

Corrigendum

On the relation between constraint regularization, level sets and shape optimization

A Leitão and O Scherzer 2003 *Inverse Problems* **19** L1-L11

We correct lemma 2.2 in this paper. The most important consequences are: (i) the level set evolution derived from this lemma has to be corrected; (ii) the relation to shape optimization presented in section 2.2 must be changed.

Lemma 2.2 (corrected) Let ϕ be a smooth C^2 function having no critical point in a compact neighborhood M of the level set $\phi^{-1}(0)$. Then,

$$\lim_{\varepsilon \rightarrow 0^+} P'_\varepsilon(\phi) = \delta(\phi),$$

where $\delta(\phi)$ is the one-dimensional δ -distribution centred at the level line $\phi^{-1}(0)$ in the normal direction.

The unique modification in the assumptions of lemma 2.2 concerns the exigence of C^2 regularity of the level set function w.r.t. the space variable. Notice that the factor $|\nabla\phi|^{-1}$, which appeared in the original formulation of this lemma, has to be removed. In the original proof, the last equality in the evaluation of $\lim_{\varepsilon \rightarrow 0^+} \int_\Omega P'_\varepsilon(\phi)v$ is not correct.

We observe that the velocity $v = \partial\phi/\partial t$ satisfies, instead of (11) in the original paper, the equation

$$\begin{cases} (I - \Delta)v &= \delta(\phi(t)) F'(u(t))^*(F(u(t)) - y), \text{ in } \Omega \\ \frac{\partial v}{\partial n} &= 0, \text{ at } \partial\Omega. \end{cases} \quad (1)$$

Therefore, the level set function ϕ is the solution of the Hamilton–Jacobi equation

$$\frac{\partial\phi}{\partial t} + V|\nabla\phi| = 0$$

with velocity

$$V = (I - \Delta)^{-1} \left[\delta(\phi) F'(u)^*(F(u) - y) \right] \frac{\nabla\phi}{|\nabla\phi|^2}.$$

This evolution equation is comparable to the one derived by Santosa in [1], who suggested the velocity

$$V = -F'(u(t))^*(F(u(t)) - y) \frac{\nabla\phi}{|\nabla\phi|}.$$

References

- [1] Santosa F 1996 A level-set approach for inverse problems involving obstacles *ESAIM Control Optim. Calc. Var.* **1** 17–33 (electronic)