TRANSFER FUNCTION BASED ON GREEN'S FUNCTION METHOD (TFBGF) APPLIED TO THE THERMAL PARAMETER ESTIMATION

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Summary

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- Heating and measurements of temperature and heat flux occur on the same surface. The method uses transfer function identification to solve inverse heat conduction problems.
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- Different objective functions are proposed to estimate thermal conductivity and diffusivity.

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- Problems such as contact resistance;
- Low sensitivity due to the small temperature gradient;
- As in any experimental method, the identification of thermal properties is sensitive to measurement uncertainty. Thus, to guarantee accuracy in the estimation, the design of the experiments should be optimized.

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- Two distinct problems are then established$\rightsquigarrow$
  Experimental and thermal model developments.
- In situ applications (only one access surface)

Summary

Dynamic System

- The technique proposed here is based on the use of an input/output dynamical system.
- The dynamic characteristics of a constant-parameter linear system can be described by an impulse response function $h(\tau)$, which is defined as the output of the system at any time to a unit impulse input applied a time τ before.

  For any arbitrary input $x(t)$, the system output $y(t)$ is given by the *convolution integral*

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{1}$$

Dynamic System

- In order for a constant-parameter linear system to be physically realizable (causal), it is necessary that the system respond only to past inputs. This implies that

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{2}$$

$$h(t) = 0 \text{ for } \tau < 0 \tag{3}$$

Frequency Response Functions

The dynamic characteristics of the system can be described by a frequency response function $H(f)$, which is defined as the Fourier transform of $h(\tau)$. That is

$$H(f) = \int_0^\infty h(\tau) e^{-j2\pi f \tau} d\tau \tag{4}$$

The convolution integral in Eq. (2) reduces to the simple algebraic expression in Equation Eq. (5).

$$Y(f) = H(f)X(f) \tag{5}$$

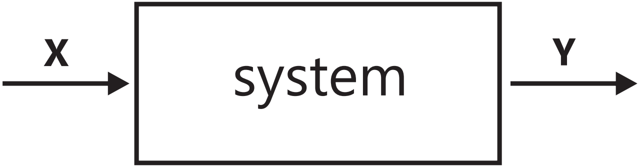
Frequency Response Functions

The frequency response function is generally a complex-valued quantity. It means, it has a magnitude and an associated phase angle that in complex polar notation can be written by

$$H(f) = \mid H(f) \mid e^{-j\phi(f)} \tag{6}$$

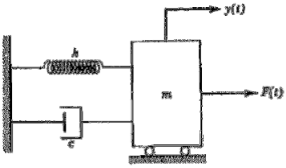
The absolute value $\mid H(f) \mid$ is called the system gain factor, and the associated phase angle $\phi(f)$ is called the system phase factor. In these terms, the frequency response function takes on a direct physical interpretation as follows.

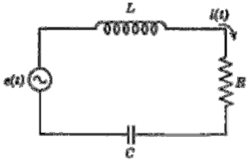
Dynamic System



This representation can be observed on different systems

Mechanical, Electrical and Thermal Systems





$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) \Leftrightarrow L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = e(t)$$

$$H(f) = [k - (2\pi f)^2 m + j2\pi fc]^{-1} \Leftrightarrow H(f) = \left[\frac{1}{C} - (L2\pi f)^2 + j2\pi fR\right]^{-1}$$

Analogous Characteristics for Several Physical Systems

System	Input	Output	Constant Parameters		
Electrical	Voltage	Current	Inductance	Resistance	Capacitance
Mechanical (translational)	Force	Velocity	Mass	Damping	Compliance
Mechanical (rotational)	Torque	Angular velocity	Moment of inertia	Angular damping	Angular compliance
Acoustical	Pressure	Particle velocity	Inertance (acoustical mass)	Acoustical damping	Acoustical capacitance
Thermal	Temperature	Heat flow	—	Thermal resistance	Thermal capacitance
Magnetic	Magneto-motive force	Flux	—	Reluctance	—

Thermal Equivalent System

As mentioned, the technique proposed here is based on the use of an input/output dynamical system, given by the convolution integral.

$$Y(t) = \int_0^\infty H(t-\tau)X(t)d\tau \tag{7}$$

or

$$T_1(t) - T_2(t) = \int_0^\infty H(t-\tau)q(t)d\tau \tag{8}$$

or in transformed frequency-plane

$$Y(f) = H(f) \times X(f) \tag{9}$$

Thermal Equivalent System

A dynamic model can be obtained from a thermal model shown in Figure, where $q(t)$ represents the heat flux, T the temperature

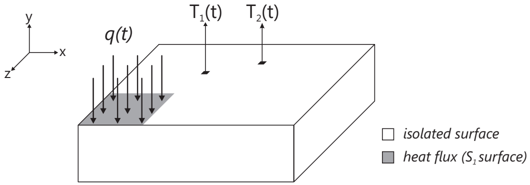


Figura: 3D thermal equivalent model

It will be shown that if we have the heat flux/temperature pair we can obtain the expression

$$\mathcal{L}[T(x,y,z,t)] = \mathcal{L}[h(\mathbf{r},t) * q(\mathbf{r},t)] \Rightarrow T(\mathbf{r},s) = H(\mathbf{r},s) \cdot q(x,z,s)$$

Thermal Equivalent System

The three-dimensional thermal model can be obtained by the solution of the diffusion equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{10}$$

subjected to the boundary conditions:

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q(t);\ \text{in } S_1 \text{ region} \tag{11}$$

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = 0;\ \text{in } S_2 \text{ region} \tag{12}$$

$$\frac{\partial T}{\partial x}\bigg|_{x=L} = \frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{\partial T}{\partial y}\bigg|_{y=W} = \frac{\partial T}{\partial z}\bigg|_{z=0} = \frac{\partial T}{\partial z}\bigg|_{z=W} = 0 \tag{13}$$

and the initial

$$T(x,y,z,0) = T_0 \tag{14}$$

Thermal Equivalent System

If T(**r**,t) represents T(x,y,z,t), the solution of Eqs.(10-14) can be given in terms of Green´s function as

$$T(\mathbf{r},t) - T_0 = \frac{\alpha}{k} \int_{\tau=0}^{t} \int_{L_1}^{L_2} \int_{R_1}^{R_2} \left[q(\tau) G(r,t|x',W,z',\tau) \right] dx' dz' d\tau \tag{15}$$

or in frequency domain

$$\theta_{eq}(\mathbf{r},f) = q(x,z,f) G_{eq}(\mathbf{r},f) \tag{16}$$

Thermal Equivalent System

Since Green's function is available and exists, the solution of the problem defined by Eqs. (10-14) can be performed numerically or analytically.

Different equivalent thermal model can be obtained as the convolution product in the frequency domain. For example,

$$H(f) = G_H(f) = \frac{T_1(f) - T_2(f)}{q(f)} \tag{17}$$

where the variable f indicates that Fourier transform was applied to the variables $T(t)$, $q(t)$, and $G_H(t)$.

Thermal Equivalent System

It means

$$G_H(t/\tau) = G_H(t-\tau) = \frac{\alpha}{k}[G(x_1, y_1, z_1, t-\tau) - G(x_2, y_2, z_2, t-\tau)] \tag{18}$$

It can be observed that as $T_1(t)$ and $T_2(t)$ are obtained by discrete measurements, Fourier transformed can be performed numerically by using the Cooley-Tukey algorithms (Discrete Fast Fourier Transform) for these data.

Thermal Equivalent System

Therefore, an equivalent thermal system to the dynamic system can be represented by:

$$H(f) = G_H(f) = \frac{T_1(f) - T_2(f)}{q(f)} = \frac{Y(f)}{X(f)} \tag{19}$$

where the function $H(f)$ is equivalent to the response in frequency H(f) defined by Eq. (9). Observing Eqs. (10) adn (11) it can be concluded that the frequency response H(f) is strongly dependent of the thermal properties

$$H(f) = G_H(f) = function(\alpha, k) \tag{20}$$

Thermal Equivalent System

It also should be observed that the transformed impedance in the f-x plane, is a complex variable which in a polar form can be written by

$$H(f) = G_H(f) = |H(f)|e^{-j\varphi(f)} \tag{21}$$

where $|H(f)|$ and $\varphi(f)$ represent, respectively the modulus and the phase factor of H. The phase factor can be written by

$$\varphi(f) = \arctan\left(\frac{\Im H(f)}{\Re H(f)}\right) \tag{22}$$

where $\Im H(f)$ and $\Re H(f)$ are the imaginary and real parts of $H(f)$, respectively.

Inverse problem or Identification problem

The question is: How to obtain T(x,y,t) without knowing q(t) or without knowing G(t)?

$$T(\mathbf{r},t) - T_0 = \frac{\alpha}{k}\int_{\tau=0}^{t}\int_{L_1}^{L_2}\int_{R_1}^{R_2}\left[q(\tau)G(\mathbf{r},t|x',W,z',\tau)\right]dx'dz'd\tau \tag{23}$$

or in frequency domain

$$T_{eq}(\mathbf{r},f) = q(x,z,f)G_{eq}(\mathbf{r},f) \tag{24}$$

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- Obtaining of (unknown) $q(t)$ defines an inverse problem.

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- Obtaining of (unknown) $q(t)$ defines an inverse problem.
- Obtaining of (unknown) $G(t)$ defines an Identification problem.
- Note that the convolution in time domain corresponds with multiplication in the frequency domain.

Transfer function based on Green's function (TFBGF) method

Obtaining of (unknown) G(t) explicitly

The solution of the homogeneous problem, described by Equations (10-14) in terms of Green's functions is given by

$$\theta(x,y,z,t) = \frac{\alpha}{k}\int_0^t \int_{L_1}^{L_2} \int_{R_1}^{R_2} q(\tau) G(x,y,z,t|x',W,z',t-\tau) dx' dz' d\tau \tag{25}$$

Analytical solution of the direct problem with Green's functions

The Green's function $G(x, y, z, t|x', y', z', t-\tau)$ is obtained, observing the types of boundary conditions in the directions of x, y and z, as the product of three independent one-dimensional problems. *For example, if all faces are subjected to* **heat exchange by convection** *except the region with heat flux*

$$G_{X33}(x, t|x', \tau) = \frac{2}{L} \sum_{m=1}^{\infty} e^{-\alpha_m^2 \alpha (t-\tau)/L^2} \left[\alpha_m \cos\left(\frac{\alpha_m x}{L}\right) + B_1 \sin\left(\frac{\alpha_m x}{L}\right)\right] \times \frac{\left[\alpha_m \cos\left(\frac{\alpha_m x'}{L}\right) + B_1 \sin\left(\frac{\alpha_m x'}{L}\right)\right]}{\left(\alpha_m^2 + B_1^2\right)\left[1 + \frac{B_2}{(\alpha_m^2 + B_2^2)}\right] + B_1} \tag{26}$$

3D Analytical solution

The analytical expression in terms of the variable θ.

$$\theta(x,y,z,t) = 8\theta_0 \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}$$

$$\times \frac{\left[\alpha_m \cos\left(\frac{\alpha_m x}{L}\right) + B_1 \sin\left(\frac{\alpha_m x}{L}\right)\right]}{(\alpha_m^2+B_1^2)\left[1+\frac{B_2}{(\alpha_m^2+B_2^2)}\right]+B_1} \times \frac{\left[\beta_n \cos\left(\frac{\beta_n y}{W}\right) + B_3 \sin\left(\frac{\beta_n y}{W}\right)\right]}{(\beta_n^2+B_3^2)\left[1+\frac{B_4}{(\beta_n^2+B_4^2)}\right]+B_3}$$

$$\times \frac{\left[\gamma_p \cos\left(\frac{\gamma_p z}{R}\right) + B_5 \sin\left(\frac{\gamma_p z}{R}\right)\right]}{(\gamma_p^2+B_5^2)\left[1+\frac{B_6}{(\gamma_p^2+B_6^2)}\right]+B_5} \times \frac{1}{\alpha_m\beta_n\gamma_p}$$

$$\times \left[\alpha_m \sin\alpha_m - B_1(\cos\alpha_m - 1)\right]\left[\beta_n \sin\beta_n - B_3(\cos\beta_n - 1)\right]\left[\gamma_p \sin\gamma_p - B\right.$$

$$+ \frac{\alpha}{k}\frac{8}{LWR}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}$$

3D Analytical solution

$$\times \frac{\left[\gamma_p \cos\left(\frac{\gamma_p z}{R}\right) + B_5 \sin\left(\frac{\gamma_p z}{R}\right)\right]}{(\gamma_p^2 + B_5^2)\left[1 + \frac{B_6}{(\gamma_p^2+B_6^2)}\right] + B_5} \times \frac{1}{\alpha_m \beta_n \gamma_p}$$

$$\times \left[\alpha_m \sin\alpha_m - B_1(\cos\alpha_m - 1)\right]\left[\beta_n \sin\beta_n - B_3(\cos\beta_n - 1)\right]\left[\gamma_p \sin\gamma_p - B\right.$$

$$+ \frac{\alpha}{k}\frac{8}{LWR}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}$$

$$\times \frac{\left[\alpha_m \cos\left(\frac{\alpha_m x}{L}\right) + B_1 \sin\left(\frac{\alpha_m x}{L}\right)\right]}{(\alpha_m^2 + B_1^2)\left[1 + \frac{B_2}{(\alpha_m^2+B_2^2)}\right] + B_1}$$

$$\times \frac{\left[\beta_n \cos\left(\frac{\beta_n y}{W}\right) + B_3 \sin\left(\frac{\beta_n y}{W}\right)\right]}{(\beta_n^2 + B_3^2)\left[1 + \frac{B_4}{(\beta_n^2+B_4^2)}\right] + B_3}\left[\beta_n \cos(\beta_n) + B_3 \sin(\beta_n)\right]$$

3D Analytical solution

$$\times \left\{ L \left[\sin\left(\frac{\alpha_m L_2}{L}\right) - \sin\left(\frac{\alpha_m L_1}{L}\right)\right] - \frac{B_1 L}{\alpha_m}\left[\cos\left(\frac{\alpha_m L_2}{L}\right) - \cos\left(\frac{\alpha_m L_1}{L}\right.\right.\right.$$
$$\times \left\{ R \left[\sin\left(\frac{\gamma_p R_2}{R}\right) - \sin\left(\frac{\gamma_p R_1}{R}\right)\right] - \frac{B_5 R}{\gamma_p}\left[\cos\left(\frac{\gamma_p R_2}{R}\right) - \cos\left(\frac{\gamma_p R_1}{R}\right)\right.\right.$$
$$\times \int_0^t \left[q(\tau) e^{\left(\frac{\alpha_m^2}{L^2} + \frac{\beta_n^2}{W^2} + \frac{\gamma_p^2}{R^2}\right)\alpha\tau} \right] d\tau \tag{29}$$

The solution in terms of the original variable T is given by $T = \theta + T_\infty$. Note that the solution of the direct problem of heat conduction $X33Y33Z33$ is determined once the heat flux, $q(t)$, and also the coefficients of heat transfer by convection, h_i are known.

Identification of the analytical impulse response

From the knowledge of the Green's function which describes the problem, it is possible to identify the impulse response of the system and therefore its transfer function.
The general solution of the problem $X33Y33Z33$, can be rewritten as

$$\Theta(x,y,z,t) = \frac{\alpha}{k}\int_0^t \int_{L_1}^{L_2} \int_{R_1}^{R_2} q(\tau) G(x,y,z,t|x',W,z',\tau) dx' dz' d\tau \tag{30}$$

Identification of the analytical impulse response

As the transfer function is independent of the input / output pair, it is proposed as the input signal (heat flux), the Dirac Delta function, $q(t) = \delta(t)$. In tis case

$$\Theta(x, y, z, t) = \int_0^t h(x, y, z, t - \tau)\delta(\tau)d\tau = h(x, y, z, t) \qquad (31)$$

Since $h * \delta = h$, the impulse response is obtained without the need to solve the integral. It means

$$h(x, y, z, t) = \Theta(x, y, z, t) = \frac{\alpha}{k}\int_{L_1}^{L_2}\int_{R_1}^{R_2} G(\mathbf{r}, t|x', W, z', \tau) \quad (32)$$

$$h(x,y,z,t) = \frac{\alpha}{k}\frac{8}{LWR}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} e^{-\left(\frac{\alpha_m^2}{L^2}+\frac{\beta_n^2}{W^2}+\frac{\gamma_p^2}{R^2}\right)\alpha t}$$

$$\times \frac{\left[L\left(\sin\left(\frac{\alpha_m L_2}{L}\right) - \sin\left(\frac{\alpha_m L_1}{L}\right)\right) - \frac{B_1 L}{\alpha_m}\left(\cos\left(\frac{\alpha_m L_2}{L}\right) + \cos\left(\frac{\alpha_m L_1}{L}\right)\right)\right]}{(\alpha_m^2 + B_1^2)\left[1 + \frac{B_2}{(\alpha_m^2+B_2^2)}\right] + B_1}$$

$$\times \frac{\left[R\left(\sin\left(\frac{\gamma_p R_2}{R}\right) - \sin\left(\frac{\gamma_p R_1}{R}\right)\right) - \frac{B_5 R}{\gamma_p}\left(\cos\left(\frac{\gamma_p R_2}{R}\right) + \cos\left(\frac{\gamma_p R_1}{R}\right)\right)\right]}{(\gamma_p^2 + B_5^2)\left[1 + \frac{B_6}{(\gamma_p^2+B_6^2)}\right] + B_5}$$

$$\times \left[\alpha_m \cos\left(\frac{\alpha_m x}{L}\right) + B_1 \sin\left(\frac{\alpha_m x}{L}\right)\right] \times \left[\beta_n \cos\left(\frac{\beta_n y}{W}\right) + B_3 \sin\left(\frac{\beta_n y}{W}\right)\right]$$

$$\times \left[\gamma_p \cos\left(\frac{\gamma_p z}{R}\right) + B_5 \sin\left(\frac{\gamma_p z}{R}\right)\right] \times \frac{[\beta_n \cos(\beta_n) + B_3 \sin(\beta_n)]}{(\beta_n^2 + B_3^2)\left[1 + \frac{B_4}{(\beta_n^2+B_4^2)}\right] + B_3} \tag{33}$$

Identification of the analytical impulse response

Note that the solution method of the proposed inverse problem can be applied to any type of heat conduction problem $1D$, $2D$ or $3D$ for different types of boundary conditions. The crux of the method lies in identifying the impulse response / transfer function for each particular problem of heat conduction, using the Green's function that describes the problem.

Inverse problem or Identification problem

The phase of frequency response $H(f)$ and the time evolution of superficial temperatures, $T1(t)$ and $T2(t)$ are the experimental data used for estimation of thermal diffusivity and thermal conductivity respectively.

Thermal Diffusivity Estimation: **Frequency Domain**

Guimaraes et al, (1995) observed that the phase factor, φ, is a function exclusively of the thermal diffusivity. This fact is the base of the procedure for obtaining the thermal diffusivity thorough minimization of an objective based on the difference between experimental and calculated values of φ. This objective function can be written as

$$S_\varphi = \sum_{i=1}^{Nf} (\varphi_e(i) - \varphi(i))^2 \tag{34}$$

where φ_e and φ are the experimental and calculated values of the phase factor of $H(f)$, respectively.

The theoretical values of the phase factor are obtained from the identification of $H(f)$ by Eq. (35).
In this case the output $Y(f)$ is the Fourier transform of the difference obtained by the numerical or analytical solution of Eqs.(10-14).
If numerical methods are used, this procedure avoids the necessity of obtaining an explicit and analytical model of $H(f)$.
It means, we just need to use

$$H(f) = G_H(f) = \frac{T_1(f) - T_2(f)}{q(f)} = \frac{Y(f)}{X(f)} \tag{35}$$

The values of α will be supposed to be those that minimize the square error between the experimental and calculated values of the phase factor of $H(f)$.
In this work, this minimization is done by using the golden section method with polynomial approximation (Vanderplaats, 1984).

Thermal Conductivity Estimation: **Frequency Domain**

Once the thermal diffusivity value is obtained, an objective function based on least square of temperature error can be used to estimate the thermal conductivity in time domain or in modulus error in the frequency domain. In this case, there is no identification problem as just one variable is being estimated. Therefore, the variable k will be supposed to be the parameter that minimizes the least square function, S_q, based on the difference between the calculated and experimental of the frequency response modulus defined by

$$S_{qH} = \sum_{j=1}^{s} \sum_{i=1}^{n} \left(|H_e(i,j)| - |H_t(i,j)| \right)^2 \tag{36}$$

Thermal Conductivity Estimation: **time Domain**

The variable k can also be the parameter that minimizes the least square function, S_{qT}, based on the difference between the calculated and experimental temperature defined in time domain and given by

$$S_q = \sum_{j=1}^{s} \sum_{i=1}^{n} (T_e(i,j) - T_t(i,j))^2 \tag{37}$$

where $T_e(i,j)$ is the experimental temperature and $T_t(i,j)$ is the calculated temperature, n is the total number of time measurements and s represents the number of sensors.

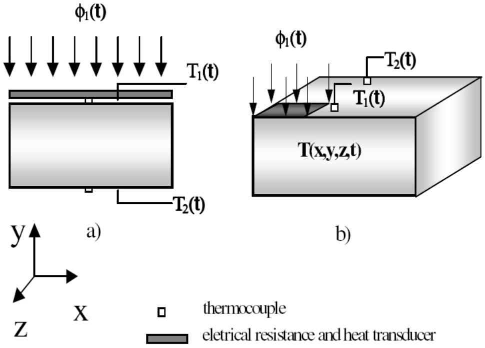
Summary

Sensitivity analysis

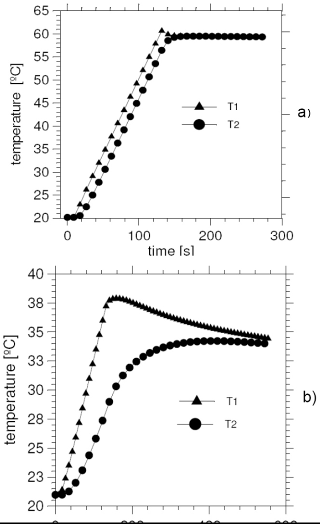
Although the thermal contact resistance and the low gradient problems do not represent any difficulties for non-metallic materials, they must be taken into account in the presence of conductor materials. This section discusses both problems.

Sensitivity analysis

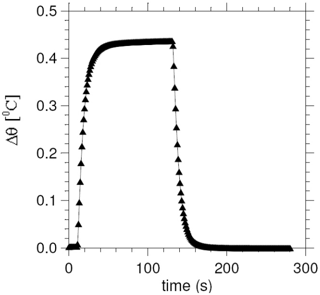
Figure presents the thermal contact resistance that can appear between sample and sensors in a one-dimensional model and the 3D alternative model to avoid this problem allowing experimental flexibility in location of the identification sensors.



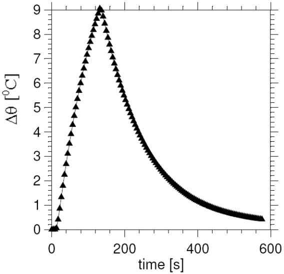
1D and 2D configurations



In one-dimensional model, a high magnitude of heat flux (input) can be necessary to establish a thermal gradient high enough for the estimation process. Figures present a simulation using the same heat flux input. While the temperature gradient is situated in the region of the uncertainty of thermocouples (0.3 K), the three-dimensional model produces a sufficient gradient to properties estimation.


0.5
0.4
0.3
0.2
0.1
0.0
Δθ [°C]
0
100
200
300
time (s)

a)


9
8
7
6
5
4
3
2
1
0
Δθ [°C]
0
200
400
600
time [s]

b)

sensitivity coefficients

This fact can be better analyzed through a sensitivity analysis. Small and/or inaccurate values of temperature difference and heat flux signals produce linear dependence or low values. The linear dependence of two or more coefficients indicates that the parameters cannot simultaneously be estimated. Low values indicate that the estimation is strongly sensitive to the measurements uncertainty.

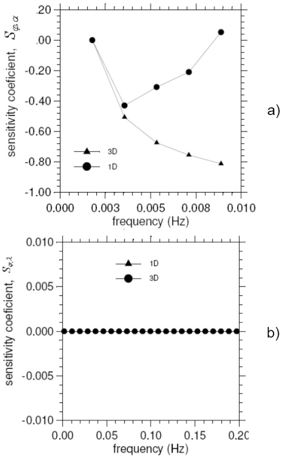
sensitivity coefficients

The sensitivity coefficients involved in this technique are defined as following and presented in Figs.

$$S_{T,\alpha} = \frac{\alpha}{T}\frac{\partial T}{\partial \alpha}, S_{T,k} = \frac{k}{T}\frac{\partial T}{\partial k}, S_{\varphi,\alpha} = \frac{\alpha}{\varphi}\frac{\partial \varphi}{\partial \alpha}, S_{\varphi,k} = \frac{k}{\varphi}\frac{\partial \varphi}{\partial k} \quad (38)$$

$$S_{|H|,\alpha} = \frac{\alpha}{|H|}\frac{\partial |H|}{\partial \alpha}, S_{|H|,k} = \frac{k}{|H|}\frac{\partial |H|}{\partial k} \quad (39)$$

sensitivity coefficients



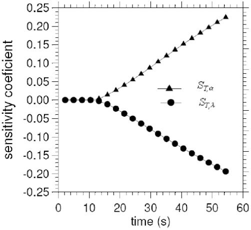
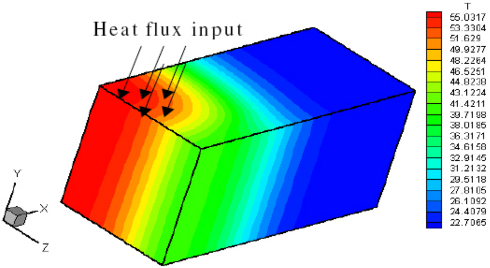


Figure reveals a linear dependency of $S_{T,\alpha}$, and $S_{T,k}$ as shown by the symmetry. This fact indicates that both thermal properties cannot be estimated simultaneously in time domain justifying the use of frequency domain for the thermal diffusivity estimation.

The values of the sensitivity of phase related to the thermal diffusivity are higher for the 3D model and there is no possibility to estimate the thermal conductivity in frequency domain due to $S_{T,k} = 0$ for any frequency value. This fact reveals that the phase dependency with thermal diffusivity is unique and exclusive.

sensitivity coefficients

Advantage of a 3D model: application to the thin sample. In the 1D case, for conductor materials, it is very hard to obtain temperature gradients with values high enough to allow a good estimation. it can be seen that no temperature variation in the direction y is observed. This fact makes the one-dimensional analysis unpractical.



sensitivity coefficients

Another important characteristic of the technique presented here is the very low sensitivity of α related to the amplitude of the signals X and Y. It means, the estimated value of the thermal diffusivity is insensitive to bias error, like uncertainty due to poor calibration of thermocouples or heat flux transducers or both. This fact can be demonstrated by verifying the figures which show the behavior of phase factor and modulus due to the same input/output signals in both versions: original data, muV/mV, and calibrated data $W/m^2/^0C$.

sensitivity coefficients

It can be observed that there are no changes in the phase factor while the modulus is strongly affected.

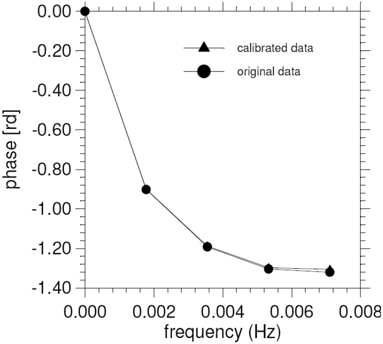


Figura: Phase factor subjected to the original and calibrated pair of input/output data

sensitivity coefficients

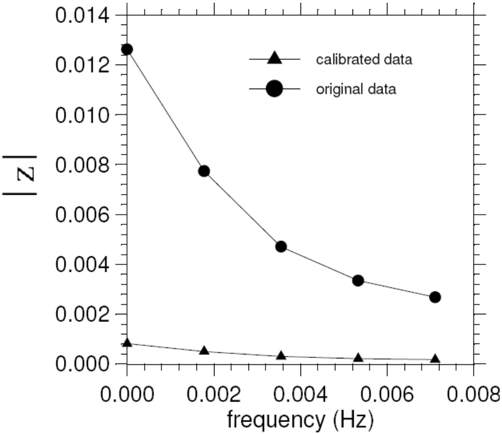


Figura: Modulus of Z for the original and calibrated pair input/output

Summary

Experimental apparatus and results: **Conductor material application**

It should be observed that the boundary conditions present in the theoretical model must be guaranteed in the experimental apparatus. It means that the isolated condition at the reminiscent surface needs to be reached for the success of the estimation techniques. A good way to reach the isolation condition in a vertical direction is the use of a symmetric experiment apparatus. Figure presents this scheme.

Conductor material application

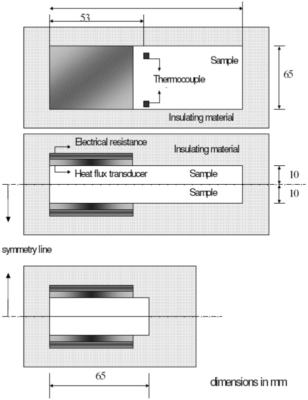


Figura: apparatus scheme

Conductor material application

Twenty independent runs were performed. In each of the experiments were acquired 1024 points at time intervals, of 0.54 s. The time duration of heating, t_h, was approximately $120s$ with a heat pulse generated at $90V(DC)$.

In Table 3 a summary of the simultaneous estimation of α and k of the AISI304 sample is presented. In this table, the value obtained for α using the Flash method and the value of k from literature are also presented.

Table 3. Summary of α and λ for AISI304 sample

Thermal properties	This work	References	Error (%)
$\alpha \times 10^{-6}$ (m^2/s)	3.762	3.82	1.54
λ (W/mK)	14.64	14.90	1.77

It can be observed an excellent agreement between the values of this work and the literature (error less than 2 percent).

The comparison between the experimental and estimated temperatures for $\alpha = 3.76 \times 10^{-06} m^2/s$ and k is shown in Fig.

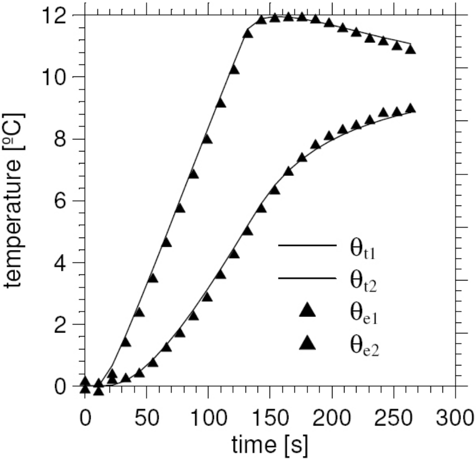


Figura: Comparison of an output of a typical run

Non conductor material application

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Non conductor material application

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- Fifty independent runs for PVC and twenty independent runs for Polythene were realized.For both samples 1024 points were taken. The time intervals, t, were 7.034 s for PVC and 6.243 s for Polythene.

- The time duration of heating, t_h, was approximately 150 s for PVC and 90 s for Polythene with a heat pulse generated at $40V(DC)$ for both samples.

Non conductor material application

Tables 4 and 5 present respectively the value estimated of α and k for the fifty runs of PVC, with 99.87 perc confidence interval.

Table 4. Statistical data of the average value of α, (initial value of α = 1.0 x 10^{-8} m^2/s).

α (m^2/s) x 10^7	Initial S_p x 10^2	Final S_p x 10^7	σ (m^2/s) x 10^{10}
1.24	6.0	3.6	7.06

Table 5. Statistical data of the average value of λ, (initial value of λ = 0,01 W/m.K).

α (m^2/s) x 10^7	λ (W/mK)	Initial S_{mq} x 10^{-6}	Final S_{mq}	σ x 10^5(W/m.K)
1.24±1.88%	0.152	1.351	5.91	4.9

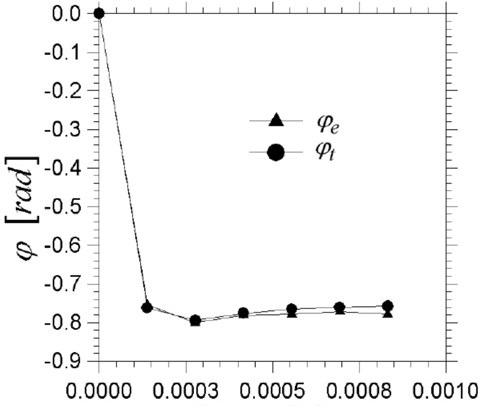
Non conductor material application

Table 6. Summary of α and λ for PVC sample

α (m^2/s)x10^7	α (m^2/s) x 10^7 (FM)	λ (W/m.K)	λ (W/m.K) (HPM)
1.24±1.88 %	1.28±3.1 %	0.152±1.1 %	0.157

Non conductor material application

In Figure a comparison between experimental and estimated phase factor is presented. It can be observed a very good agreement between them.



Non conductor material application

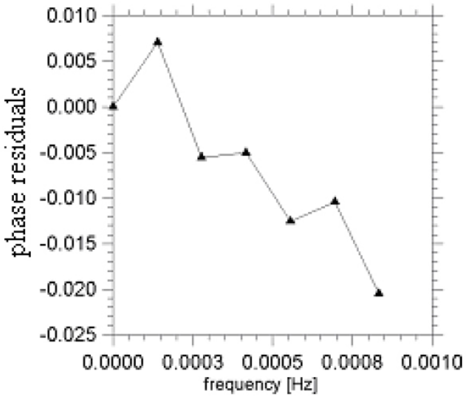
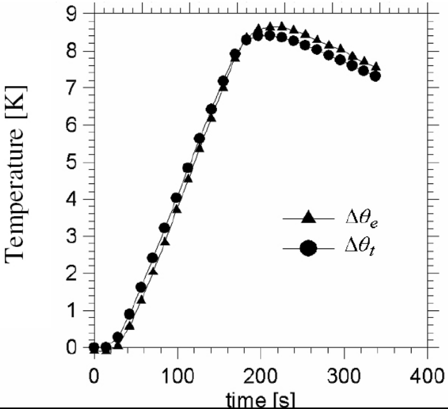


Figura: Phase factor: residuals beteewen experimental and calculated data

Non conductor material application

The comparison between the experimental and estimated temperatures for $\alpha = 1.24x10^{-7} m^2/s$ and $k = 0.152 W/m.K$ is shown in Fig.



Non conductor material application

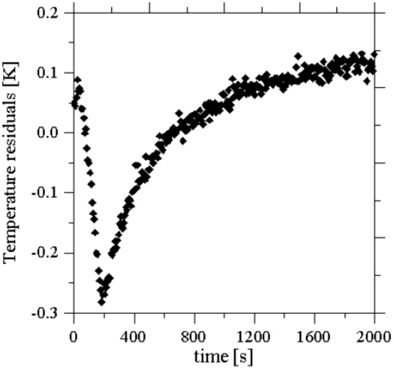


Figura: Temperature evolution: residuals of experimental and calculated data

Non conductor material application

Table 7 presents a summary of the simultaneous estimation of α and k for the Polythene sample with a confidence interval of 99.87 perc. For this sample only the reference value for k obtained by the guarded hot plate method is presented.

Table 7. Summary of α and λ for Polythene sample estimation

α (m^2/s) x 10^7	λ (W/m.K)	λ (W/m.K)(HPM)	Error (%)
2.14±1.15%	0.383±1.68 %	0.389	1.57

Summary

- In this work the transfer function (or impulse response) which describes a given thermal problem was obtained through analogies between the theories of Green's functions and dynamic systems, and then applied to the solution of inverse problems in heat conduction.

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- The procedure gives a great flexibility to the technique allowing the technique to deal with sample of small dimensions and also can be applied to conductor or non conductor materials.
- The great advantage of the dynamic observers technique is the easy and fast numerical implementation for any 1D, 2D or 3D model.

Summary

- I would like to thank CAPES, Fapemig, CNPq.
- Thanks to Fernandes A.P and Borges, V. L.
- and **Thank you for your attention!!!**