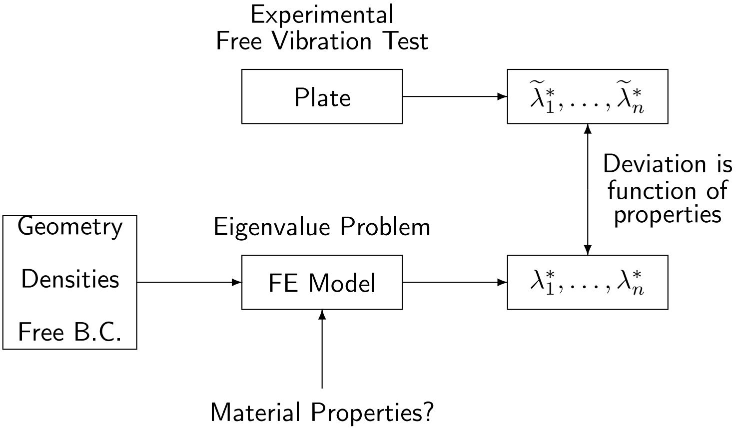
The Inverse Problem

Eigenvalue based formulation



The Inverse Problem

Problem statement

$$\min_{x_i} \Phi(x_1, \dots, x_{ndv}) \geq 0$$
$$g_j(x_1, \dots, x_{ndv}) \leq 0, \qquad j = 1, \dots, ng$$
$$x_i^l \leq x_i \leq x_i^u, \qquad i = 1, \dots, ndv$$

Objective function

$$\Phi = \sum_{n=1}^{I} \mathrm{w}_{\lambda_n} \left(1 - \frac{\lambda_n}{\tilde{\lambda}_n}\right)^2 + \sum_{n=1}^{I} \mathrm{w}_{\eta_n} \left(1 - \frac{\eta_n}{\tilde{\eta}_n}\right)^2$$

Estimation of elastic parameters

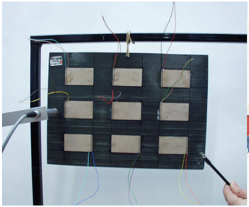
Design variables are non-dimensional elastic parameters are defined in terms of elastic engineering constants:

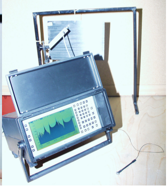
$$
\begin{aligned}
\alpha_2 &= 4 - 4E_2/E_1 \\
\alpha_3 &= 1 + (1 - 2\nu_{12})E_2/E_1 - 4\alpha_0 G_{12}/E_1 \\
\alpha_4 &= 1 + (1 + 6\nu_{12})E_2/E_1 - 4\alpha_0 G_{12}/E_1 \\
\alpha_8 &= 4(G_{13} + G_{23})\alpha_0/E_1 \\
\alpha_9 &= 4(G_{13} - G_{23})\alpha_0/E_1
\end{aligned}
$$

where $\alpha_0 = 1 - \nu_{12}^2 E_2/E_1$.

Estimation of piezoelectric parameters

The piezoelectric parameters that are retained in the finite element formulation are e^*_{31} and e^*_{32}, and these will be the only piezoelectric design variables.





The dielectric coefficient $\epsilon^{*\varepsilon}_{33}$ is not a design variable, since it can be directly related to the other elastic and piezoelectric design variables and to the dielectric constant at constant stress, which is usually provided by manufacturers and can also be easily measured following procedures similar to the ones proposed in ASTM D150-98.

Estimation of elastic and piezoelectric properties

Results

	Carbon T300	PIC 151	
	Estimated	PI Ceramic	Estimated
$E_1(GPa)$	123.9	66.7	56.5
$E_2(GPa)$	16.8	66.7	42.8
$G_{12}(GPa)$	5.8	—	17.6
$G_{13}(GPa)$	8.5	—	23.8
$G_{23}(GPa)$	8.5	—	23.9
ν_{12}	0.24	0.34	0.36
$e^*_{31}(N/Vm)$	—	—	-15.5
$e^*_{32}(N/Vm)$	—	—	-12.8
$d_{31}(10^{-12}m/V)$	—	-210	-194.5
$d_{32}(10^{-12}m/V)$	—	—	-210.0
$\epsilon^{\sigma}_{33}(10^{-9}F/m)$	—	21.2	—
$\rho(kg/m^3)$	1610	8036	

Estimation of frequency dependent viscoelastic parameters

Fractional derivative constitutive models (isotropic):

- Four parameter model:

$$G(j\omega) = G_0 \frac{1 + a(j\omega)^\alpha}{1 + b(j\omega)^\alpha}$$

where $a \geq 0$, $b \geq 0$, $0 \leq \alpha \leq 1$

- Five parameter model:

$$G(j\omega) = G_0 \left[1 + \frac{a(j\omega)^\alpha}{1 + b(j\omega)^\beta}\right]$$

where where $a \geq 0$, $b \geq 0$, $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, and $\alpha > \beta$ ($\alpha - \beta < 0.06$, from experimental evidence on polymeric damping materials)

Sensitivity calculations

Analytic sensitivities

$$\frac{\partial \Phi}{\partial x_i} = -2 \sum_{n=1}^{I} \mathrm{w}_{\lambda_n} \left(1 - \frac{\lambda_n}{\tilde{\lambda}_n}\right) \frac{1}{\tilde{\lambda}_n} \frac{\partial \lambda_n}{\partial x_i} - 2 \sum_{n=1}^{I} \mathrm{w}_{\eta_n} \left(1 - \frac{\eta_n}{\tilde{\eta}_n}\right) \frac{1}{\tilde{\eta}_n} \frac{\partial \eta_n}{\partial x_i}$$

$$\frac{\partial \eta_n}{\partial x_i} = \frac{1}{\lambda_n} \left(\frac{\partial \Im(\lambda_n^*)}{\partial x_i} - \eta_n \frac{\partial \Re(\lambda_n^*)}{\partial x_i} \right)$$

$$\frac{\partial \lambda_n^*}{\partial x_i} = \frac{\mathbf{u}_n^T \left[\frac{\partial \mathbf{K}_{uu}(\omega, x_i)}{\partial x_i} - \lambda_n^* \frac{\partial \mathbf{M}_{uu}(x_i)}{\partial x_i} \right] \mathbf{u}_n}{\mathbf{u}_n^T \left[\mathbf{M}_{uu} - \frac{\omega_n}{2\lambda_n^*} \frac{\partial \mathbf{K}_{uu}(\omega, x_i)}{\partial \omega} \Big|_{\omega = \omega_n} \right] \mathbf{u}_n}$$

Comparison of fractional derivative models

- Clamped plate of in plane dimensions 300 mm $\times$ 200 mm, made of carbon fibre laminate faces and a central ISD-112 viscoelastic material core;
- Stacking sequence: $[0^\circ_c/90^\circ_c/+45^\circ_c/0^\circ_v/+45^\circ_c/90^\circ_c/0^\circ_c]$;
- The thickness of each carbon fiber ply is 0.5 mm, and the viscoelastic core is 2.5 mm thick;
- ISD-112 properties for the frequency range $f = 5 \ldots 1600$ Hz (Barkanov et al. (2009)):

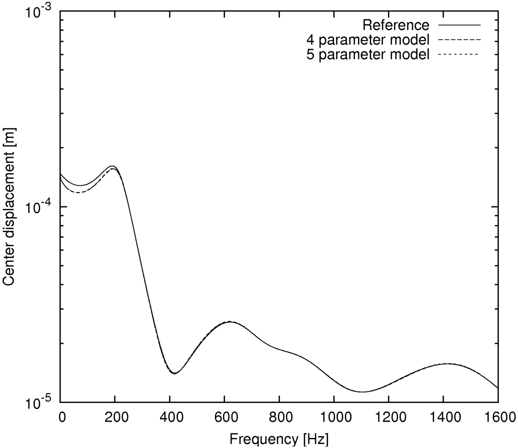
$$G = 4.759 - 0.9266/z + 2.405z^2 \quad \text{[MPa]}$$
$$\text{with} \quad z = 0.1918 + 0.0005148f$$
$$\eta_G = \eta_E = 1.385 - 0.03673z - 0.01342/z$$
$$\text{with} \quad z = 0.01 + 0.0006306f$$
$$\nu = 0.49,\ \rho = 1300\,\text{kg/m}^3$$

Comparison of fractional derivative models

Identified parameters by the present technique:

	4 parameter model	5 parameter model
a	0.542	0.561
b	1.302×10^{-5}	1.402×10^{-5}
α	0.597	0.597
β	—	0.597
G_0 [MPa]	0.078	0.075
Φ	1.106×10^{-3}	1.111×10^{-3}

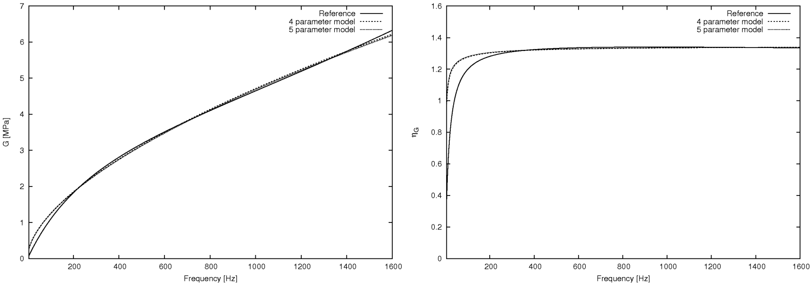
Comparison of fractional derivative models



Central displacement due to a 10 N impulse load, applied at $t = 0$.

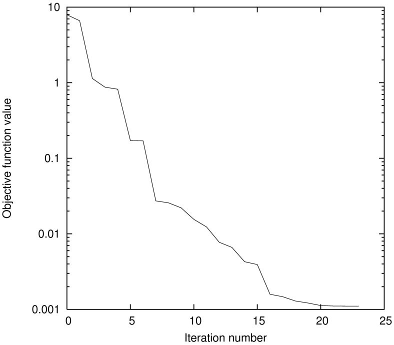
Comparison of fractional derivative models

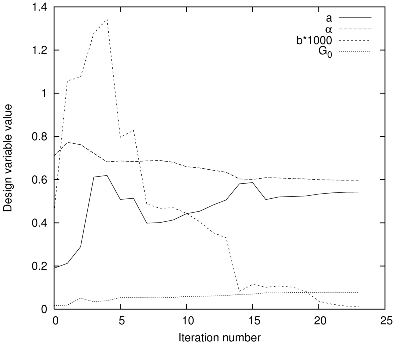
Curves for the material storage modulus and loss factor



Convergence behaviour

Objective function and design variables for the best fit identification (4 parameter model)





Journal Papers in Parameter Estimation

- A.L. Araújo, C.M. Mota Soares, C.A. Mota Soares, J. Herskovits, *Characterisation by inverse techniques of elastic, viscoelastic and piezoelectric properties of anisotropic sandwich adaptive structures*, Applied Composite Materials, 17(5), pp. 543-556, 2010.
- A.L. Araújo, C.M. Mota Soares, C.A. Mota Soares, J. Herskovits, *Optimal design and parameter estimation of frequency dependent viscoelastic laminated sandwich composite plates*, Composite Structures, 92(9), pp. 2321-2327, 2010.
- A.L. Araújo, C.M. Mota Soares, J. Herskovits, P. Pedersen, *Estimation of Piezoelectric and Viscoelastic Properties in Laminated Structures*, Composite Structures, 87, pp. 168-174, 2009.
- A.L. Araújo, C.M. Mota Soares, J. Herskovits, P. Pedersen, *Visco-Piezo-Elastic Parameters Estimation in Laminated Structures*, Inverse Problems in Science and Engineering, 17(2), pp. 145-157, 2009.
- A. L. Araújo, C. M. Mota Soares, J. Herskovits, P. Pedersen, *Parameter Estimation in Active Plate Structures using Gradient Optimisation and Neural Networks*, Inverse Problems in Science and Engineering, 14(5), pp. 483-493, 2006.
- A.L. Araújo, H.M.R. Lopes, M.A.P. Vaz, C.M. Mota Soares, J. Herskovits, P. Pedersen, *Parameter Estimation in Active Plate Structures*, Computers and Structures, 84, pp. 1471-1479, 2006.
- J. Herskovits, V. Dubeux, C.M. Mota Soares, A.L. Araújo, *Interior Point Algorithms for Nonlinear Least Squares Problems*, Inverse Problems in Science and Engineering, 12(2), pp. 211-223, 2004.
- A.L. Araújo, C.M. Mota Soares, J. Herskovits, P. Pedersen, *Development of a Finite Element Model for the Identification of Mechanical and Piezoelectric Properties Through Gradient Optimisation and Experimental Vibration Data*, Composite Structures, 58, pp. 307-318, 2002.
- A.L. Araújo, C.M. Mota Soares, M.J. Moreira de Freitas, P. Pedersen, J. Herskovits, *Combined Numerical-experimental Model for the Identification of Mechanical Properties of Laminated Structures*, Composite Structures, 50, pp. 363-372, 2000.