Reconstruction of Coefficients and Source Parameters in Elliptic Systems

by

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Thanks to Carlos J.S. Alves from CEMAT-Instituto Superior Tecnico- Lisbon University for the help full discussion.

Objectives

- The objective of this presentation is describes some important aspects related with the reconstruction of parameters in models described with elliptic partial differential equations.
- We will not discuss regularization strategies.
- Incomplete information about coefficients and source is compensated by an overprescription of Cauchy data at the boundary.
- The methodology we propose explores concepts as:
  1. Lipschitz Boundary Dissection;
  2. Complementary Mixed Problems with trial parameters;
  3. Internal Discrepancy Fields.
- The main techniques are variational formulation, boundary integral equations and Calderon projector.

The engineering problem

- Most of the stationary engineering models can be represented as elliptic system of partial differential equations.
- Those models are mathematically elaborated with continuous thermomechanics and the constitutive theories of materials.
- Constitutive equations removes ambiguity in the model and frequently presents incomplete information about parameters.
- To assure uniqueness of model solution we must combine boundary information with correct parameters values.
- Estimation of missing parameters in diffusion reaction convection like systems of equations are the main problem.

Introduction

- In this work we study the problem of reconstruction of coefficients and source parameters in second order strongly elliptic systems [1], [2].
- Let Ω be a Lipschitz domain. Its boundary can be locally as the graph of a Lipschitz function, that is, a Holder continuous $C^{0,1}$ function.
- Let $F_\alpha = [f_\alpha, ..., f_\alpha] \in (L^2(\Omega))^{m \times N_p}$ be the source and
- $(H, H_\nu) \in (H^{\frac{1}{2}}(\partial\Omega) \times (H^{-\frac{1}{2}}(\partial\Omega)))^{m \times N_p}$ the Cauchy data for N_p problems based on the $m-$fields model.

The inverse problem

- The inverse boundary value problem for parameter determination investigated here is: To find $(U, \alpha) \in H^1(\Omega)^{m \times N_p} \times \mathbb{R}^{N_a}$ such that

$$P^{\alpha}_{F_\alpha, H, H_\nu} \begin{cases} \mathcal{L}_\alpha U = F_\alpha & \text{if } x \in \Omega; \\ \gamma[U] = H & \text{if } x \in \partial\Omega; \\ \mathcal{B}_\nu[U] = H_\nu & \text{if } x \in \partial\Omega; \end{cases} \tag{1}$$

- Here γ is the boundary trace and
- $\mathcal{B}_\nu$ is the conormal trace.
- The coefficients of the strongly elliptic operator $\mathcal{L}_\alpha$, self-adjoint, and the source depend on the parameters α.

The Parameters to Cauchy Data Implicit Function

- The main question is how Cauchy data (H, H_ν) are related with the constitutive parameters α and the source F_α? Some functional equation $\mathcal{C}(\alpha, H, H_\nu, F_\alpha) = 0$ which could solve our problem, or, at least, conduct to a good framework to analysis.
- What are the consequence of incorrect values on the parameters?
- What are the consequence of incorrect values of the Cauchy Data?
- By using properties the Calderon Projector we will develop a methodology for the parameters determination problem by solving only direct problems and optimization problems.

Auxiliary mixed problem

- Let $\partial\Omega = \partial\Omega_D \cup \Pi \cup \partial\Omega_N$ a Lipschitz dissection of the boundary.
- The auxiliary mixed boundary value problem for inverse problem (1) is given by the well posed problem $P^{\alpha}_{f_\alpha, g^D, g^N_\nu}$: For given Dirichlet and Neumann data $(g^D, g^N_\nu) \in (H^{\frac{1}{2}}(\partial\Omega_D) \times H^{-\frac{1}{2}}(\partial\Omega_N))^m$, find $u \in H^1(\Omega)^m$ such that

$$P^{\alpha}_{f_\alpha, g^D, g^N_\nu} \begin{cases} \mathcal{L}_\alpha u = f_\alpha & \text{if } x \in \Omega; \\ \gamma[u] = g^D & \text{if } x \in \partial\Omega_D; \\ \mathcal{B}_\nu u = g^N_\nu & \text{if } x \in \partial\Omega_N; \end{cases} \tag{2}$$

Complementary Problems and Internal Discrepancy Field

- The over prescription of data at the boundary is used to introduce the concept of complementary problems with the same constitutive equations.
- **Internal discrepancies** between **solutions of complementary problems** are observed when the model is supplied with **parameters values in the constitutive equations** that are **inconsistent with the Cauchy data** prescribed at the boundary.
- This **discrepancy field** measures the deviation from uniformity in sets of points in the interior of Ω. It is due to the solution of the complementary problems with incorrect parameters values.

Complementary Problems on Lipschitz Domains

Definition

Let us consider two mixed boundary value problems $P^{\alpha}_{f^{(1)},g^{(1)},g_{\nu}^{(1)}}$ *and* $P^{\alpha}_{f^{(1)},g^{(2)},g_{\nu}^{(2)}}$ *defined on the same Lipschitz domain* Ω *with* **boundary dissection** $\partial\Omega = \Gamma_D^{(1)} \cup \Pi \cup \Gamma_N^{(1)}$.

We say that these problems are complementary if $f_{\alpha}^{(1)} = f_{\alpha}^{(2)}$, $\Gamma_D^{(2)} = \Gamma_N^{(1)}$, $\Gamma_N^{(2)} = \Gamma_D^{(1)}$ *and there exist a Cauchy data* $(g^{\alpha}, g_{\nu}^{\alpha})$ *such that*

$$g^{(1)} = g^{\alpha}\chi_{\Gamma_D^{(1)}} \text{ and } g^{(2)} = g\chi_{\Gamma_D^{(2)}}.$$

$$g_{\nu}^{(1)} = g_{\nu}^{\alpha}\chi_{\Gamma_N^{(1)}} \text{ and } g_{\nu}^{(2)} = g_{\nu}^{\alpha}\chi_{\Gamma_N^{(2)}}.$$

where χ_{Γ} *is the characteristic function for set* Γ.

Definition

The Calderon operator is the 2×2 linear operator $\mathcal{C} : (H^{\frac{1}{2}}(\Omega))^m \times (H^{-\frac{1}{2}}(\Omega))^m \to (H^{\frac{1}{2}}(\Omega))^m \times (H^{-\frac{1}{2}}(\Omega))^m$ defined by

$$\mathcal{C}[\gamma[u], \mathcal{B}_\nu[u]]^T = \begin{bmatrix} -\gamma DL[\gamma[u]] & \gamma SL[\mathcal{B}_\nu[u]] \\ -\mathcal{B}_\nu DL[\gamma[u]] & \mathcal{B}_\nu SL[\mathcal{B}_\nu[u]] \end{bmatrix}$$

- The Calderon operator is a projector: $\mathcal{C}[g, g_\nu] = [g, g_\nu]^T = \mathcal{C}^2[g, g_\nu]^T$
- It has the following index zero Fredholm representation:

$$\mathcal{C} = \begin{bmatrix} \frac{1}{2}(I - T) & S \\ R & \frac{1}{2}(I + T^*) \end{bmatrix}.$$

Theorem on Complementary Solution

Theorem

Suppose that $P^{\alpha}_{f^{(1)},g^{(1)},g_\nu^{(1)}}$ *and* $P^{\alpha}_{f^{(1)},g^{(2)},g_\nu^{(2)}}$ *are the two mixed complementary boundary value problems with respective solutions* $u^{(1)}$ *and* $u^{(2)}$*, then*

$$u^{(1)} = u^{(2)}$$

independent of the Cauchy Data Lipschitz dissection.

- The proof is based on the concept of Calderon projector [1].
- As consequence, for the same Cauchy data, we can make different Lipschitz dissections.

Sketch of the Proof: By noting that $P^{\alpha}_{f^{(1)},g^{(1)},g^{(1)}_{\nu}}$ and $P^{\alpha}_{f^{(1)},g^{(2)},g^{(2)}_{\nu}}$ are the two mixed Complementary boundary value problems with solutions $u^{(1)}$ and $u^{(2)}$, respectively. the solution will be given by

$$u^{(p)}(x) = \int_{\Omega} G^{\alpha}_{x}(\xi) f^{\alpha}(\xi) d\xi - DL^{\alpha}[\gamma[u^{(p)}]](x) + SL^{\alpha}[\mathcal{B}_{\nu}[u^{(p)}]](x) \ ,$$

for $p = 1, 2$ the two complementary problems.

- Note that the unknown part of boundary trace and conormal trace can be calculated via boundary integral equation methods.
- It is not difficult, by using one of BIE formulations that the unknown information about the traces of one problem coincides with the known information of the other complementary problem.
- The correct parameter α assures that the Calderon projector has no gap.

Matrix equation with some Lipschitz Boundary Dissection:

$$\begin{bmatrix} \frac{1}{2}(I^{DD}_{x\to\xi} - T^{DD}_{x\to\xi}) & -T^{ND}_{x\to\xi} & S^{DD}_{x\to\xi} & S^{ND}_{x\to\xi} \\ -T^{DN}_{x\to\xi} & \frac{1}{2}(I^{NN}_{x\to\xi} - T^{NxN}_{x\to\xi}) & S^{DN}_{x\to\xi} & S^{NN}_{x\to\xi} \\ R^{DD}_{x\to\xi} & R^{ND}_{x\to\xi} & \frac{1}{2}(I^{DD}_{x\to\xi} + T^{*DD}_{x\to\xi}) & T^{*ND}_{x\to\xi} \\ R^{DN}_{x\to\xi} & R^{NN}_{x\to\xi} & T^{*DN}_{x\to\xi} & \frac{1}{2}(I^{NN}_{x\to\xi} + T^{*NN}_{x\to\xi}) \end{bmatrix}$$

$$\times \begin{bmatrix} \gamma u(x)|_{\Gamma_D} \\ \gamma u(x)|_{\Gamma_N} \\ \mathcal{B}_\nu u(x)|_{\Gamma_D} \\ \mathcal{B}_\nu u(x)|_{\Gamma_N} \end{bmatrix} + \begin{bmatrix} \int_\Omega \gamma_\xi G_\xi|_{\Gamma_D}(y)f(y)dy \\ \int_\Omega \gamma_x iG_\xi|_{\Gamma_N}(y)f(y)dy \\ \int_\Omega \mathcal{B}_{\nu_\xi} G_\xi|_{\Gamma_D}(y)f(y)dy \\ \int_\Omega \mathcal{B}_{\nu_\xi} G_\xi|_{\Gamma_D}(y)f(y)dy \end{bmatrix} = \begin{bmatrix} \gamma u(\xi)|_{\Gamma_D} \\ \gamma u(\xi)|_{\Gamma_N} \\ \mathcal{B}_\nu u(\xi)|_{\Gamma_D} \\ \mathcal{B}_\nu u(\xi)|_{\Gamma_N} \end{bmatrix}.$$

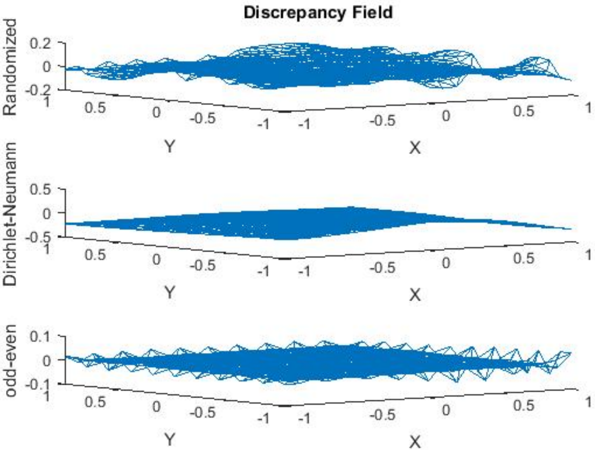
So, Cauchy data obtained by the extension formulates a unique problem with integral representation, which is independent of the Cauchy data dissection.

Fact (Existence of Internal Discrepancy Fields)

(i) *For a given association of a Lipschitz domain with a model given by the operator $\mathcal{L}_\alpha$, the Calderon projector is as a restriction which the Cauchy data must satisfy in order to be a consistent data with boundary value problems.*

(ii) *If the inverse problem $P^{\alpha}_{F_\alpha,H,H_\nu}$ is solved with trial parameters values different from the exact value, $\alpha^{(0)} \neq \alpha$, the associated Calderon projector will present a gap.*

(iii) *Then the solutions of complementary problems associated with the Cauchy data will be different, and the* **internal discrepancy field** $D^{(1,2)} = u^{(1)} - u^{(2)}$ *is calculated.*


Discrepancy Field
Randomized
0.2
0
-0.2
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
Y
X
Dirichlet-Neumann
0.5
0
-0.5
odd-even
0.1
0
-0.1

The Dirichlet Functional

- Diffusion ($c_\alpha(x) > 0$) absorption ($a_\alpha(x) > 0$) problem:

$$\begin{array}{ll} \mathcal{L}_\alpha u(x) = -\nabla \cdot c_\alpha(x) \nabla u(x) + a_\alpha(x) u(x) & \text{if } x \in \Omega; \\ \gamma[u](x) = u(x) \text{ Trace Operator} & \text{if } x \in \partial\Omega_D; \\ \mathcal{B}_\nu[u](x) = c_\alpha(x) \nabla u(x) \text{ Conormal Trace} & \text{if } x \in \partial\Omega_N; \end{array}$$

- Dirichlet Functional and the First Green Identity:

$$\Phi_\alpha(u, v) := \int_\Omega [c_\alpha(x) \nabla u(x) \nabla v(x) + a_\alpha(x) u(x) v(x)] dx$$

$$= \int_\Omega \mathcal{L}_\alpha u(x) v(x) dx + \int_{\partial\Omega} \mathcal{B}_\nu[u](x) \gamma[v](x)$$

- For fixed α , $\Phi_\alpha(u, v)$ defines a bilinear symetric form.

- If $u = v$ are functions in a normed space, then $\Phi_\alpha(u, u)$ is the energy norm.
- If $u = \phi_i$ and $v = \phi_j$, $i, j = 1, 2, 3, \ldots$ are finite elements basis in a Galerkin approximation, then $\Phi_\alpha(u, v)$ is the sum of stiffness and absorption matrix.
- If $u = v = \phi_i$, $i = 1, 2, 3, \ldots$ are orthonormal eigenfunctions in the spectral problem for this model, then $\lambda_i := \Phi_\alpha(\phi_i, \phi_i)$ are the respective eigenvalue.
- The rate of decay of the sequence $\{\mu_i := 1/\Phi_\alpha(\phi_i, \phi_i), i = 1, 2, \ldots\}$ gives information about the ill-conditioning of this system of the inverse coefficients problems associated with this model. Moderately ill-posed for polynomial decay and severely ill posed for exponential decay.

- The weak formulation $W^{\alpha}_{f_\alpha, g^D, g^N_\nu}$ for the mixed problem (2):
  To find $(u, \lambda) \in H^{\frac{1}{2}}(\partial\Omega_D) \times H^{-\frac{1}{2}}(\partial\Omega_N)$

$$\begin{cases} \Phi_\alpha(u, v) - \langle \gamma^*[\lambda], v\rangle_{\partial\Omega_D} = \langle f_\alpha, v\rangle_\Omega + \langle g^N, \gamma[v]\rangle_{\partial\Omega_N} \\ \langle \gamma[u], \mu\rangle = \langle g^D, \mu\rangle_{\partial\Omega_D} \\ \forall (v, \mu) \in H^{\frac{1}{2}}(\partial\Omega_D) \times H^{-\frac{1}{2}}(\partial\Omega_N). \end{cases}$$

  where the Lagrange multiplier λ is a conormal trace of some $H^1(\Omega)$ function.

- Note tha tthe extension operator
  $\gamma^*[.] : H^{-\frac{1}{2}}(\partial\Omega) \to (H^1(\Omega))^*$ is defined by
  $\langle \gamma[v], \lambda\rangle_{\partial\Omega} = \int_{\partial\Omega} \gamma[v]\lambda ds_x = \int_\Omega \gamma^*[\lambda] v dx = \langle \gamma^*[\lambda], v\rangle_\Omega$.

Theorem (Lagrangian Functional Saddle Critical Point)

$(u,\lambda) \in H^1(\Omega) \times H^{-\frac{1}{2}}(\partial\Omega)$ is solution of the mixed problem

$$W^{\alpha}_{f_\alpha, g^D, g^N_\nu} \Leftrightarrow \mathcal{A}_\alpha(u,\mu) \leq \mathcal{A}_\alpha(u,\lambda) \leq \mathcal{A}_\alpha(v,\lambda)$$

for all $(v,\mu) \in H^1(\Omega) \times H^{-\frac{1}{2}}(\Omega)$. The Lagrangian functional is

$$\mathcal{A}_\alpha(v,\lambda) := \frac{1}{2}\Phi_\alpha(v,v) - \langle \gamma[v], \lambda\rangle_{\partial\Omega_D} - \langle f, v\rangle_\Omega + \langle g_D, \lambda\rangle_{\partial\Omega_D} - \langle \gamma[v], g_N\rangle_{\partial\Omega_N}$$

- For a fixed Lipschitz boundary dissection,
- to find $(U_\alpha, \Lambda_{dir}) \in \mathbb{R}^{N_v \times Np} \times \mathbb{R}^{N_{dir} \times Np}$ such that

$$\begin{cases} (K_\alpha + A_\alpha)U_\alpha - Tr_{dir}^T \Lambda_{dir} = F_\alpha + Tr_{neu}^T G_{neu} \\ Tr_{dir} U_\alpha = G_{dir} \end{cases}$$

- $size(K_\alpha) = size(A_\alpha) = [N_v, N_v]$
- $size(F_\alpha) = [N_v, N_p]$
- $size(G_{dir}) = [N_{dir}, N_p]$ and $size(G_{neu}) = [N_{neu}, N_p]$
- $size(Tr_{dir}) = [N_{dir}, N_v]$ and $size(Tr_{neu}) = [N_{neu}, N_v]$

where N_v , N_{dir} , N_{neu} and N_p are respectively the number of vertices on $\Omega \cup \partial\Omega$, $\partial\Omega_D$, $\partial\Omega_N$ and the number of problems with the same parameters α values .

Optimization Problem based on the Discrepancy Fields

Problem (1) can now be posed as the following optimization problem:

Problem

In the guess set of parameters $\alpha^{(0)} \in \{[\alpha_1, \alpha_2] \subset \mathbb{R}^{N_\alpha}\}$, *to find* α *that minimizes the distance between complementary solutions*

$$u^{(1)}_{\alpha, Ld, Cauchy} \text{ and } u^{(2)}_{\alpha, Ld, Cauchy}$$

for all Cauchy data and all respective Lipschitz dissected solutions.

Remark (The Reciprocity Gap Method for Discrepancy Field)

Let $H_{\mathcal{L}_\alpha}(\Omega) = \{v \in H^1(\Omega) : \mathcal{L}_\alpha v = 0\}$ *the set of* $\mathcal{L}_\alpha$*-harmonics functions.*

(i) *The discrepancy field* $D^{(1,2)}_{\alpha_0, L_d, Cauchy}$ *associated with the wrong parameter* α_0 *and exact parameter* α *is* $\mathcal{L}_{\alpha_0}$*-harmonic.*

(ii) *Let* (U, α) *a solution of problem* $P^{\alpha}_{F_\alpha, H, H_\nu}$ *as in equation (1). Then they satisfy the following Reciprocity Gap equation:*

$$\int_\Omega ((\mathcal{L}_\alpha - \mathcal{L}_{\alpha_0})[U](x) - F_\alpha(x)) D^{(1,2)}_{\alpha_0, L_d, Cauchy}(x) dx =$$

$$\int_{\partial\Omega} H_\nu(x) \gamma[D^{(1,2)}_{\alpha_0, L_d, Cauchy}](x) dS_x - \int_{\partial\Omega} H(x) \mathcal{B}_\nu [D^{(1,2)}_{\alpha_0, L_d, Cauchy}](x) ds_x \tag{3}$$

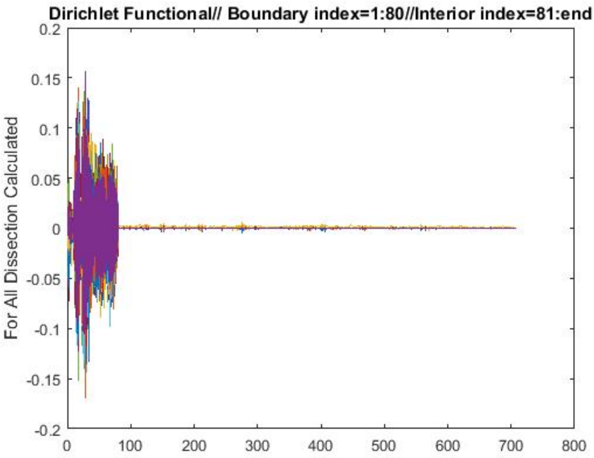
Proposition (The Variational Method for Discrepancy Field)

Let $\alpha_0 \neq \alpha$ a parameter trial in the inverse parameter problem $P^{\alpha}_{F_\alpha,H,H_\nu}$ given by equation (1).
Then for all $v \in H^1(\Omega)$, for all Lipschitz Boundary Dissection L_d and for all Cauchy datum:

$$\Phi_\alpha(D^{(1,2)}_{\alpha_0,L_d,Cauchy}, v) - \langle \mathcal{B}_\nu[D^{(1,2)}_{\alpha_0,L_d,Cauchy}], \gamma[v]\rangle_{\partial\Omega} = 0 \ .$$

With $\gamma^*[.] : H^{-\frac{1}{2}}(\Omega) \to (H^1(\Omega))^*$ the adjoint of the trace map, we have for all $v \in H^1(\Omega)$:

$$\Phi_\alpha(D^{(1,2)}_{\alpha_0,L_d,Cauchy}, v) - \langle \gamma^*[\mathcal{B}_\nu[D^{(1,2)}_{\alpha_0,L_d,Cauchy}]], v\rangle_\Omega = 0$$


Dirichlet Functional// Boundary index=1:80//Interior index=81:end
For All Dissection Calculated
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
100
200
300
400
500
600
700
800

Corollary (The Annihilator set for parameters)

For the Cauchy Data consistent with parameters α and discrepancy field $D^{(1,2)}_{\alpha_0,L_d,Cauchy} = u^{(1)}_{\alpha_0,L_d,Cauchy} - u^{(2)}_{\alpha_0,L_d,Cauchy}$ between complementary problems calculated with trial parameters α_0 and Lipschitz Boundary Dissection on $\partial\Omega$ indexed as L_d, the Dirichlet Functional

$$\Phi_\alpha(D^{(1,2)}_{\alpha_0,L_d,Cauchy}, v) = \langle \gamma^*[\mathcal{B}_\nu[D^{(1,2)}_{\alpha_0,L_d,Cauchy}]], v\rangle_\Omega = 0$$

for all test function $v \in H^1_0(\Omega) = \{v \in H^1(\Omega) : \gamma[v] = 0\}$.

Distance based on the Discrepancy

- Based on Theorem of Complementary Solutions we create some discrepancy function that measures observed differences for guess value of the parameters. Norms in the solution space for the direct problems can be adopted, that is,

$$d_{\alpha^{(0)},L_d,Cauchy} = ||u^{(1)}_{\alpha^{(0)},L_d,Cauchy} - u^{(2)}_{\alpha^{(0)},L_d,Cauchy}||_V, \tag{4}$$

- where V can be some norm.
- The Bregmann distance can also be experimented,
- but the simplest are obviously the Least Squares and the $d^{\infty}_{\alpha^{(0)},L_d,Cauchy} = \sup(u^{(1)}_{\alpha^{(0)},L_d,Cauchy} - u^{(2)}_{\alpha^{(0)},L_d,Cauchy}, x \in \Omega)$.

Least Squares

- First order α expansion of

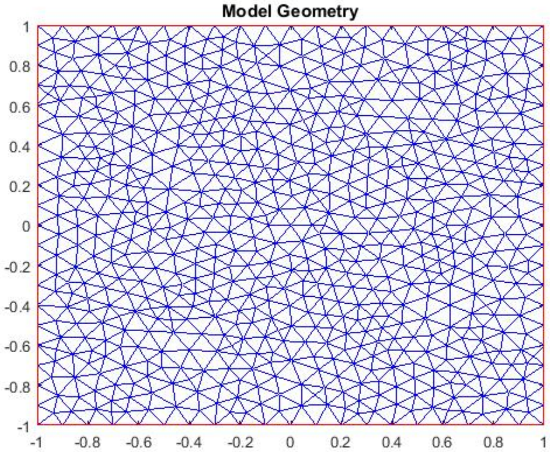
$$u^{(1)}_{j,\alpha,L_d} - u^{(2)}_{j,\alpha,L_d} = 0 \tag{5}$$

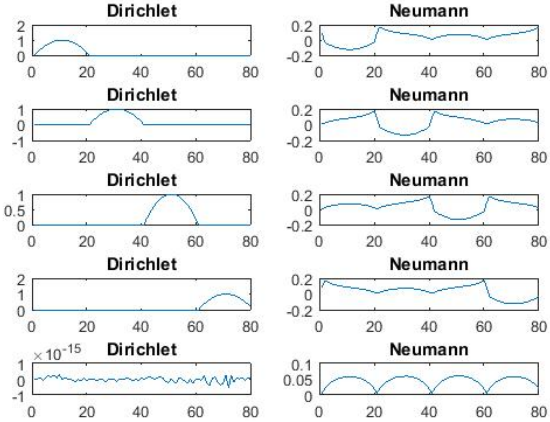
- suggest least squares solve of the system

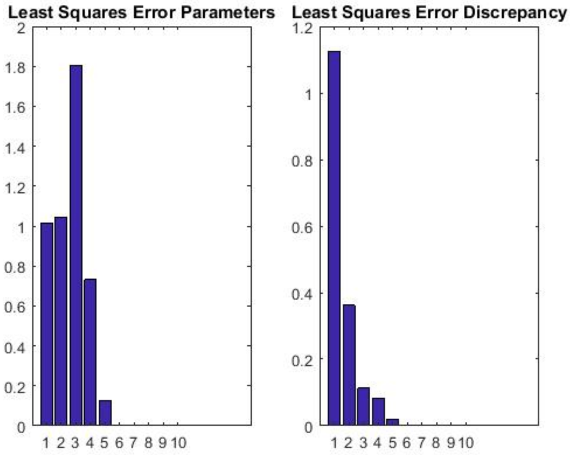
$$u^{(1)}_{j,\alpha^{(0)},L_d} - u^{(2)}_{j,\alpha^{(0)},L_d} + \sum_{k=1}^{N_\alpha} \frac{\partial}{\alpha_k}\left(u^{(1)}_{j,\alpha,L_d} - u^{(2)}_{j,\alpha,L_d}\right)\Big|_{\alpha^{(0)}} \Delta\alpha_k = 0 \tag{6}$$

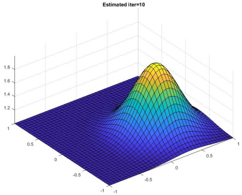
for all $j = 1, \ldots, N_v,\ L_d = 1, \ldots, N_{L_d}$ and Cauchy data

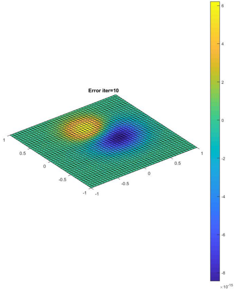
- and an appropriated choose of a regularization methodology.

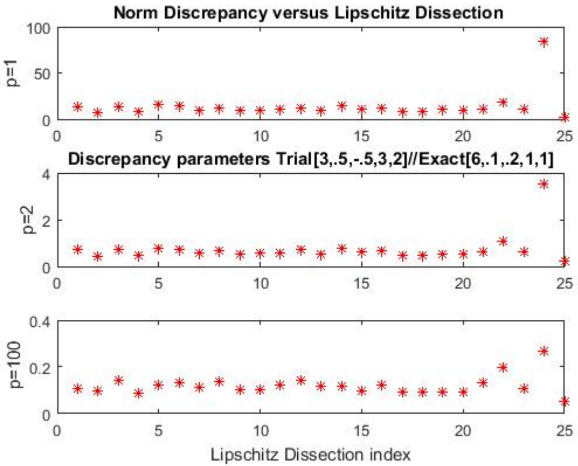

Model Geometry
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

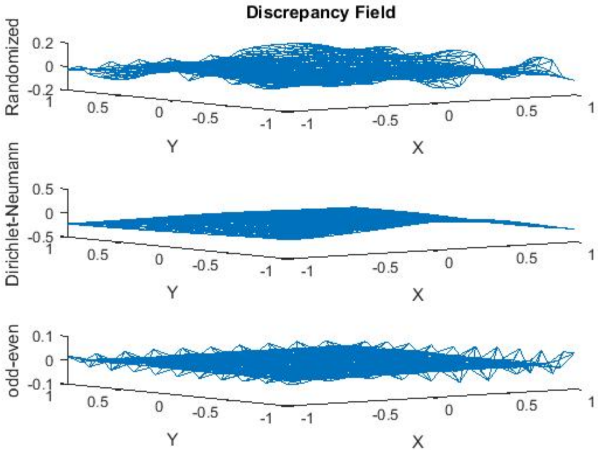

Dirichlet
Neumann
Dirichlet
Neumann
Dirichlet
Neumann
Dirichlet
Neumann
×10^-15
Dirichlet
Neumann
0
20
40
60
80


Least Squares Error Parameters
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10
Least Squares Error Discrepancy
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10


Estimated iter=10


Error iter=10
×10^-15


Norm Discrepancy versus Lipschitz Dissection
p=1
Discrepancy parameters Trial[3,.5,-.5,3,2]//Exact[6,.1,.2,1,1]
p=2
p=100
Lipschitz Dissection index


Discrepancy Field
Randomized
0.2
0
-0.2
Dirichlet-Neumann
0.5
0
-0.5
odd-even
0.1
0
-0.1
1
0.5
0
-0.5
-1
Y
X

Unknown Rectangle inside a square

- The numerical experiment that illustrate this experiment is a model in which the square $[-1, +1] \times [-1, +1]$
- has in its interior a small rectangle with has unknown center, unknown edges a and b, which supports unknown parameters related with the
- conductivity, c, the potential, a, and the source intensity, f.
- Cauchy data are synthetically generated with a problem in which parameters value are known equal to 1 in the exterior of the small rectangle, and all equal 2 in the interior.
- Also the unknown information about the rectangle used are center at the origin and side length $= .2$.

Operator parameters

The operator $\mathcal{L}$ has only one equation and its set parameters is given by $\alpha = (x_0, y_0, x_1, y_1, c, a, f) \in \mathbb{R}^7$. The constitutive equations are

$$c(x) = 1 + (c-1)\chi_{\alpha_0}(x,y)\ ,\ a(x) = 1 + (a-1)\chi_{\alpha_0}(x,y) \text{ and}$$

$$f(x) = 1 + (f-1)\chi_{\alpha_0}(x,y)\ ,$$

where

$$\chi_{\alpha_0}(x,y) = \chi(\alpha(1) - \frac{1}{2}x_1, \alpha(1) + \frac{1}{2}\alpha(3)(x)\chi(\alpha(2) - \frac{1}{2}\alpha(4), \alpha(2) + \frac{1}{2}\alpha(4))(y)$$

and $\chi(s1, s2)(x)$ is the characteristic function of $[s1, s2] \subset \mathbb{R}^1$

Nelder-Mead reconstruction of parameters-I

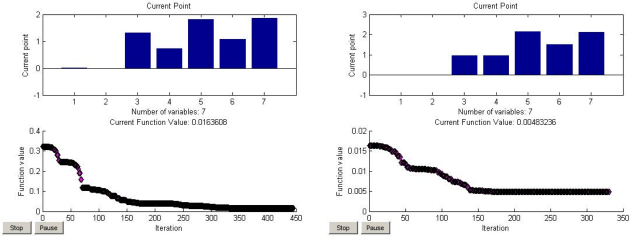


Figura: Iterative simultaneous reconstruction of rectangle shape, conductivity, absorption and source

Nelder-Mead reconstruction of parameters-II

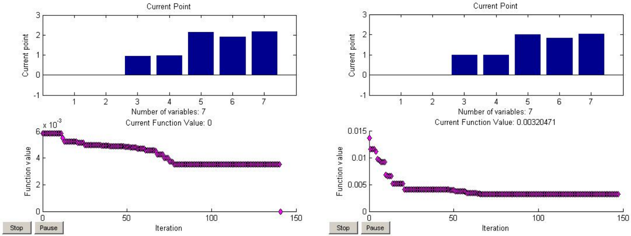


Figura: Iterative simultaneous reconstruction of rectangle shape, conductivity, absorption and source

- The main results in this work are:
- It is based on:
  1. Over prescription of Cauchy data;
  2. Lipschitz Boundary Dissection;
  3. A specialized Finite Elements formulation for this class of problems;
  4. Solution of Multiple Complementary Direct Mixed Problems with wrong values of trials parameters.

- We demonstrate:
  1. A Theorem on Complementary Solutions;
  2. The existence of Discrepancy Fields for trials with wrong parameters values;
  3. The Reciprocity Gap equation for Discrepancy fields parameter determination;
  4. The Variational Method for Discrepancy Fields parameter determination;
  5. A annihilator set condition for Discrepancy fields parameter determination.

- The optimization methodology is numerically investigate
- with the nonlinear least squares method for Discrepancy Fields
- and with non differentiable Nelder-Mead minimum search algorithm with a C^0 norm of Discrepancy Fields.
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