MINI-COURSES

Thematic Program on Parameter identification in mathematical models

October 02 to November 30, 2017

IMPA – National Institute of Pure and Applied Mathematics
Rio de Janeiro, Brazil

	Monday Oct/02	Tuesday Oct/03	Wednesday Oct/04	Thursday Oct/05	Friday Oct/06
14:00 - 15:30					
15:30 - 16:00					
16:00 - 17:30	**Mini-Course 01** J. Baumeister		**Mini-Course 01** J. Baumeister		**Mini-Course 01** J. Baumeister
17:30 - 19:00					

	Monday Oct/09	Tuesday Oct/10	Wednesday Oct/11	Thursday Oct/12	Friday Oct/13
14:00 - 15:30					
15:30 - 16:00					
16:00 - 17:30	**Mini-Course 01** J. Baumeister		**Mini-Course 01** J. Baumeister		**Mini-Course 01** J. Baumeister
17:30 - 19:00					

	Monday Oct/16	Tuesday Oct/17	Wednesday Oct/18	Thursday Oct/19	Friday Oct/20
14:00 - 15:30					
15:30 - 16:00					
16:00 - 17:30		**Mini-Course 03** U. Ascher		**Mini-Course 03** U. Ascher	
17:30 - 19:00				**Mini-Course 02** B. Hofmann	

	Monday Oct/23	Tuesday Oct/24	Wed Oct/25	Thursday Oct/26	Friday Oct/27
14:00 - 15:30		**Mini-Course 04** S. Siltanen		**Mini-Course 04** S. Siltanen	**Mini-Course 04** S. Siltanen
15:30 - 16:00					
16:00 - 17:30		**Mini-Course 03** U. Ascher		**Mini-Course 03** U. Ascher	
17:30 - 19:00		**Mini-Course 02** B. Hofmann		**Mini-Course 02** B. Hofmann	

Mini-Course 01: Johann Baumeister (Frankfurt Univ., baumeist@math.uni-frankfurt.de)

Title: Iterative regularization methods for parameter identification
Period: 02/Oct - 13/Oct (2 weeks)
Lectures: 2, 4, 6, 9, 11, 13/Oct (from 16:00 to 17:30)

Abstract: Mathematical models for a physical processes contain parameters which cannot be measured directly, in general. They must be determined by using recorded consequences they produce. Such problems are called inverse problems.
Identification problems are inverse problems since one wants to reconstruct causes (parameter in a model) from consequences (output variables). Ill-posedness is a characteristic feature of inverse problems. Ill-posedness means that at least one of the following properties is violated:

existence, uniqueness, stability

Mostly, we are concerned with the case that the property of stability is violated, i.e. that the solution does not depend continously on the data. This is connected to the fact that in inverse problems the consequences are determined by some measure process and therefore corrupted by noise. It is a challence to solve such problems without that a priori the stability property holds. The approach which makes it possible to handle such situations is regularization, i.e. restoration of stability by an appropriate approach. Such regularization approaches may be realized in many different ways:

- Change of the solution concept
- Truncation of small singular values in systems of linear equations
- Discretization with an appropriate discretization level
- Stopping rules in iterative computational schemes
- . . . . . .

As a rule, the degree of efficiency of restoration of stability is deeply connected to the quality and quantity of a priori knowledge that we put in in the approach. Usually, a priori knowledge is introduced by adding qualitative and quantitative constraints to the solution we want to find out.
The aim of this mini-course is to present some topics which are relevant for studying parameter identification problems for ordinary and partial differential equations. The following outline for the lectures in the course is meant to describe a tentative rather than a final plan. Here are issues which we want to present:

(1) Systems theory, modelling, identification
(2) Parametric distributed systems - problems and examples
(3) Functionalanalytic tools
(4) Special concepts for regularizing (nonlinear) problems
(5) Solving parameter identification problems by the model reference adaptive systems appoach
(6) Identification of parameters by the model refernce adaptive systems–approach from a projected observation

Mini-Course 02: Bernd Hofmann (TU Chemnitz, hofmannb@mathematik.tu-chemnitz.de)

Title: Regularization methods in Banach spaces
Period: 16/Oct - 27/Oct (2 weeks)
Lectures: 19, 24, 26/Oct (from 17:30 to 19:00)

Abstract: During the last ten years, there has been a significant progress with respect to the development and analysis of solutions to nonlinear inverse problems. This rapid expansion has been caused by requirements of applications that arise in natural sciences, engineering, imaging, and finance. The inverse problems can be written as operator equations $F(x) = y$ with a nonlinear forward operator F mapping between the Hilbert or Banach spaces X and Y. In this context, $x \in X$ is the non-observable element to be determined from noisy observations of the element $y \in Y$, which can be interpreted as an effect caused by x. Unfortunately, it is an intrinsic property of F to be 'smoothing', which means that during the transition from x to y information gets lost as is typical for procedures including integration. Consequently, the retrieval of x from y tends to become unstable as is typical for procedures including differentiation. This is the phenomenon of ill-posedness. We will outline in the first lecture different concepts of ill-posedness applying to inverse problems. As a consequence of ill-posedness the stable approximate solution of inverse problems requires regularization based on the substitution of the ill-posed original problem by a well-posed auxiliary problem. For a wide range of regularization methods that have been studied, an error analysis could be developed. In particular, to derive convergence rates for regularization methods some kind of smoothness of the solution is necessary. From this perspective, smoothness is a welcome property occurring in the solution process of inverse problems. This lecture series presents, in addition to theoretical ingredients, examples of the treatment of nonlinear applied inverse problems from technology, laser optics and from the financial markets. Extensions of regularization theory from a Hilbert space to a Banach space setting have also been established for linear inverse problems in recent years. We include this kind of theoretical progress in our discussions.

One important approach for the treatment of inverse problems is based on Tikhonov regularization, in which case a one-parameter family of regularized solutions is obtained. We present in the second lecture the novel theory of variational source conditions for obtaining convergence rates in Tikhonov regularization for Hilbert spaces, reflexive Banach spaces and also some recent results on regularization in the non-reflexive Banach space L^1. Moreover, it is crucial to choose the parameter appropriately. In this talk, a sequential variant of the discrepancy principle is analyzed. In many cases such parameter choice exhibits the so-called regularization property, that the chosen parameter tends to zero as the noise tends to zero, but slower than the noise level. It will be shown that such regularization property occurs under two natural assumptions. First, exact penalization must be excluded (exact penalization veto), and secondly, the discrepancy principle must stop after a finite number of iterations.

For inverse problems with monotone forward operators, the simpler Lavrentiev regularization can be exploited for the stable approximate solution of corresponding linear and nonlinear operator equations. We present in the third lecture the theoretical background of this specific regularization method and recent research results in combination with concepts of conditional stability.

Mini-Course 03: Uri Ascher (British Columbia, ascher@cs.ubc.ca)

Title: Computational methods in applied inverse problems
Period: 16/Oct - 27/Oct (2 weeks)
Lectures: 17, 19, 24, 26/Oct (from 16:00 to 17:30)

Abstract: Below is a brief description of my planned short course at IMPA, given as part of the Thematic Program on Parameter Identification in Mathematical Models. It consists of four lectures, at most 90 minutes each. In the past two decades there have been many developments in computational methods for applied inverse problems. These include PDE constrained optimization, sparsity-enhancing methods, level set methods, probabilistic methods, randomized algorithms, machine learning techniques (e.g., deep learning) and more. Optimization techniques play a prominent role, as do PDE discretization methods and fast solution techniques. I will attempt to shed some light on several of the challenges and solution techniques in these computational areas, using my own research to demonstrate and highlight issues.
This document is meant to describe a tentative rather than a final plan. The lectures will be adjusted according to audience level of interest and needs as well as the instructor's limitations.

1 Calibration and simulation of deformable objects.
2 Data manipulation and completion
3 Estimating the trace of a large implicit matrix and applications
4 Numerical Analysis and Visual Computing: not /too little, not too much

Mini-Course 04: Samuli Siltanen (University of Helsinki, samuli.siltanen@helsinki.fi)

Title: Reconstruction methods for sparse-data X-ray tomography
Period: 23/Oct - 27/Oct (1 week)
Lectures: 24, 26, 27/Oct (from 14:00 to 15:30)

Abstract: X-ray tomography is an imaging method where an unknown physical body is photographed from many directions using X-rays. The X-rays passing through the object lose their intensity exponentially in proportion to the density of the material along the ray according to the Beer-Lambert law. After a calibration step one arrives at the following mathematical problem: can one recover a non-negative, compactly supported function from the knowledge of integrals of that function along lines? Johann Radon showed in his seminal 1917 article how to do that in dimension two when all possible line integrals are known. Radon's geometric reconstruction formula serves as the foundation of today's Computerized Tomography (CT) scanners in hospitals in the form of the Filtered Back-Projection (FBP) algorithm. FBP is based on inverting the so-called Radon transform.

Recently, there is growing interest in X-ray tomography imaging based on limited data. The main reason for this is the need to limit the harmful radiation dose to the patient. Mathematically, the problem of recovering a function from an incomplete set of line integrals is an example of a linear ill-posed inverse problem. Ill-posedness means that the reconstruction problem is extremely sensitive to measurement noise and modelling errors. In such situations the FBP algorithm is not optimal. This course discusses variational regularisation methods for limited-data X-ray tomography, including classical Tikhonov regularisation and modern sparsity-promoting algorithms such as Total Variation regularization. The core idea behind these methods is complementing the insufficient measurement data by additional information about the unknown function.

The methods presented in the course are applicable to any linear ill-posed inverse problems. Also, they can be extended to nonlinear cases.