An inverse problem of electromagnetic shaping of liquid metals

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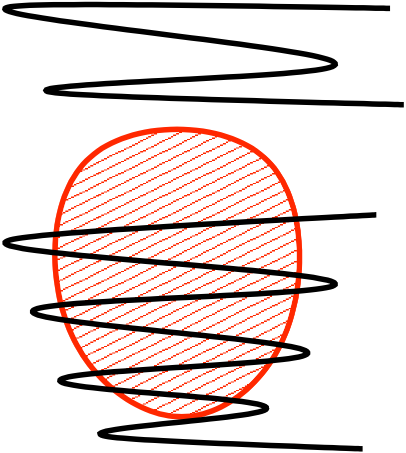
IMPA-2017

Model Problem in 3d, the electromagnetic casting problem.

$$
\begin{aligned}
\nabla \times B &= \mu_0 \overline{j_0} \quad \text{in } \Omega \\
\nabla . B &= 0 \quad \text{in } \Omega \\
B.\nu &= 0 \quad \text{on } \partial\omega = \Gamma \\
||B|| &\to 0 \text{ at } \infty \\
\frac{||B||^2}{2\mu_0} &+ \sigma\mathcal{H} + \rho g \cdot x_3 = p_0 \text{ on } \Gamma
\end{aligned}
$$



μ_0 the magnetic permeability.

ν the unit normal vector.

σ the surface tension.

$\mathcal{H}$ the mean curvature of $\Gamma = \partial\Omega$.

p_0 a constant.

j_0 is the current density.

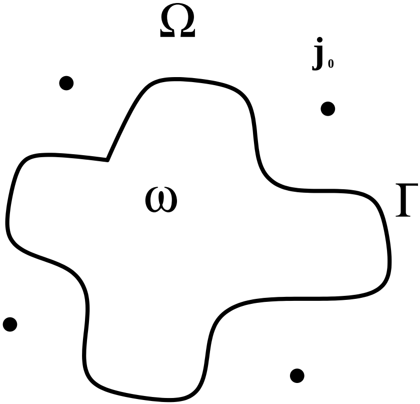
example

Model Problem in 2d

$$
\begin{aligned}
\nabla \times B &= \mu_0 \overline{j_0} \quad \text{in } \Omega \\
\nabla . B &= 0 \quad \text{in } \Omega \\
B.\nu &= 0 \quad \text{on } \partial\omega = \Gamma \\
||B|| &\to 0 \text{ at } \infty \\
\frac{\|B\|^2}{2\mu_0} &+ \sigma\mathcal{C} = p_0 \text{ on } \Gamma
\end{aligned}
$$

$\overline{j_0} = (0, 0, j_0)$ is the current density.

$$j_0 = I(\sum_{p=1}^{m} \alpha_p \delta_{x_p})$$



Conditions $\nabla \times B = \mu_0 \overline{j_0}$ in Ω and $\nabla . B = 0$ in Ω imply that there exist a potential function $\varphi : \Omega \to \mathbb{R}$ such that $B = (\varphi_y, -\varphi_x, 0)$ and φ is the solution of:

$$\begin{aligned} -\Delta\varphi &= \mu_0 j_0 \quad \text{in } \Omega \\ \varphi &= 0 \quad \text{on } \Gamma \\ \varphi(x) &= O(1) \text{ as } ||x|| \to \infty \end{aligned}$$

Under suitable assumptions, the equilibrium configurations are given by a local critical point w.r.t. the domain of the following total energy:

$$E(\omega) = -\frac{1}{2\mu_0} \int_\Omega ||\nabla\varphi||^2 + \sigma P(\omega)$$

where $P(\omega)$ is the perimeter of $\omega = \Omega^c$.

The variational formulation of the direct problem consists in considering the equilibrium domain ω as a stationary point for the total energy $E(\omega)$ under the constraint that measure of ω is given by S_0.

The shape optimization inverse problem in 2d

Given Ω^* the target shape, we want to compute j_0 solution of the following optimization problem:

$$\min_{j_0} \delta(\Omega, \Omega^*)$$

where $\Omega \in \mathcal{O}$ the set of admissible domains, with the following constraints:

$$\begin{aligned}
-\Delta\varphi &= \mu_0 j_0 \quad \text{in } \Omega \\
\varphi &= 0 \quad \text{on } \partial\Omega \\
\varphi(x) &= O(1) \text{ as } ||x|| \to \infty \\
\frac{1}{2\mu_0}||\frac{\partial\varphi}{\partial\nu}||^2 &+ \sigma\mathcal{C} = p_0 \quad \text{on } \Gamma \\
\int_\omega dx &= S_0
\end{aligned}$$

First formulation of the inverse problem

Let V be a regular vector field with compact support in an open neighborhood of Ω^* and $\Gamma = (I + V)(\Gamma^*)$. Then the inverse problem formulation is the following :

$$\min_{j_0} ||V||^2_{L^2(\Gamma^*)}$$

with the following constraints:

$$\int_\Gamma \left(\frac{1}{2\mu_0}||\frac{\partial\varphi}{\partial\nu}||^2 + \sigma\mathcal{C}\right)Z.\nu d\gamma = \int_\Gamma p_0 Z.\nu d\Gamma$$

for all Z in $C^1(\mathbb{R}^2, \mathbb{R}^2)$ and

$$\begin{aligned} -\Delta\varphi &= \mu_0 j_0 \quad \text{in } \Omega \\ \varphi &= 0 \quad \text{on } \Gamma \\ \varphi(x) &= O(1) \text{ as } ||x|| \to \infty \end{aligned} \tag{1}$$

$$\int_\omega dx = S_0$$

Second formulation, indirect approach

An indirect approach of the inverse problem can be considered if we introduce a slack variable function $P(x) : \Gamma^* \to \mathbb{R}$ in the equilibrium equation. Then we obtain the following formulation of the problem:

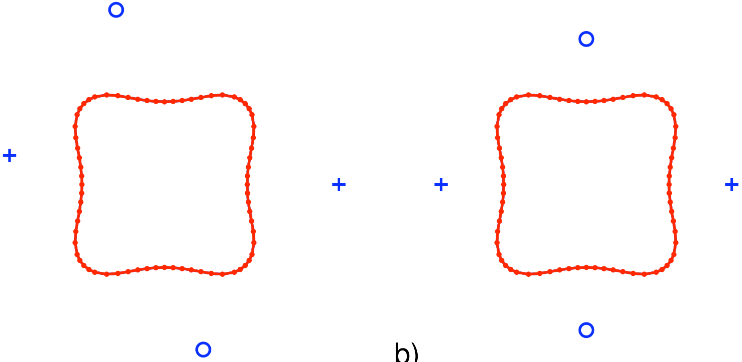
$$\min_{j_0} ||P||^2_{L^2(\Gamma^*)}$$

such that:

$$\int_{\Gamma^*} \left(\frac{1}{2\mu_0}||\frac{\partial\varphi}{\partial\nu}||^2 + \sigma\mathcal{C} + P\right) Z.\nu d\Gamma = \int_{\Gamma^*} p_0 Z.\nu d\Gamma \quad \forall\ Z \in C^1(\mathbb{R}^2, \mathbb{R}^2)$$

with the constraints (3). In this formulation the shape is no more a unknown of the problem.

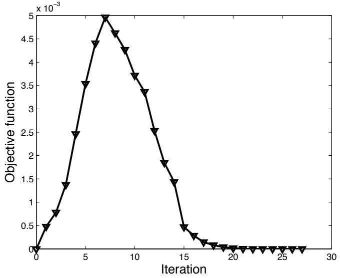
Test example:



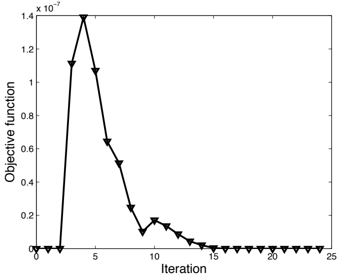
a) b)

Figure: Example 1,a) initial distribution of the inductors, b) final distribution of the inductors with formulation one and two.

Numerical results



a)



b)

Numerical results

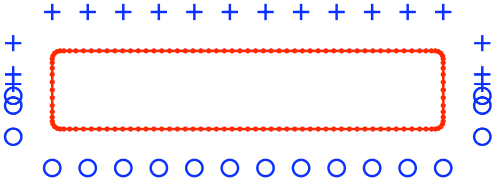


Figure: Example 2 - Target shape and initial configuration of the inductors.

Numerical results

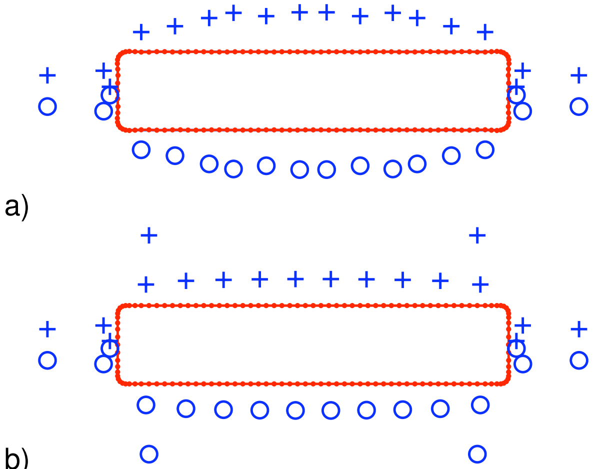


Figure: Example 2, final distribution of the inductors and final shape, a) formulation one, b) formulation two.

Numerical results

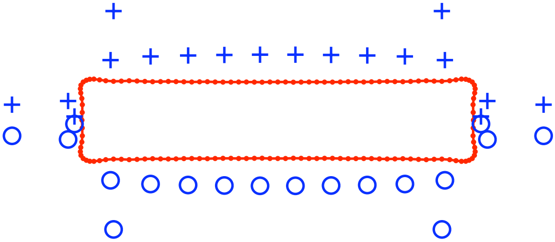
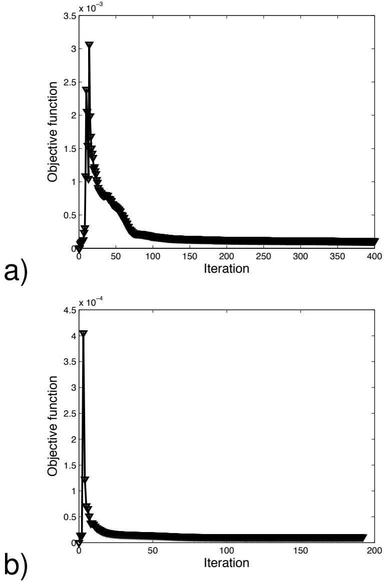


Figure: Example 2 , Equilibrium shape obtained using he inductors resulting from the solution of the inverse problem by formulation two.

Numerical results



Numerical results

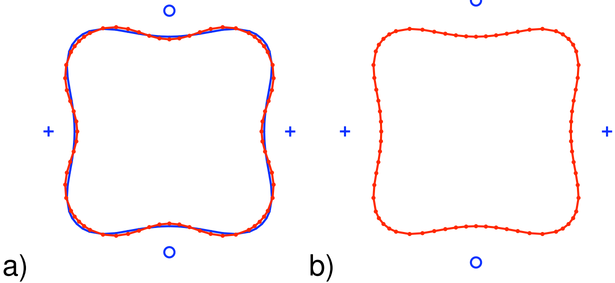


Figure: Example 1b, a) initial distribution of the inductors, b) final distribution of the inductors.

Numerical results

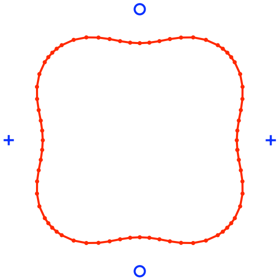
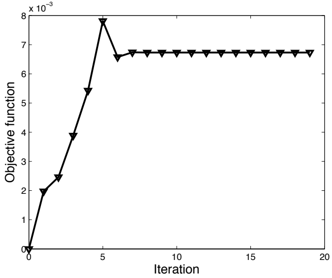
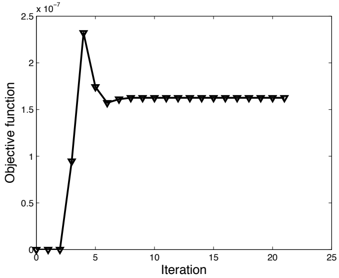


Figure: Example 1b , Equilibrium shape obtained using he inductors resulting from the solution of the inverse problem by formulation two.


x 10^-3
8
7
6
5
4
3
2
1
0
0
5
10
15
20
Objective function
Iteration

a)


x 10^-7
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25
Objective function
Iteration

b)

Numerical results

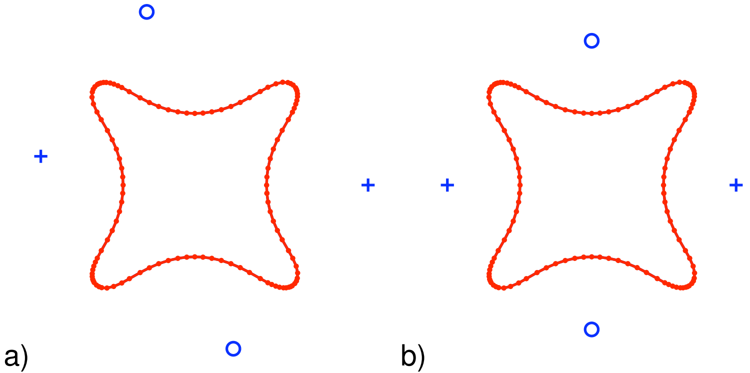
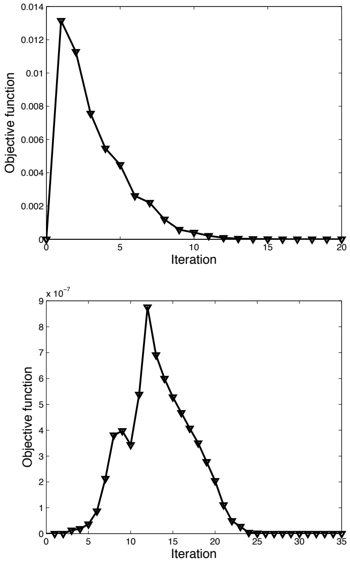


Figure: Example 2a, a) initial distribution of the inductors, b) final distribution of the inductors.

Numerical results



a)

b)

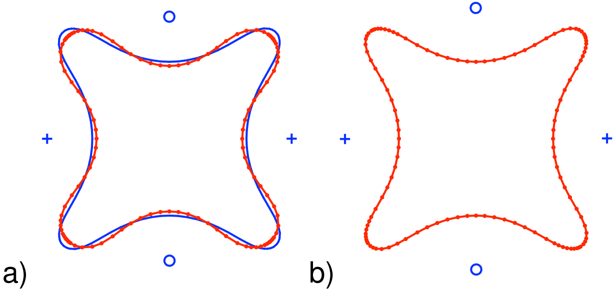


Figure: Example 2b, a) initial distribution of the inductors, b) final distribution of the inductors.

Numerical results

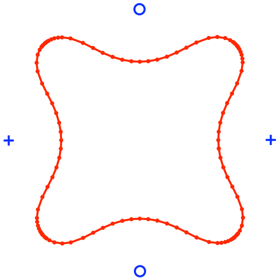
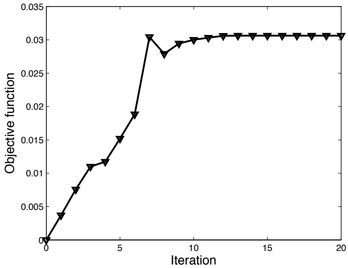
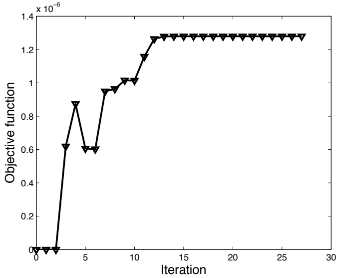


Figure: Example 2b , Equilibrium shape obtained using he inductors resulting from the solution of the inverse problem by formulation two.

Numerical results



a)



b)

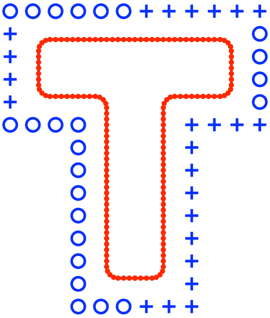


Figure: Example 6, Target shape and initial configuration of the inductors.

Numerical results

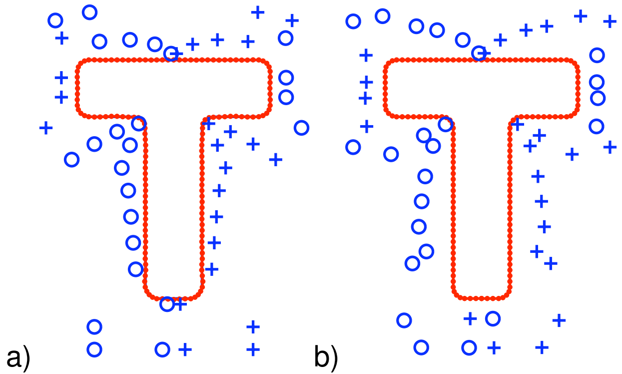


Figure: Example 6, final distribution of the inductors and final shape, a) formulation one, b) formulation two.

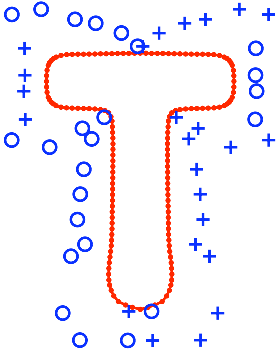


Figure: Example 6, Equilibrium shape obtained using he inductors resulting from the solution of the inverse problem by formulation two

Numerical results