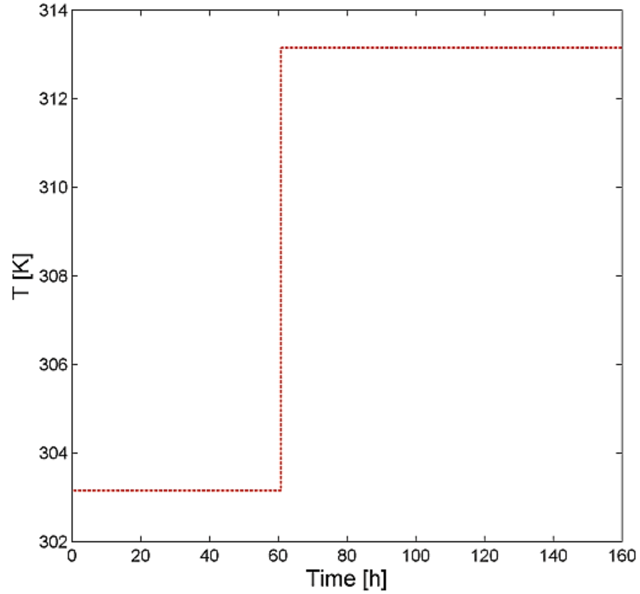
Histograms for effective sample size (Neff)



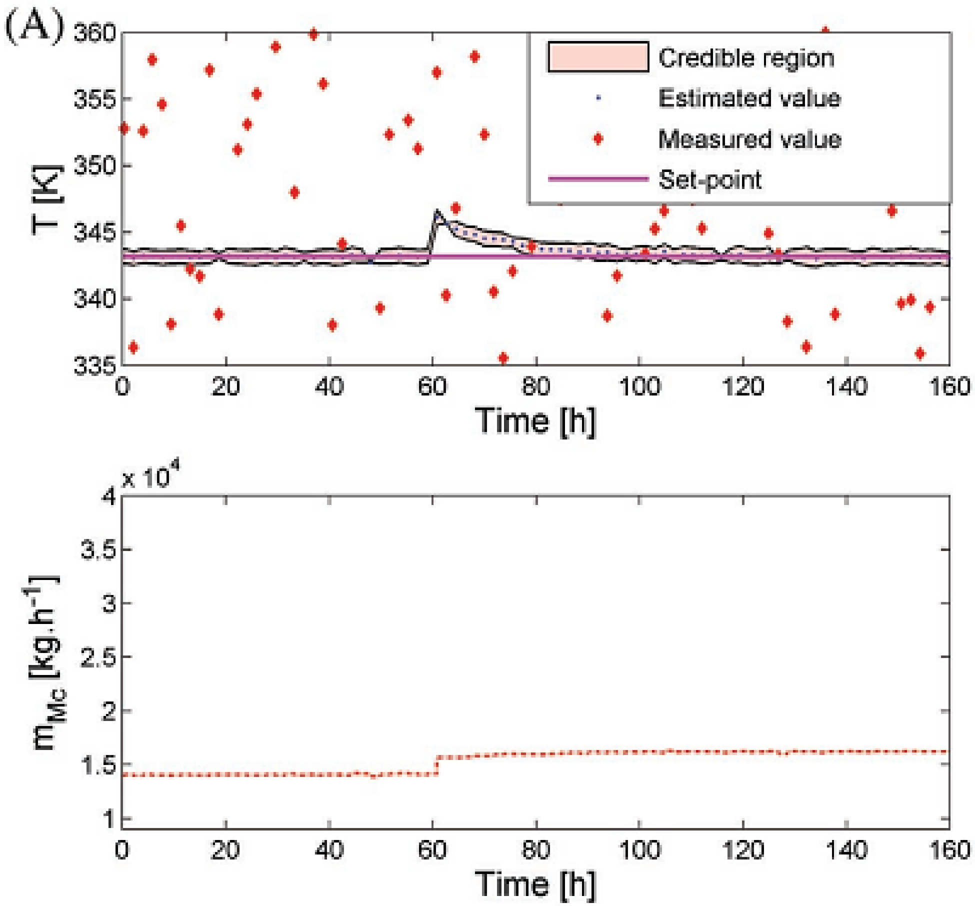
2 – PP Reactor

Numerical results

Regulator test: +10K in T_{feed}



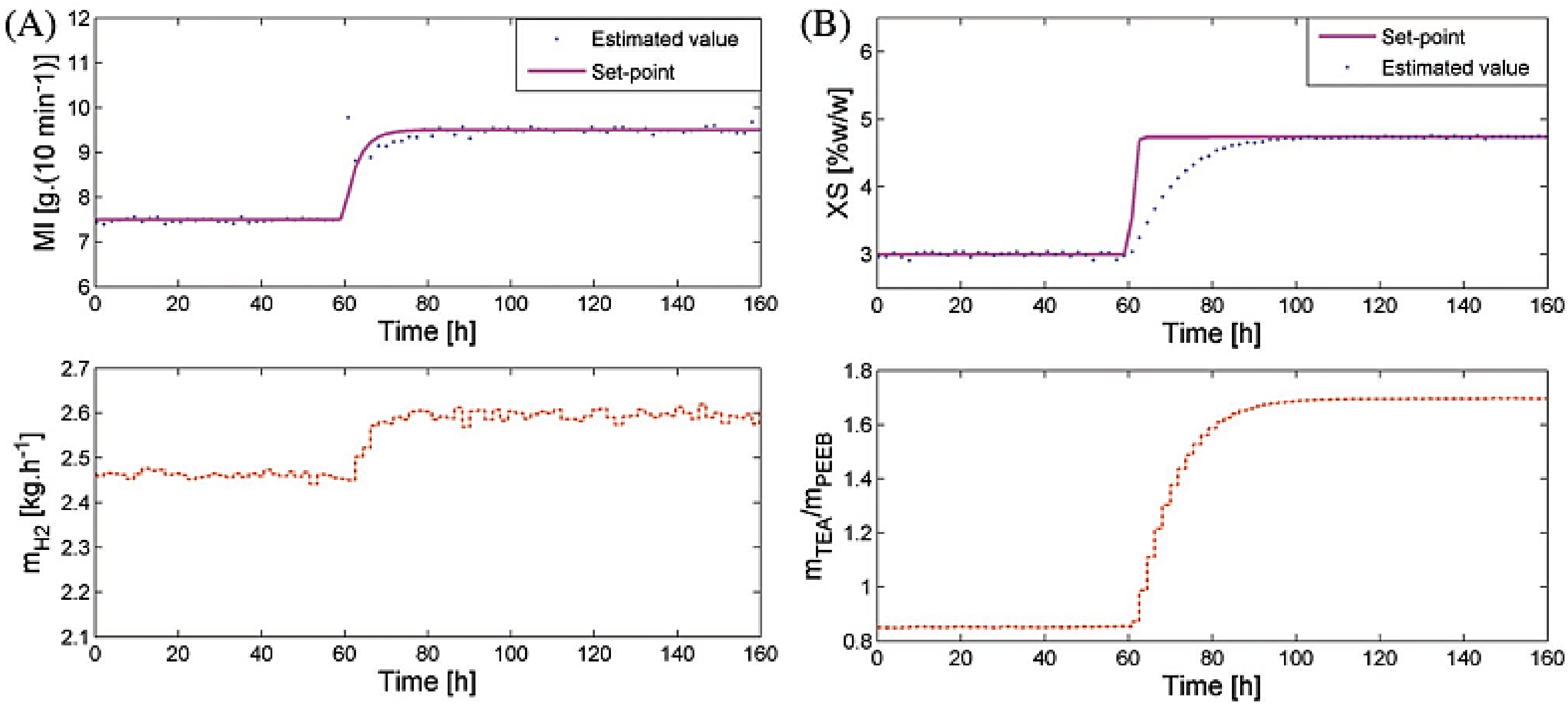
Monomer feed temperature



2 – PP Reactor

Numerical results

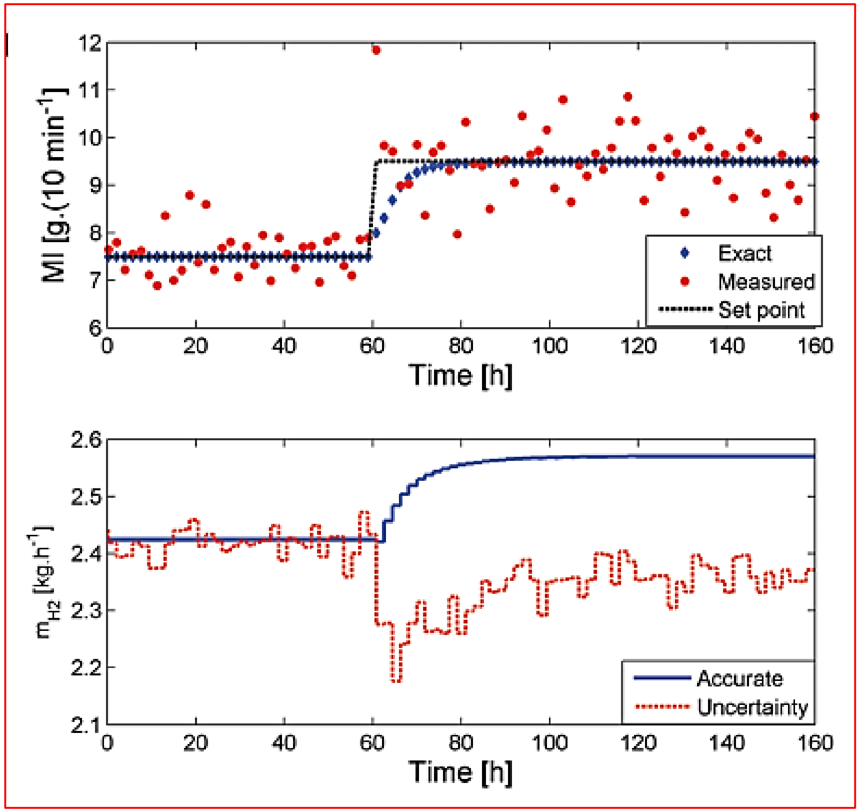
Servo test: +2 in MI and XS

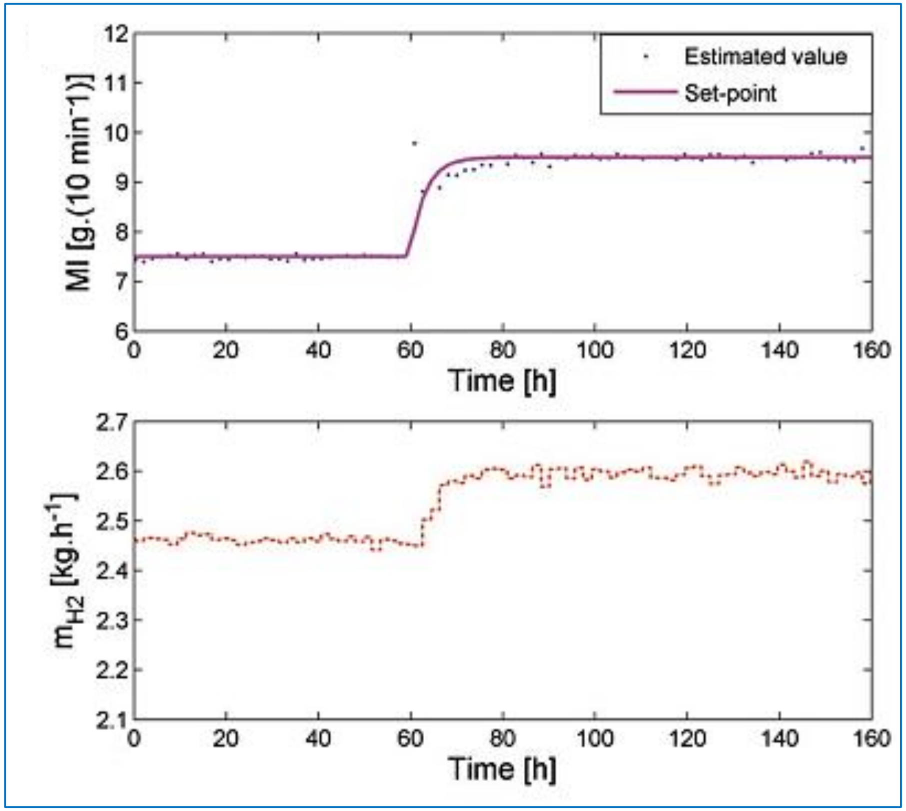


2 – PP Reactor

Numerical results

Servo test: +2 in MI





Control actions were based on purely noisy measurements which did not have matched the actual values of the process.

SIR + control loop Remarks

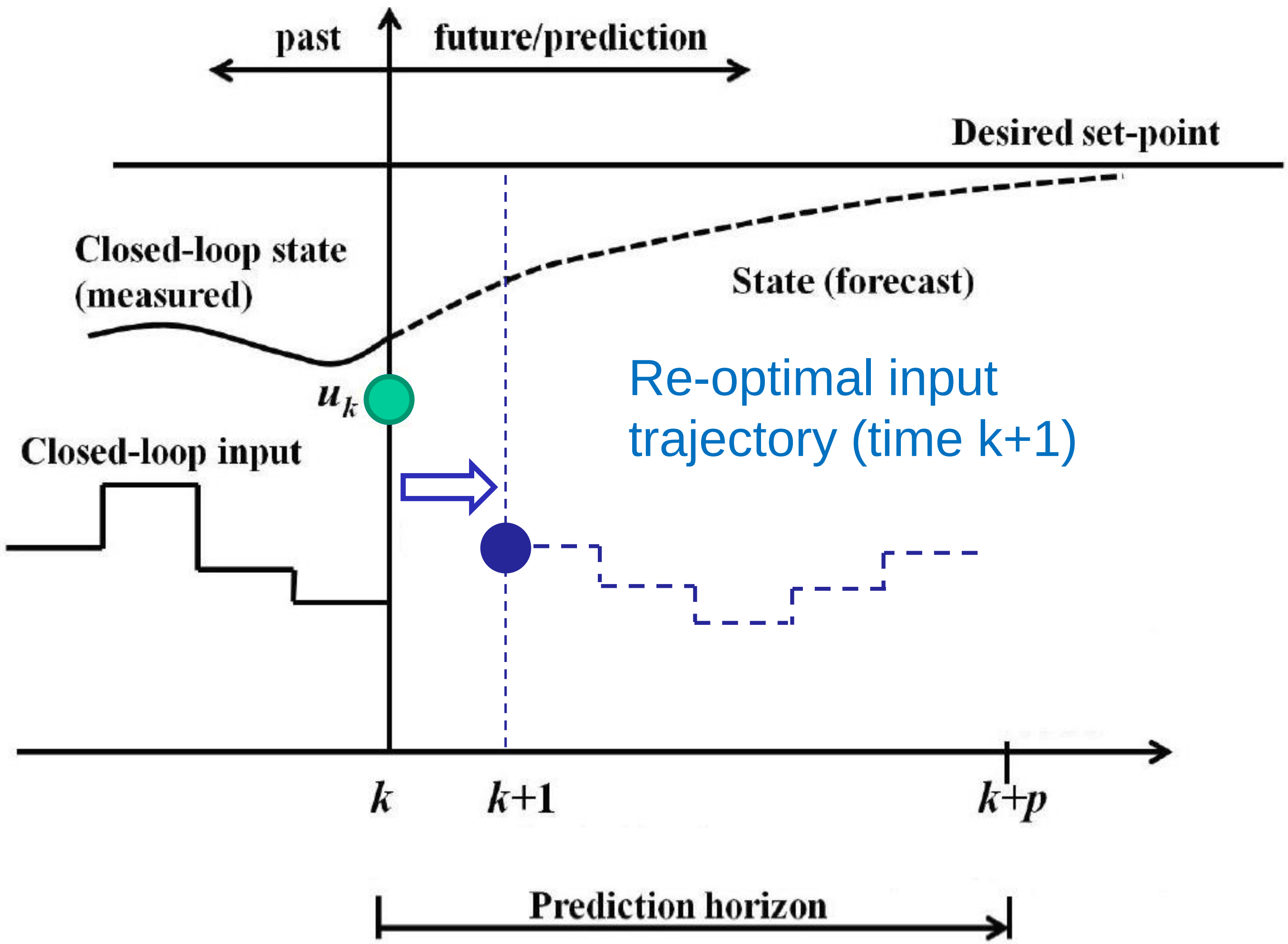
The lack of instruments and finite precision of the existing devices are real problems faced in industry that may leave the process at risk of failure, affect safety, …

Particle filters (PF) can be used as an observer to estimate latent variables and to reduce uncertainties, in order to take control actions with accurate information.

Advanced soft sensor schemes can be proposed with PF associated to machine learning algorithms, as artificial neural network (ANN), to address extremely nonlinear processes, which are very common in chemical engineering.

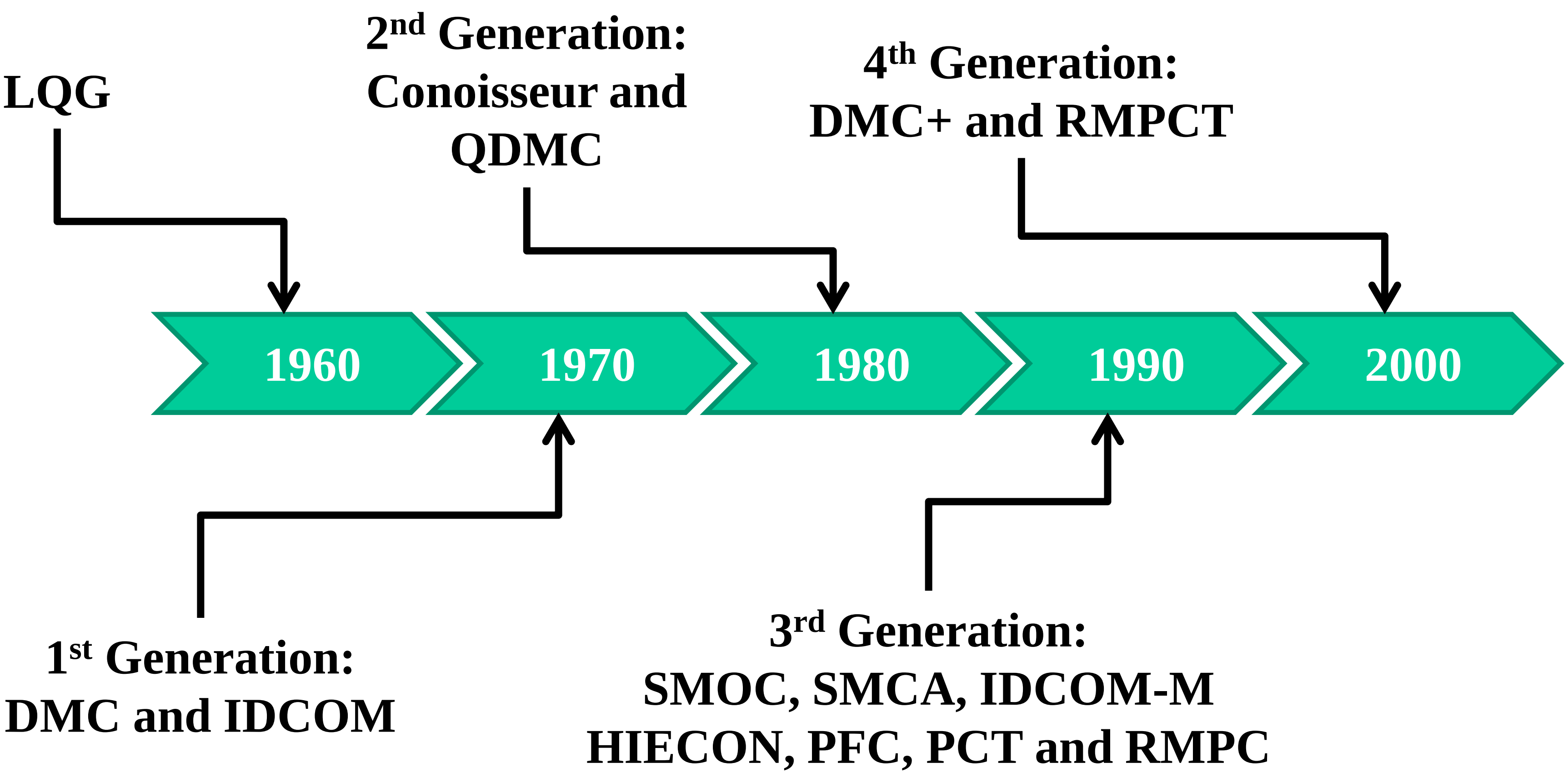
MPC using Particle Filters

MPC basics



past
future/prediction
Desired set-point
Closed-loop state
(measured)
State (forecast)
u_k
Re-optimal input
trajectory (time k+1)
Closed-loop input
k
$k+1$
$k+p$
Prediction horizon

MPC basics


LQG
2nd Generation: Conoisseur and QDMC
4th Generation: DMC+ and RMPCT
1960
1970
1980
1990
2000
1st Generation: DMC and IDCOM
3rd Generation: SMOC, SMCA, IDCOM-M HIECON, PFC, PCT and RMPC

Deterministic formulation

Discrete-time system model

$$x_{k+1} = f(x_k, u_k)$$
$$\hat{y}_k = g(x_k)$$

$$u^{\min} \leq u_{\mathrm{k}} \leq u^{\max}$$
$$\Delta u^{\min} \leq \Delta u_k \leq \Delta u^{\max}$$
$$y^{\min} \leq \hat{y}_k \leq y^{\max}$$

Although having applications in many real systems, **MPC is a mature technique only for linear systems**.

Linear systems with quadratic objective function is a **convex problem**, whose solution is analytic and/or recursive.

Objective function

$$J = \sum_{j=k}^{k+N_P} \left\| \bar{u}_j - \bar{u}_{j-1} \right\|_Q^2 + \sum_{j=k+1}^{k+N_P} \left\| s_j - \hat{x}_j \right\|_R^2$$

MPC and NMPC approaches

Classical MPC may fail due to the fact that it usually only works on linearized or approximated models.

Since nonlinearity are often significant in chemical and biological applications, **non-linear MPC** (**NMPC**) become essential for better performance and stable operation under dynamic conditions.

It is necessary:

- at each time k, to **estimate the initial conditions** to integrate the state equations along the prediction horizon N_P.
- to define an **integration strategy** of the state equations (which are function of variables $\bar{u}(k)$ that will only be known when solving the optimization problem) and an **optimization strategy**.

MPC and NMPC approaches

Integration strategy

Biegler (2007):

$$[\bar{u}(k), \cdots, \bar{u}(k+P-1)]$$

- **Sequential approach** — **Control vector parametrization**: prediction and optimization are solved sequentially until convergence.
- **Simultaneous approach** — **State and control profile discretization**: equations are incorporated as algebraic constraints to the optimization problem

Alternatives for approximate solution

- Successive linearization
- Combined use of linear and non-linear models
- Adaptive linearization
- Feedback linearization
- Use of block oriented nonlinear model
- Use of multilinear models

There are few NMPC approaches accounting for estimation (and global optimal solution) using particle filters.

MPC and particle filters

Andrieu *et al.* (2004)
Botchu e Ungarala (2007)

PF was used to estimate current state as initial condition on a classical MPC problem.

Blackmore (2006)

Approached the original stochastic problem as a determinist using a large number of particles.

Kantas, Maciejowski e Lecchini-Visintini (2009)

Sequential Monte Carlo (SMC) was used as non convex optimizer for stochastic NMPC, but at the expense of high computational cost.

Stahl e Hauth (2011)
Lopez (2014)

More basic implementation using a stochastic approach to solve the optimization problem using 2 particle filters, called PF-MPC.

PF-MPC

Stochastic general state space model

corresponding conditional probability densities

State transition $x_k = f(x_{k-1}, u_{k-1}, v_{k-1})$ $\Rightarrow$ $a_k(xk|x_{k-1}, u_{k-1})$

Observation $y_k = g(x_k, n_k)$ $\Rightarrow$ $b_k(yk|x_k)$

Density distributions

Estimation PF#1

$$x_k^{(i)} \sim a_k\left(x_k \middle| x_{k-1}^{(i)}\right) \quad i = 1, \dots, N_{part}$$

$$w_k^{(i)} \propto b_k\left(y_k \middle| x_k^{(i)}\right)$$

Prediction PF#2

$(j = 1, \dots, P\ )$

$$\bar{a}_j(\bar{x}_j, \bar{u}_j | \bar{x}_{j-1}, \bar{u}_{j-1})$$

$$\bar{b}_j(s_j | \bar{x}_j, \bar{u}_j)$$

$\Rightarrow$

$$\bar{x}_j = f\left(\bar{x}_{j-1}, \bar{u}_{j-1}, \bar{v}_{j-1}\right)$$

$$\bar{u}_j = \bar{f}_u\left(\bar{x}_{j-1}, \bar{u}_{j-1}, \tilde{v}_{j-1}\right)$$

$$s_j = \bar{g}\left(\bar{x}_j, \bar{u}_j, \hat{v}_j\right)$$

PF-MPC

First particle filter: <u>standard estimator</u>

Estimation of the current values of hidden variables from observed data.

<u>SIR filter</u>: $\pi_{posterior} = \{\mathbf{x}_k^i \cdot \mathbf{w}_k^i\}_{i=1\ldots N} \Rightarrow \hat{\boldsymbol{x}}_k$

Second particle filter: <u>control predictor</u> (over j=1,…, P)

It works on different states $(\bar{x}_j, \bar{u}_j)$ and "observations" (s_j).

Joint Process: $(\bar{U}_{k:k+P}, \bar{X}_{k:k+P})$

Transition density:

$$\bar{a}_j(\bar{x}_j, \bar{u}_j|\bar{x}_{j-1}, \bar{u}_{j-1}) = \bar{a}_{u,j}(\bar{u}_j|\bar{u}_{j-1})\bar{a}_j(\bar{x}_j|\bar{x}_{j-1})$$

Control input: $\Rightarrow \boldsymbol{u}_k^*$

PF-MPC

Input transition density $\bar{a}_{u,j}\left(\bar{u}_j|\bar{u}_{j-1}\right) \to \bar{u}_j = \bar{f}_u\left(\bar{x}_{j-1}, \bar{u}_{j-1}, \tilde{v}_{j-1}\right)$

$$\bar{u}_j = \bar{u}_{j-1} + \tilde{v}_{j-1}$$
$$or$$
$$\bar{u}_j = \tilde{v}_{j-1} \text{ (free)}$$

$\tilde{\boldsymbol{v}} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$: The control input is constrained in a sense that the effort $\Delta\bar{u}_j$ s kept small depending on Σ.

Quite similar to $\|\Delta\bar{u}\|_Q^2$ of a usual MPC objective function.

$$\bar{u}_j = \tilde{v}_{j-1}$$

$\tilde{\boldsymbol{v}} \sim \boldsymbol{U}\left(\boldsymbol{u}^{\boldsymbol{min}}, \boldsymbol{u}^{\boldsymbol{max}}\right)$: case it is necessary to meet hard constraints $\mathrm{u}^{\mathrm{min}} < \bar{\mathrm{u}}_j < u^{\mathrm{max}}$.

PF-MPC

Control performance density $\bar{b}_j(s_j|\bar{x}_j, \bar{u}_j)$ or $\bar{b}_j(s_j|\bar{x}_j)$

$$J = \sum_{j=k}^{k+N_P} \|s_j - \bar{x}_j\|_R^2$$

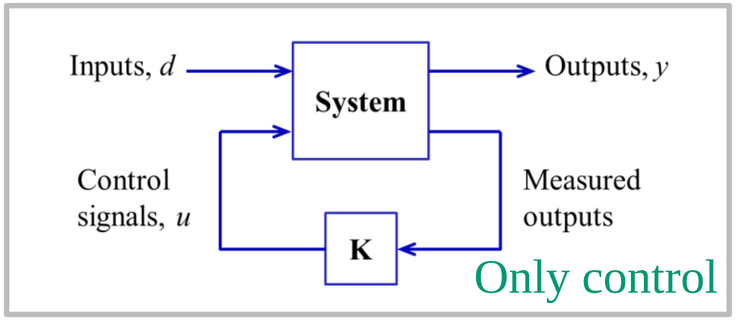
$$\bar{b}_j(s_j|\bar{x}_j) = \frac{1}{(2\pi)^{n/2}|R|^{-1/2}} exp\left(-\|s_j - \bar{x}_j\|_R^2\right)$$

Setpoint equation $s_j = \bar{g}(\bar{x}_j, \bar{u}_j, \hat{v}_j)$

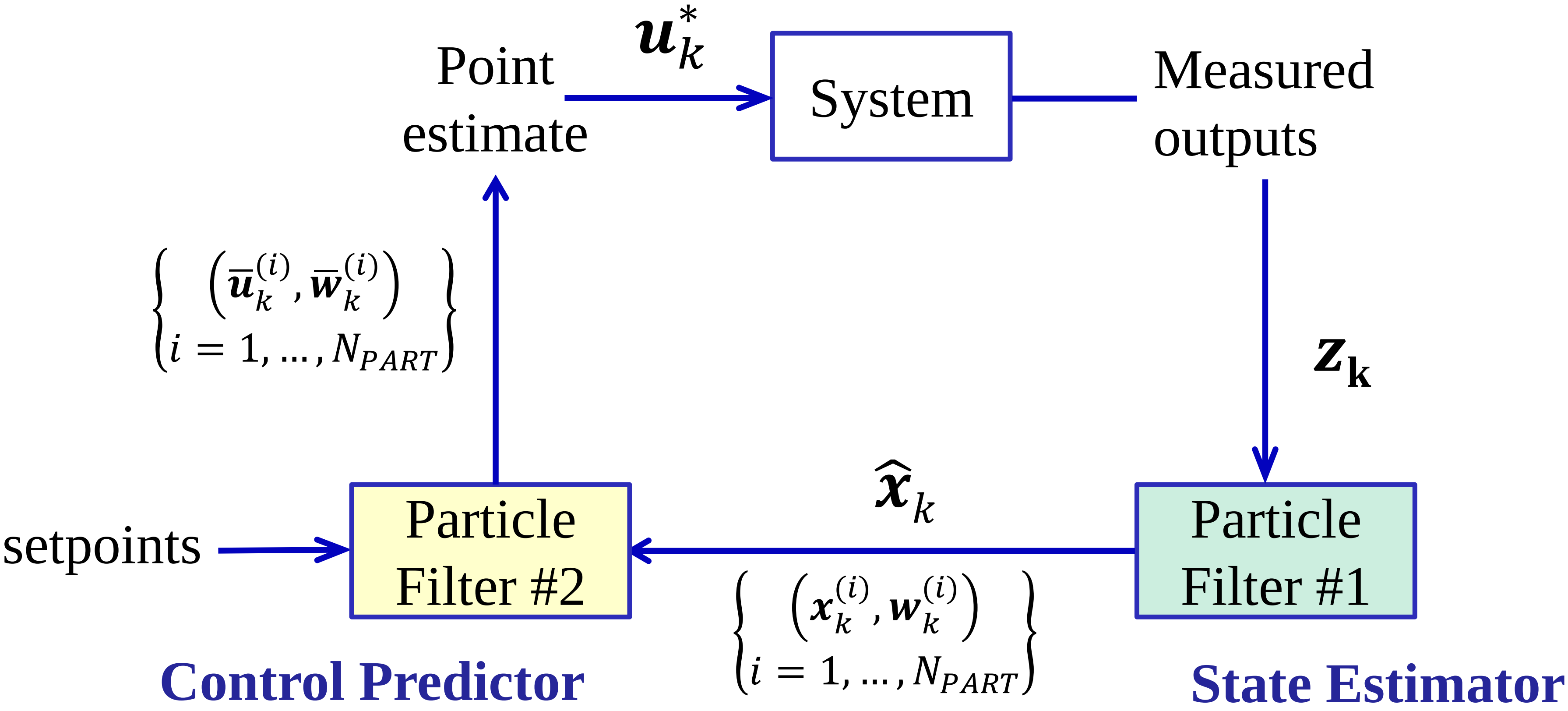
$$s_j = s_{desired} + \hat{v}_j \quad \hat{v}_j \sim N(0, \sigma^2)$$

If it is to keep at rest, $\boldsymbol{s_{desired}} = 0$.

PF-MPC

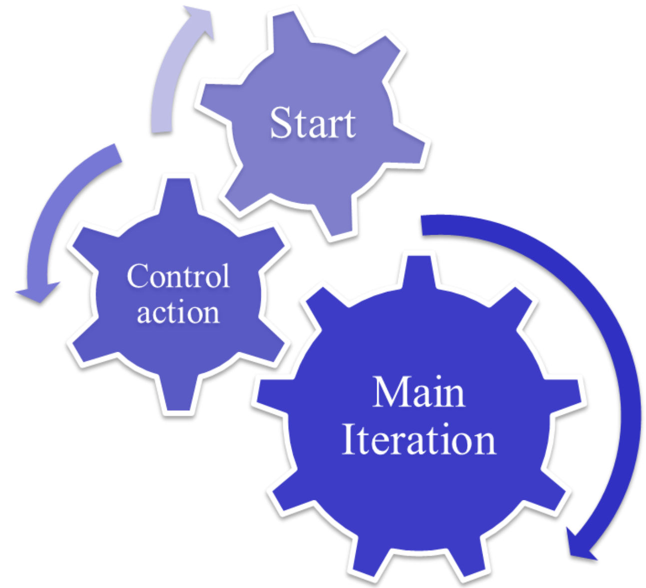


Adapted from
Stahl e Hauth (2011)



PF-MPC Algorithm

Particle Filter #2



1. Start $i = 1, \ldots, N_{part}$

$$\bar{x}_k^{(i)} = x_k^{(i)}, \bar{w}_k^{(i)} = w_k^{(i)}$$

$$\bar{u}_k^{(i)} = \bar{f}_u\left(u_{k-1}^*, \tilde{v}_{k-1}^{(i)}\right)$$

2. Iteration $j = k+1, \ldots, k+P$

$$\bar{x}_j = f(\bar{x}_{j-1}, \bar{u}_{j-1}, \bar{v}_{j-1})$$

$$\bar{u}_j = \bar{f}_u(\bar{x}_{j-1}, \bar{u}_{j-1}, \tilde{v}_{j-1})$$

$$s_j = \bar{g}(\bar{x}_j, \bar{u}_j, \hat{v}_j)$$

$$\tilde{u}_j = \tilde{u}_{j-1}$$

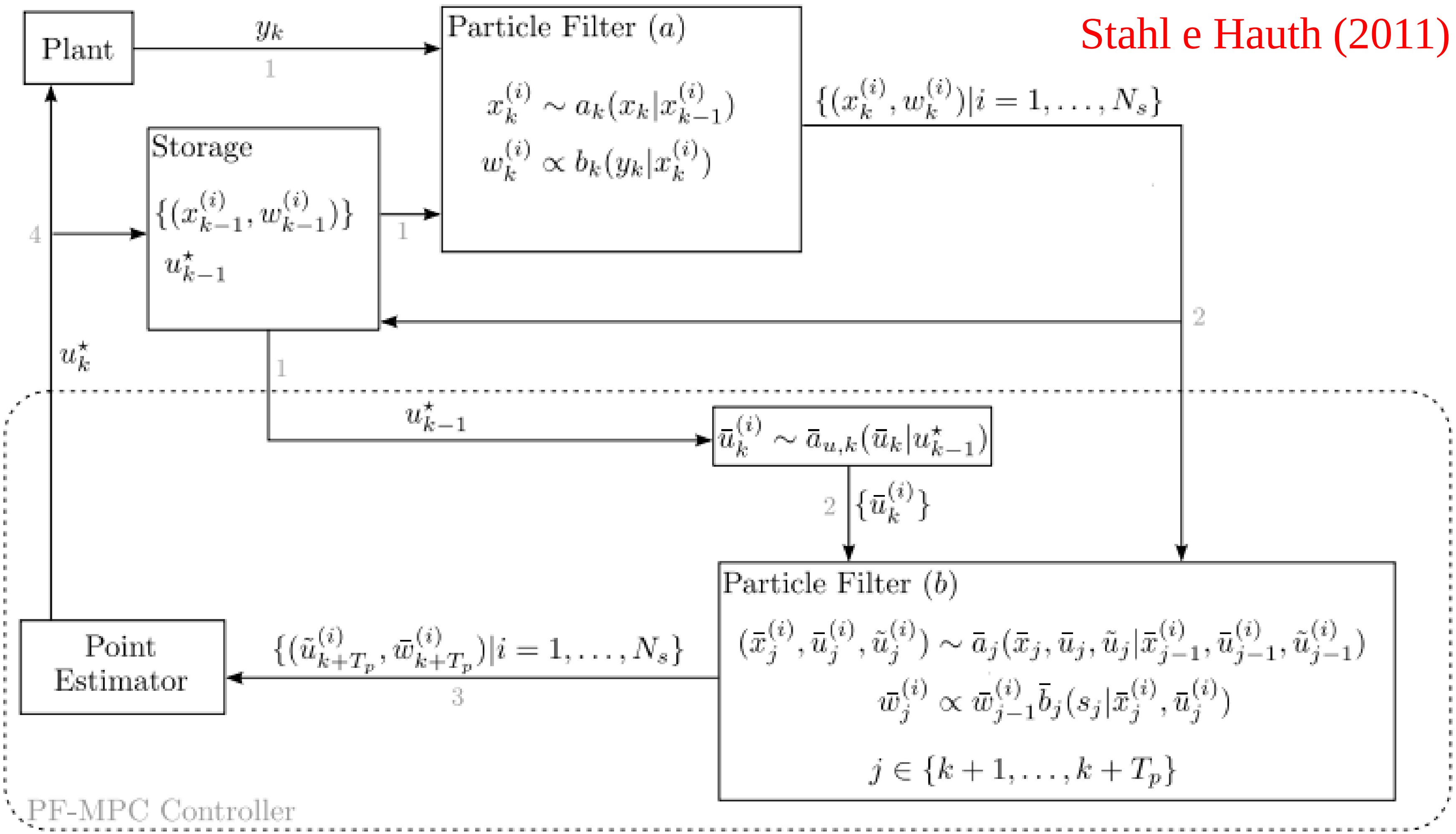
$$\bar{w}_j \propto \bar{w}_{j-1}\, \bar{b}_j(s_j | \bar{x}_j, \bar{u}_j)$$

3. Control action

$$\left(\bar{x}_{k+P}^{(i)}, \bar{u}_{k+P}^{(i)}, \tilde{u}_{k+P}^{(i)}, \bar{w}_{k+P}^{(i)}\right)_{i=1}^{N_{part}}$$

$$u_k^* = \sum_{i=1}^{N_{part}} \tilde{u}_{k+P}^{(i)} \bar{w}_{k+P}^{(i)}$$

PF-MPC Algorithm



Stahl e Hauth (2011)

Improvements to PF-MPC

Constrained particle filter

Zhao et al. (2012)

1. <u>Acceptance/rejection</u> : constraints on the prior particles x_k^{i-}

$$w_k^i = \begin{cases} 0. & if\ (x_k^{i-} \notin \Omega) \\ \propto p\left(y_k \middle| x_k^{i-}\right) & if\ (x_k^{i-} \in \Omega) \end{cases}$$

Main advantage	Disadvantages
It can guarantee no violation of particles and need no extra computation	Sample impoverishment
	Inconsistency if no particle meets the constraints
	It does not suffice for nonlinear constraint, equality, and inequality constraints

Improvements to PF-MPC

Constrained particle filter

Zhao et al. (2012)

2. Optimization approach: constraints on prior particles x_k^{i-}, posterior particles x_k^i, and/or the estimated value $\hat{x}_k$.

$$\min_{x_k^{i-}} -\log\left(p_{x_k^e}\left(\tilde{x}_k^{i-} - x_k^{i-}\right)\right)$$

$$\min_{x_k^{i-}} -\log\left(p_{x_k^e}\left(\tilde{x}_k^{i-} - x_k^{i-}\right)\right) - \log\left(p_{v_k}\left(y_k - h_k\left(\tilde{x}_k^{i-}\right)\right)\right)$$

$$\min_{x_k^{i-}} -\log\left(p_{x_k^e}\left(\tilde{x}_k^{i} - x_k^{i}\right)\right) - \log\left(p_{v_k}\left(y_k - h_k\left(\tilde{x}_k^{i}\right)\right)\right)$$

$$\min_{x_k^{i-}} -\log\left(p_{x_k^e}\left(\tilde{x}_k - \hat{x}_k\right)\right) - \log\left(p_{v_k}\left(y_k - h_k\left(\tilde{x}_k\right)\right)\right)$$

~ indicates projected particle or state estimation.

Improvements to PF-MPC

Constrained particle filter

Zhao et al. (2012)

2. Optimization approach: constraints on prior particles x_k^{i-}, posterior particles x_k^i, and/or the estimated value $\hat{x}_k$.

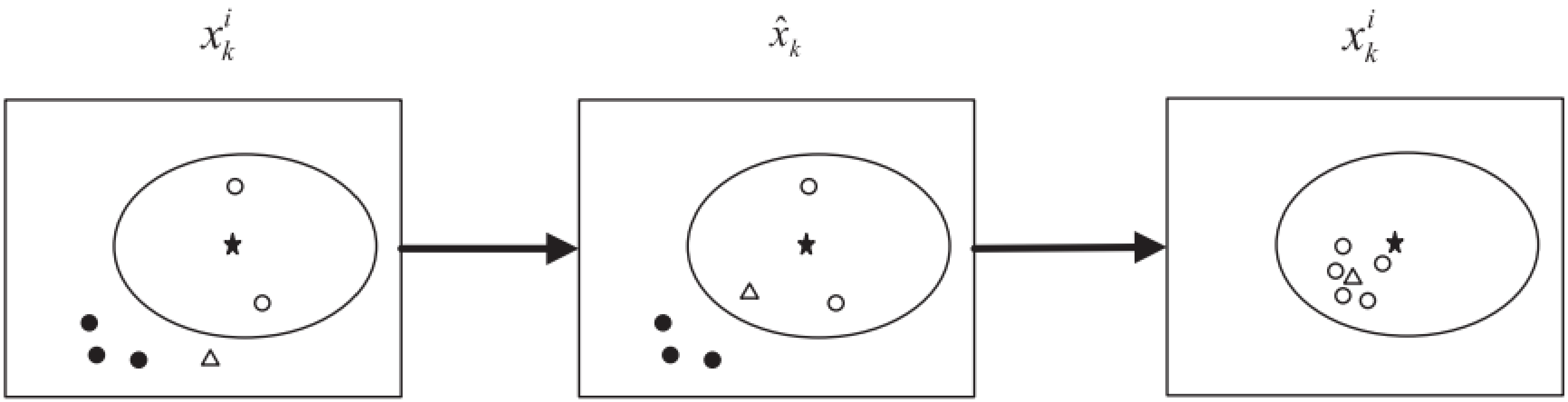


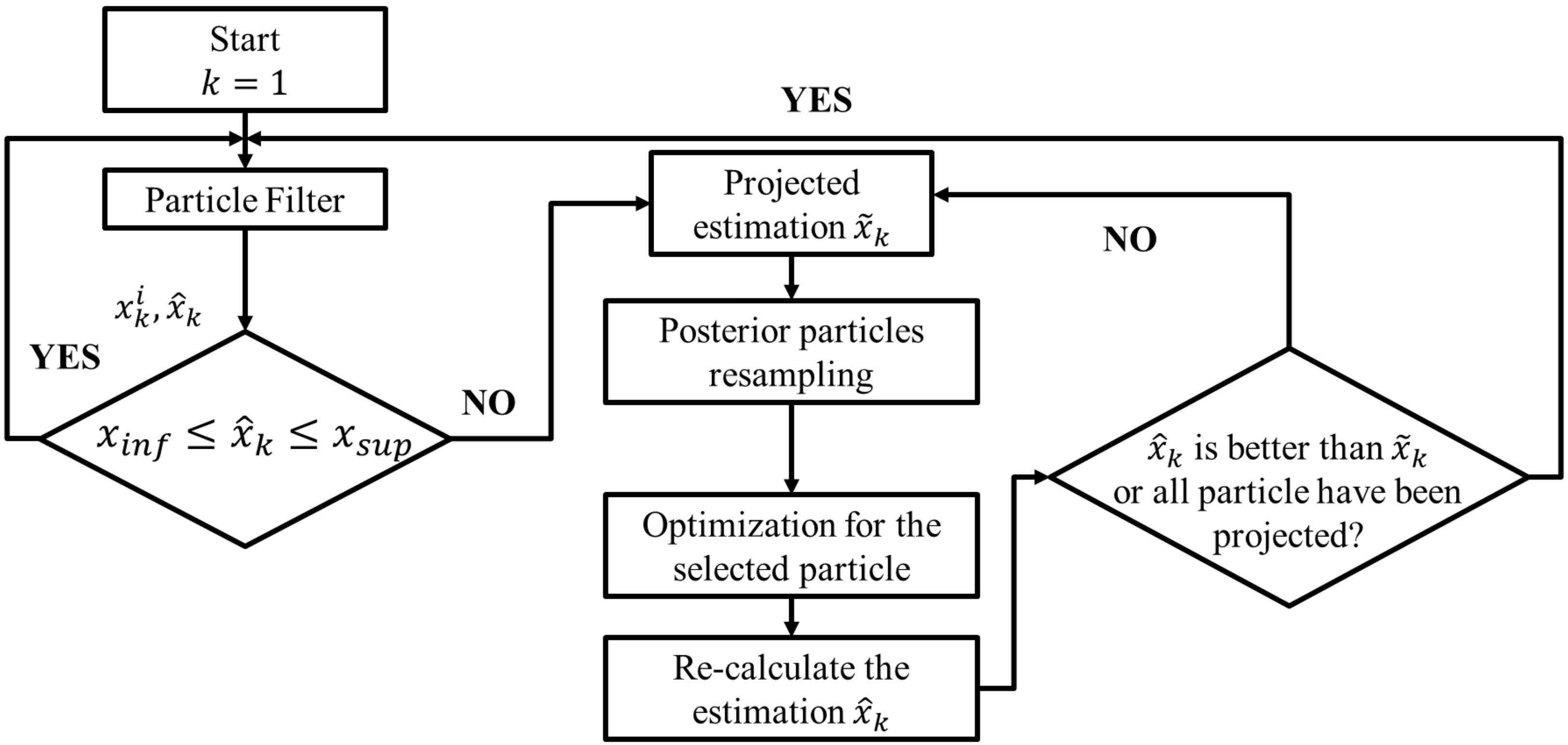
Illustration of the estimation projection (○: valid particle, ●: violated particle, △: estimation, ⋆: true state)

Improvements to PF-MPC

Constrained particle filter

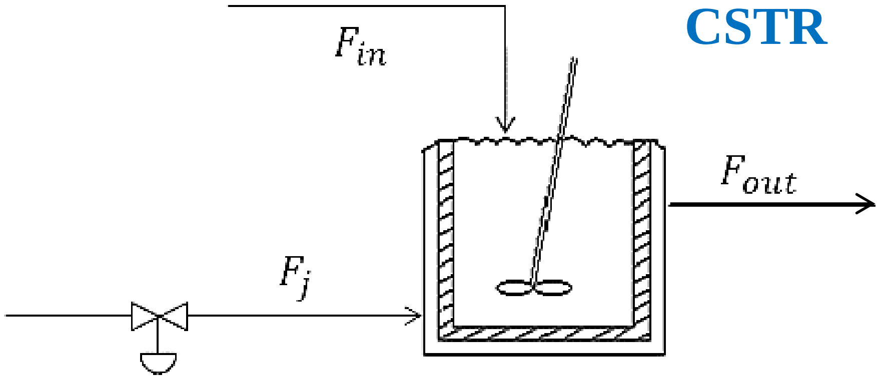
Zhao et al. (2012)

2. <u>Optimization approach</u>: procedure.



Application

Carvalho, R. F., 2017, *Controle preditivo baseado em modelo com estimação de estado restrita para controle e monitoramento de processos não lineares*. Dissertação de Mestrado (PPEQ/UFES).



PF-MPC structure

Objective: filter observed data and control Ca

R$_{\text{likelihood}}$: 0.1ºC, 0.5ºC and 5ºC

Npart: 10, 50 and 100

Performance: RMSE, AES

$$\frac{dCa}{dt} = \frac{F}{Vr}(Cao - Ca) - ko\ Ca\ e^{\frac{-E}{RTr}}$$

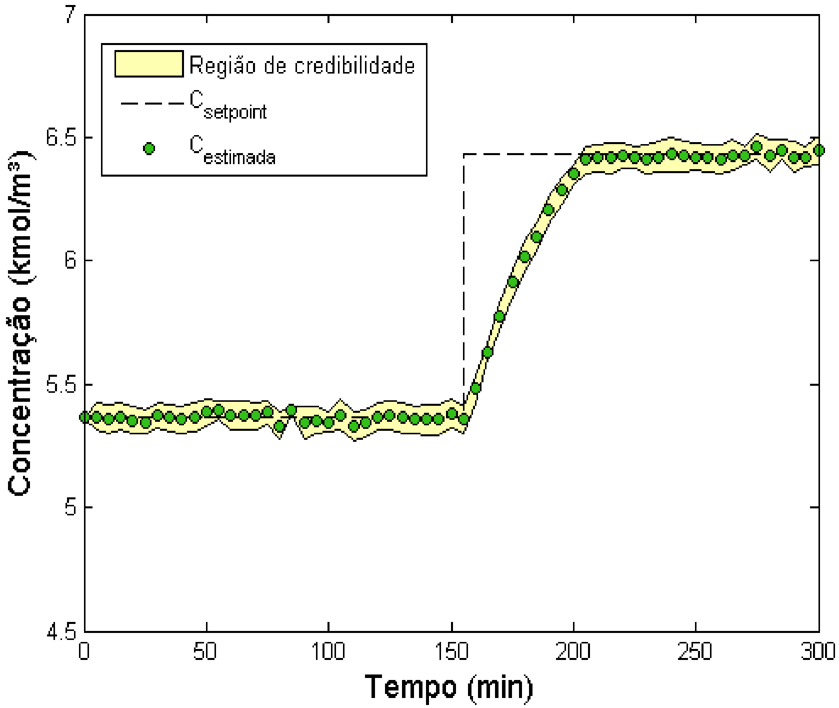
$$\frac{dTr}{dt} = \frac{F}{Vr}(Tf - Tr) - \frac{\Delta H}{\rho Cp} ko\ Ca\ e^{\frac{-E}{RTr}} - \frac{UA}{Vr\rho Cp}(Tr - Tj)$$

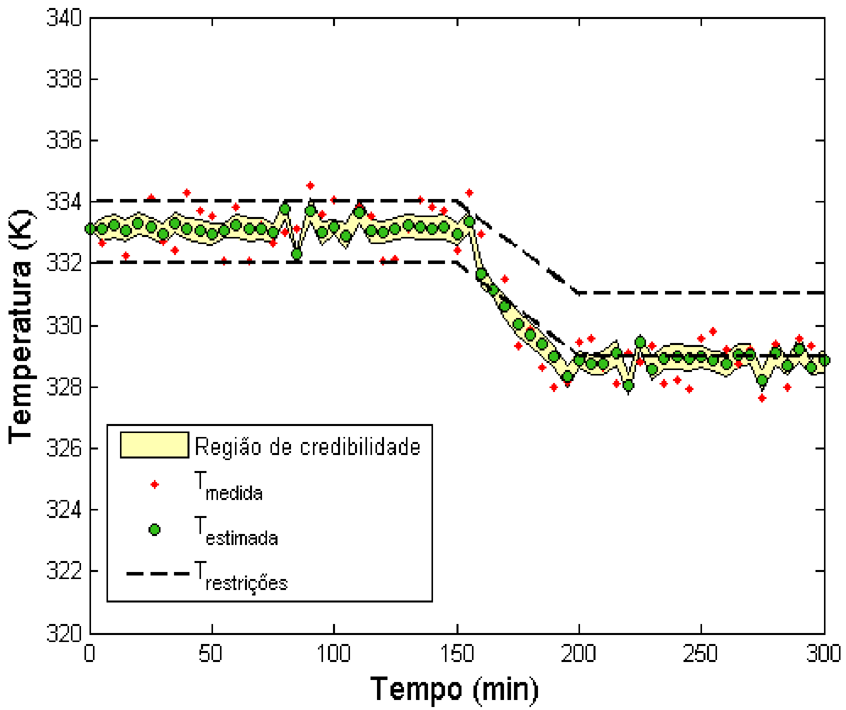
Application

Number of particles	R	RMSE	AES	Time (s)
10	0.1	0.0271	6.4795	0.2034
	0.5	0.0271	10.0073	0.1672
	5	0.0271	11.2071	0.1662
50	0.1	0.0220	6.1006	0.9453
	0.5	0.0220	9.7913	0.7375
	5	0.0220	10.8588	0.7186
100	**0.1**	**0.0217**	**6.0001**	**1.6765**
	0.5	0.0217	9.4079	1.4797
	5	0.0217	10.6891	1.4093

Application

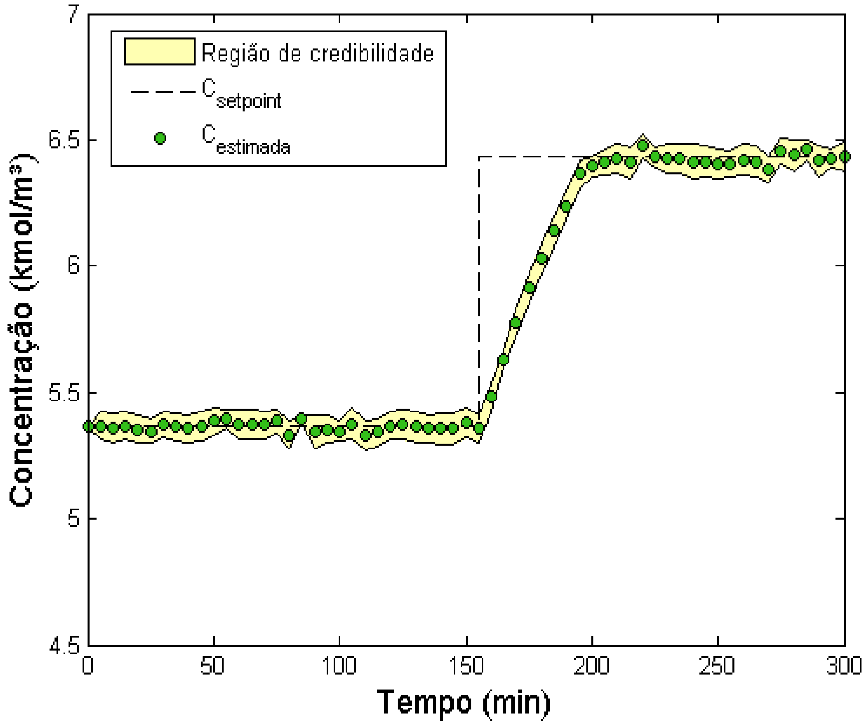
PF-MPC

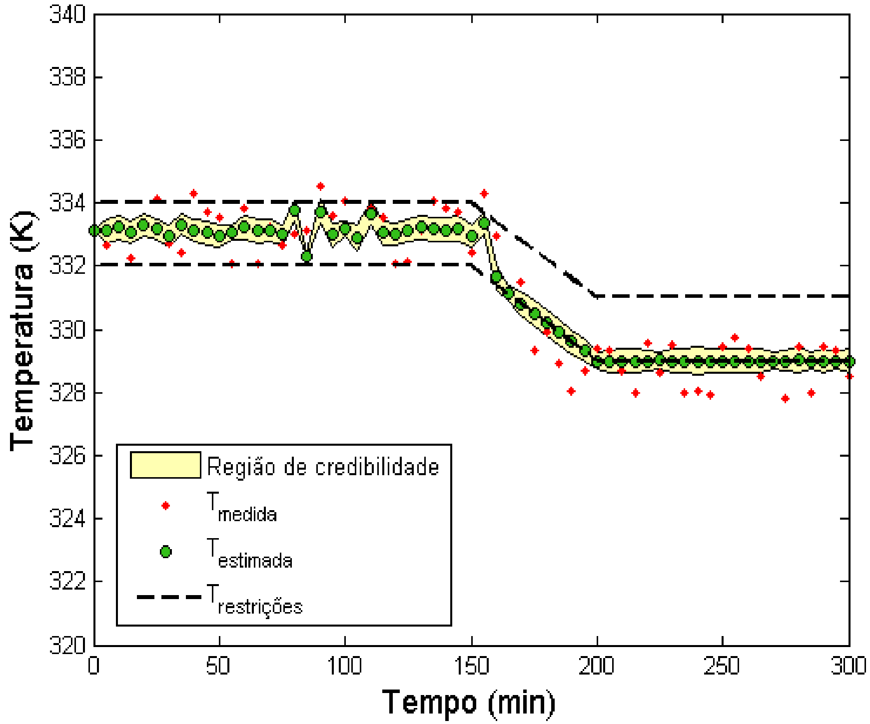




Application

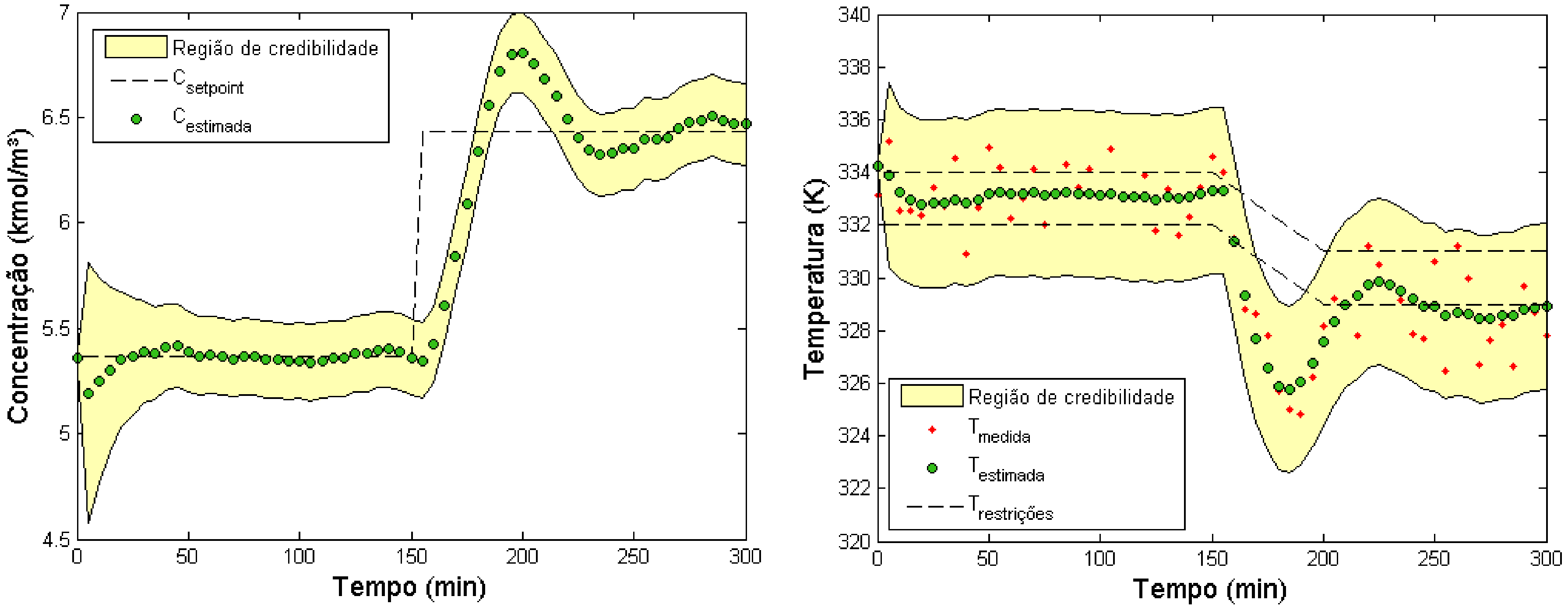
CPF-MPC





Application

CUKF-MPC



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Pura e Aplicada

References

ANDRIEU, C.; DOUCET, A.; SINGH S. S.; TADIC, V. B. Particle Methods for Change Detection, System Identification, and Control. Proceedings of the IEEE. v. 92, p. 423-438, 2004.

ARULAMPALAM, M. S.; MASKELL, S.; GORDON, N.; CLAPP, T. A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking, IEEE Transactions on Signal Processing, v. 50, p. 174-188, 2002.

ASTROM, K. J.; HAGGLUND, T. Advanced PID Control. ISA – Instrumentation, Systems, and Automation Society, 2006.

BIEGLER, L. T., "An overview of simultaneous strategies for dynamic optimization". Chemical Engineering and Processing, v. 46, pp. 1043–1053, 2007.

BLACKMORE, L. A probabilistic Particle Control Approach to Optimal, Robust Predictive Control. Computer Science and Artificial Intelligence Laboratory Techinical Report, 2006.

BOTCHU, S. K.; UNGARALA, S. Nonlinear model predictive control based on Sequential Monte Carlo state estimation. Preprints of the 8th IFAC Symposium on Dynamics and Control of Process Systems. v. 3, 2007.

CARVALHO, R. F., Controle preditivo baseado em modelo com estimação de estado restrita para controle e monitoramento de processos não lineares. Dissertação (Mestrado em Engenharia Química), PPEQ, Universidade Federal do Espírito Santo, 2017.

CARVALHO, R. F.; DIAS, A. C. S. R.; SILVA, W. B.; DUTRA, J. C. S. Filtro de partículas como observador online em um esquema de controle cascata para um reator contínuo. Anais do XIX Encontro Nacional de Modelagem Computacional, João Pessoa, 2016.

CHEN, T.; MORRIS, J.; MARTIN, E. Dynamic data rectification using particle filters, Computers & Chemical Engineering, v. 32, p. 451-462, 2008.

DIAS et al., 2017, "Propylene Polymerization Reactor Control and Estimation Particle Filter and Neural Network". Macromolecular Reaction Engineering DOI: 10.1002/mren.201700010.

DIAS, A. C. S. R.; DEMUNER, L.; RANGEL, F. D; DUTRA, J. C. S; SILVA, W. B. da. Online state estimation through particle filter for feedback temperature control. Anais do XXI Congresso Brasileiro de Engenharia Química. Fortaleza, 2016.

DOUCET, A.; GODSILL, S.; ANDRIEU, C. On sequential Monte Carlo sampling methods for Bayesian filtering. Statistics and Computing. v. 10, p. 197-208, 2000.

DUTRA, J. C. S. Controle de processos baseado em esquemas de reconfiguração. Tese (Doutorado em Engenharia Química), COPPE, Universidade Federal do Rio de Janeiro, 2012.

DUTRA, J. C. S.; FEITAL, T. S.; SKOGESTAD, S.; LIMA, E. L.; PINTO, J. C. Control of bulk propylene polymerizations operated with multiple catalysts through controller reconfiguration. Macromolecular Reaction Engineering, v. 8, p. 201-216, 2014.

FOSS, A. S., “Critique of chemical process control theory”. AIChE Journal, v. 19, p. 209-214,1973.

KAIPIO, J.; SOMERSALO, E. Statistical and computional inverse problems, Applied Mathematical Sciences 160, Springer-Verlag, 2004.KANTAS, N.; MACIEJOWSKI, J. M.; LECCHINI-VISINTINI, A. Sequential Monte Carlo for Model Predictive Control. LNCIS. v. 384, p. 263-273, 2009.

KHATIBISEPEHR, S.; HUANG, B.; KHARE, S. Design of inferential sensors in the process industry: A review of Bayesian methods. Journal of Process Control, v. 23, p. 1575-1596, 2013.

LIU, J. Developing a soft sensor with online variable reselection for unobserved multi-mode operations. Journal of Process Control, v. 42, p. 90-103, 2016.

LOPEZ, L.F. Cyber-enabled Manufacturing Systems (CEMS): Model-based estimation and control of a solidification process. Dissertation (Doctor of Philosophy) – The University of Texas at Austin, 2014

MACIEJOWSKI, J. M. Predictive Control with Constraints. Prentice Hall, 2002.

ORLANDE, H.; COLAÇO, M.; DULIKRAVICH, G.; VIANNA, F.; DA SILVA, W. B.; FONSECA, H.; FUDYM, O. State estimation problems in heat transfer. International Journal for Uncertainty Quantification, v. 2, p. 239-258, 2012.

PRATA, D. M.; SCHWAAB, M.; LIMA, E. L.; PINTO, J. C. Nonlinear dynamic data reconciliation and parameter estimation through particle swarm optimization: Application for an industrial polypropylene reactor. Chemical Engineering Science, v. 64, p. 3953-3967, 2009.

QIN, S. J.; BADGWELL, T. A. A survey of industrial model predictive control techonolgy. Control Engineering Practice. v.11, p. 733-764, 2003.

RAWLINGS, J. B., “Tutorial overview of model predictive control”. IEEE Control Systems Magazine, v. 3, n. 20, pp. 38-52, 2000.

RAWLINGS, J., MAYNE, D., Model Predictive Control Theory and Design. Nob Hill Pub, 2009.

REWAGAD & KISS, 2012, “Dynamic optimization of a dividing-wall column using model predictive control”. Chemical Engineering Science, v. 68, pp. 132-142.

RISTIC, B.; ARULAMPALAM, S.; GORDON, N. Beyond the Kalman Filter. Boston: Artech House, 2004.

ROSSITER, J. A., Model-based predictive control – a practical approach. CRC Press, New York, 2004.

SEBORG, D. E.; EDGAR, T. F.; MELLICHAMP, D. A.; Process Dynamics and Control. 2. ed. Danvers, MA: John Wiley & Sons, 2003.

SILVA, W. B.; DUTRA, J. C. S.; ABREU, L. A. S.; KNUPP, D. C.; SILVA NETO, A. J. Estimation of timewise varying boundary heat flux via bayesian filters and markov chain monte carlo method. Apresentado na XVIII Convención Científica de Ingeniería y Arquitectura, Havana, Cuba, November, 2016.

SKOGESTAD, S., 2004, “Control structure design for complete chemical plants”. Computers and Chemical Engineering, v. 28, pp. 219-234

SKOGESTAD, S., POSTLETHWAITE, L., 2005, “Multivariable Feedback Control”, New York, John Wiley and Sons.

STAHL, D.; HAUTH, J. Particle filter - model predictive control. Systems & Control Letters, v. 60, p. 632–643, 2011.

VILLIERS, J.P.; GODSILL, S.J.; SINGH, S.S. Particle predictive control. Journal of Statistical Planning and Inference. v. 141, p. 1753-1763, 2011.

ZHAO, Z.; HUANG, B.; LIU, F. A particle filter based on a constrained sampling method for state estimation, Information Fusion (FUSION), 15th International Conference on, p. 816-823, 2012.

ZHAO, Z.; HUANG, B.; LIU, F. Constrained particle filtering methods for state estimation of nonlinear process, Aiche journal, vol. 60, p.2072-2082, 2014.

ZHENG, A., MAHAJANNAMAND, R.V., DOUGLAS, J. M., 1999, “Hierarchical procedure for plant-wide control system synthesis”. AIChE Journal, v. 45, pp. 1255–1265.