

Quantitative Photoacoustics Using the Transport Equation

Simon Arridge¹

Joint work with: Ben Cox³ Teedah Saratoon³
Tanja Tarvainen^{1,2}

¹Department of Computer Science, University College London, UK

²Department of Physics and Mathematics, University of Eastern Finland, Finland

³Department of Medical Physics, University College London, UK

Colóquio Brasileiro de Matemática, July 29-August 2, 2013



Outline

- 1 Introduction
- 2 PhotoAcoustics
- 3 PhotoAcoustic Forward Model
- 4 Quantitative PhotoAcoustic Tomography
- 5 Summary
- 6 Acknowledgements

Outline

- 1 Introduction
- 2 PhotoAcoustics
- 3 PhotoAcoustic Forward Model
- 4 Quantitative PhotoAcoustic Tomography
- 5 Summary
- 6 Acknowledgements

Introduction

Outline

Photoacoustic Imaging

- outline of photoacoustic imaging
- Photoacoustic image reconstruction
- Spectroscopic photoacoustic imaging
- Artefacts in photoacoustic imaging

Quantitative Photoacoustic Imaging

- Models of light transport
- Multispectral reconstructions
- Unknown scattering: diffusion-based inversions
- Unknown scattering: using radiative transfer equation

Outline

1 Introduction

2 PhotoAcoustics

3 PhotoAcoustic Forward Model

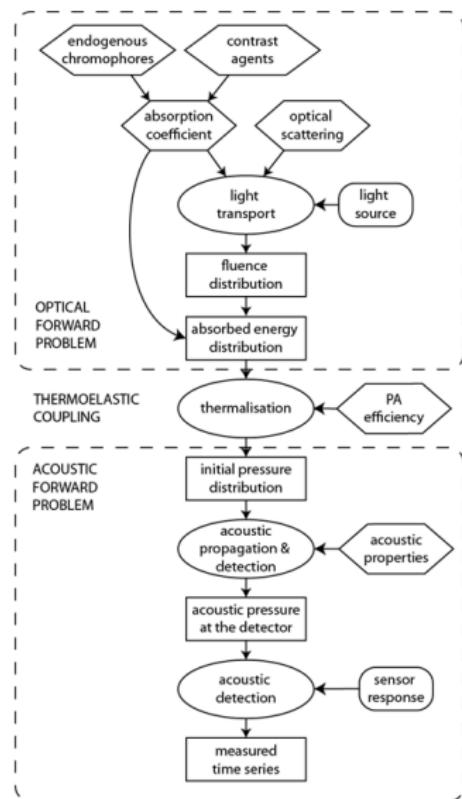
4 Quantitative PhotoAcoustic Tomography

5 Summary

6 Acknowledgements

PhotoAcoustic Tomography

PhotoAcoustic Signal Generation



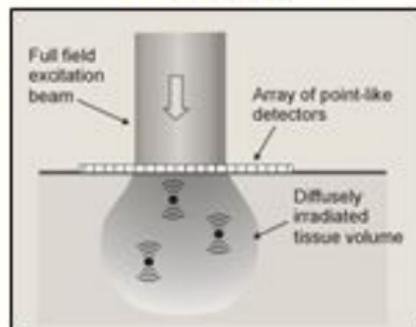
Spatially varying chromophore concentrations (naturally occurring or contrast agents) give rise to optical absorption in the medium. The absorption and scattering coefficients μ_a and μ'_s determine the fluence distribution Φ , and thence the absorbed energy distribution H . This energy generates a pressure distribution p_0 via thermalisation, which because of the elastic nature of tissue, then propagates as an acoustic pulse. The pulse is detected by a sensor resulting in the measured PA time series $p(t)$.

PhotoAcoustic Tomography

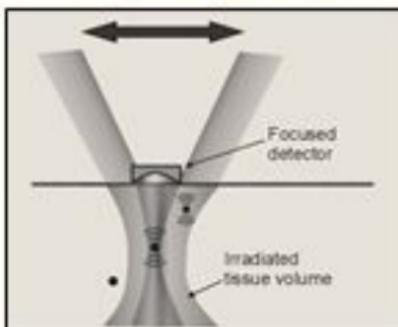
PhotoAcoustic Imaging : 3 Modes

Photoacoustic Imaging: Three Modes

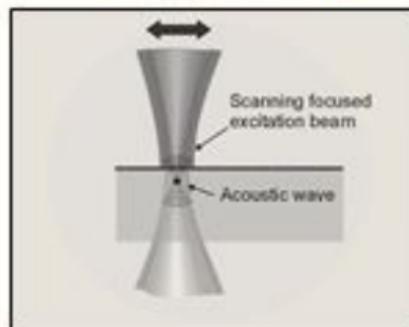
Photoacoustic
Tomography (PAT)



Photoacoustic
Microscopy (PAM)



Optical Resolution Photoacoustic
Microscopy (OR-PAM)



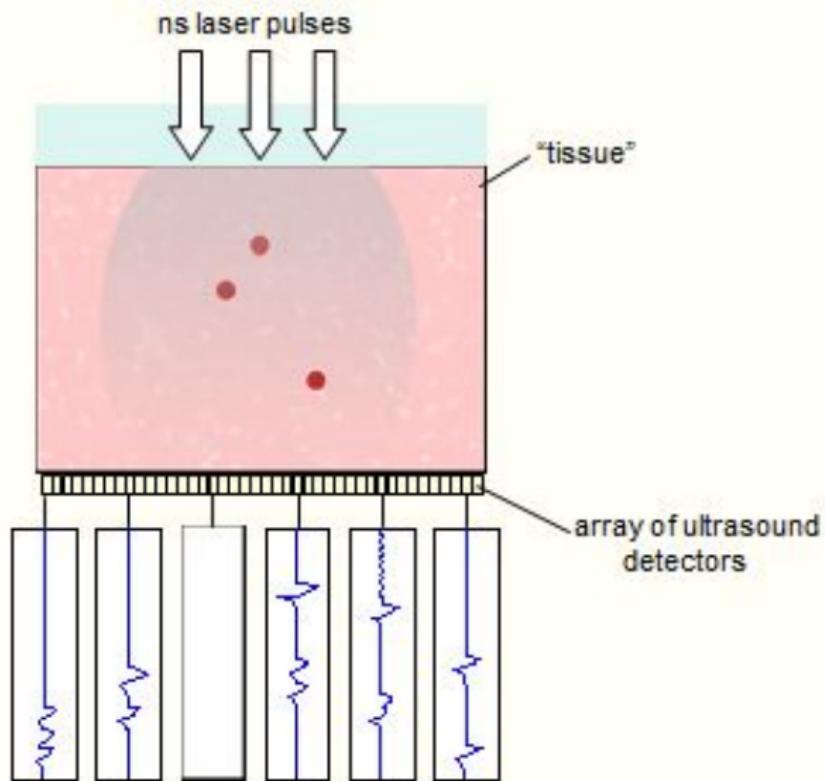
- Full field excitation
- Acoustic diffraction limited spatial resolution throughout irradiated volume

- Dark-field/full field excitation
- Acoustic diffraction limited spatial resolution at transducer focus

- Focused excitation beam
- Optical diffraction limited lateral spatial resolution

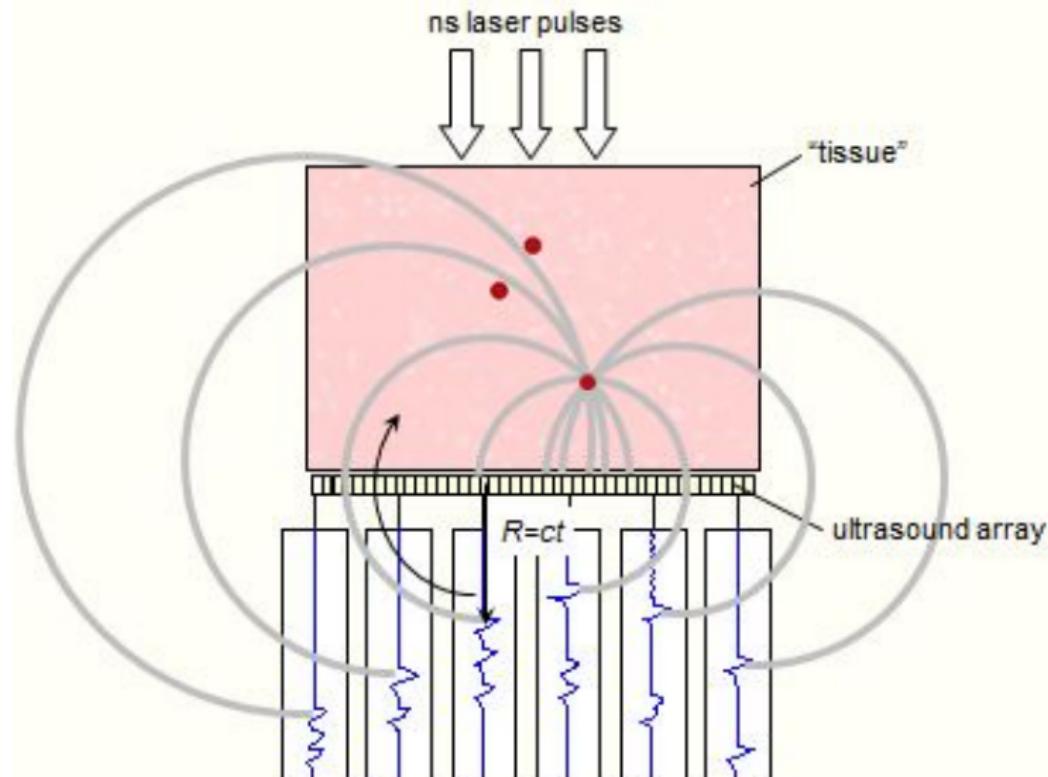
PhotoAcoustic Tomography

PhotoAcoustic Signal Generation



PhotoAcoustic Tomography

PhotoAcoustic Spherical BackProjection



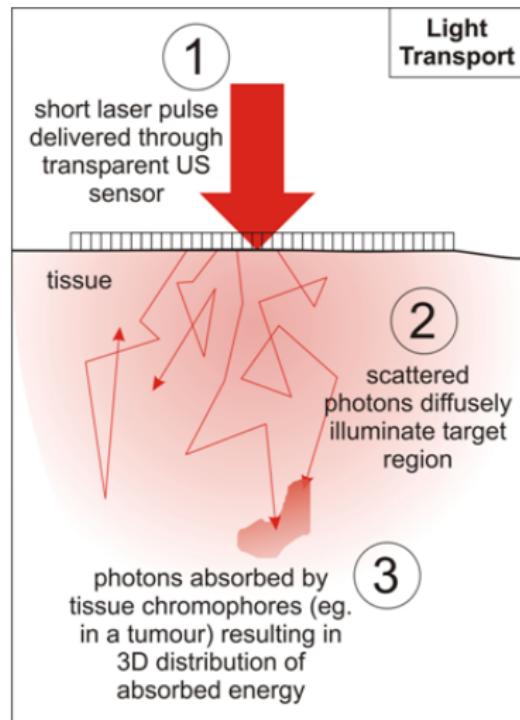
PhotoAcoustic Tomography

Optical part of the direct problem

Optical part of the direct problem

$$H(\mathbf{r}) = \mu_a(\mathbf{r})\Phi(\mathbf{r})$$

absorbed
energy density absorption coefficient
= heat per
unit volume

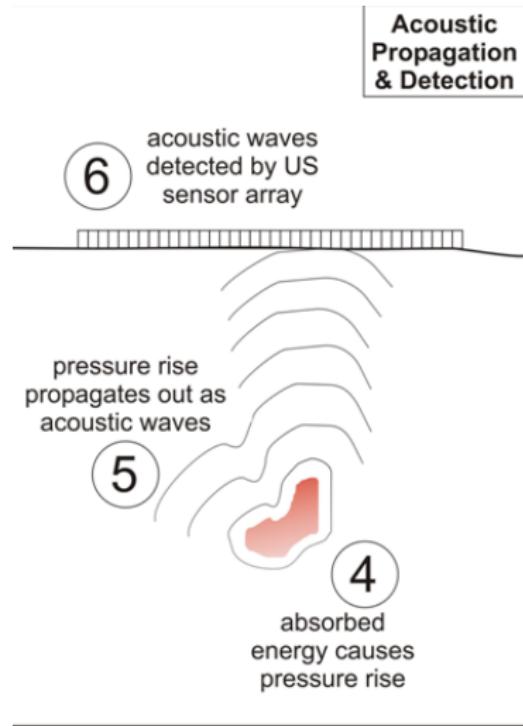


PhotoAcoustic Tomography

Acoustic part of the direct problem

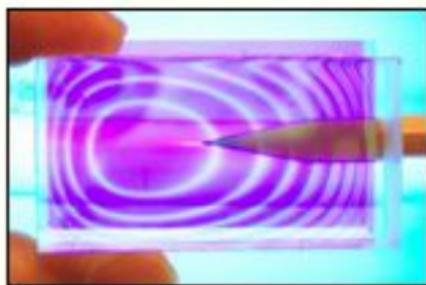
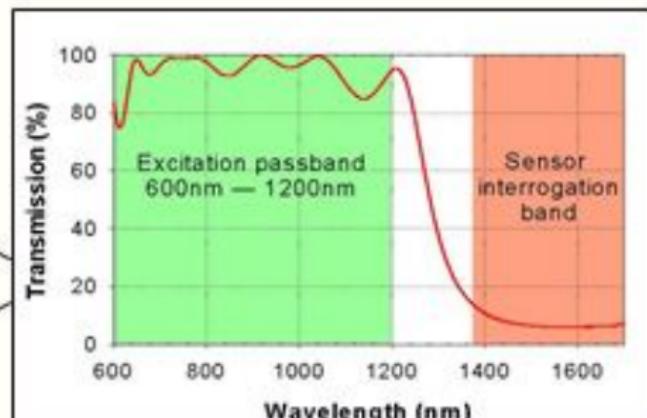
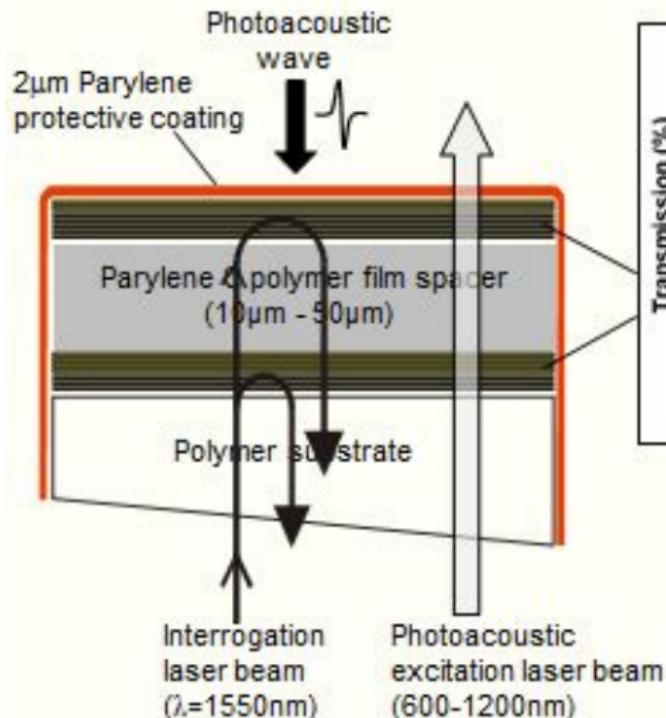
Acoustic part of the direct problem

$$\begin{aligned} p(\mathbf{r})|_{t=0} &= \Gamma(\mathbf{r})H(\mathbf{r}) \\ &= \Gamma(\mathbf{r})\mu_a(\mathbf{r})\Phi(\mathbf{r}) \\ &\text{Grüneisen} \\ &\text{parameter} \\ \left(c^2\nabla^2 - \frac{\partial^2}{\partial t^2}\right)p &= 0 \end{aligned}$$



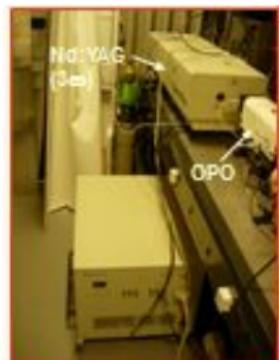
PhotoAcoustic Tomography

Fabrey-Perot Detector

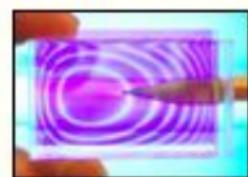


PhotoAcoustic Tomography

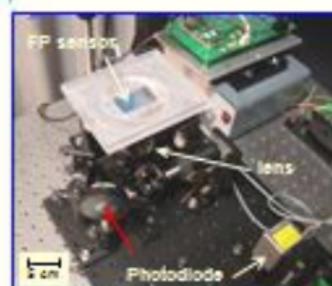
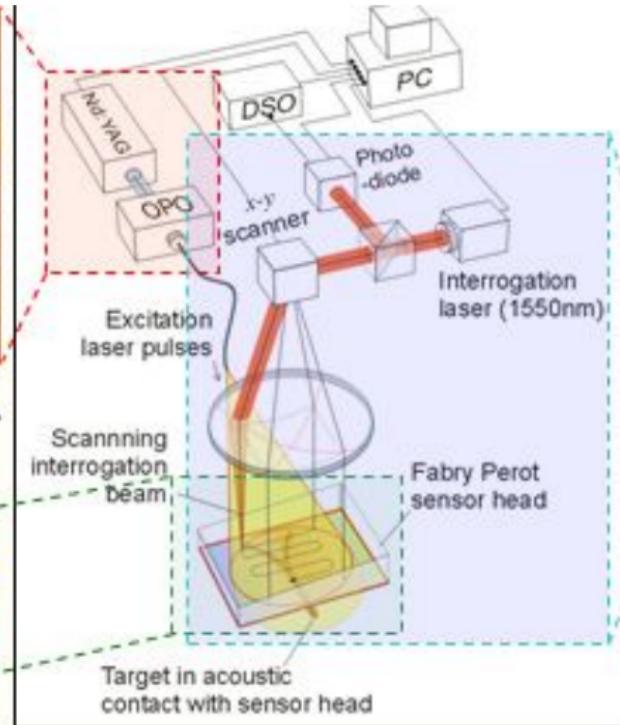
3D PhotoAcoustic Scanner



OPO excitation laser system (410 - 2100nm), 8ns pulse duration, PRF=10Hz



Fabry Perot sensor



Scanning system

- Scan area: 50mm circle
- Min step size: 10 μ m
- Optically defined "element size": 50 μ m

Zhang E, Lauter J, Beard PC, Applied Optics, Vol 47, pp561-577, 2008

PhotoAcoustic Tomography

PAT Acoustic Inversion (Image Reconstruction)

Initial value Problem

$$\left(c^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) p = 0$$

$$p|_{t=0} = \Gamma \mu_a \Phi$$

$$\left. \frac{\partial p}{\partial t} \right|_{t=0} = 0$$

PhotoAcoustic Tomography

PAT Acoustic Inversion (Image Reconstruction)

Initial value Problem

$$\left(c^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) p = 0$$
$$p|_{t=0} = \Gamma \mu_a \Phi$$
$$\frac{\partial p}{\partial t} \Big|_{t=0} = 0$$

Boundary value Problem (t running backwards from T to 0)

$$\left(c^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) p = 0$$
$$p(\mathbf{r}, t)|_{t=T} = 0$$
$$p(\mathbf{r}, t)|_{\partial\Omega} = p^{\text{obs}}(\mathbf{r}_s, t)$$

PhotoAcoustic Tomography

PAT Acoustic Inversion (Image Reconstruction)

Initial value Problem

$$\left(c^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) p = 0$$

$$p|_{t=0} = \Gamma \mu_a \Phi$$

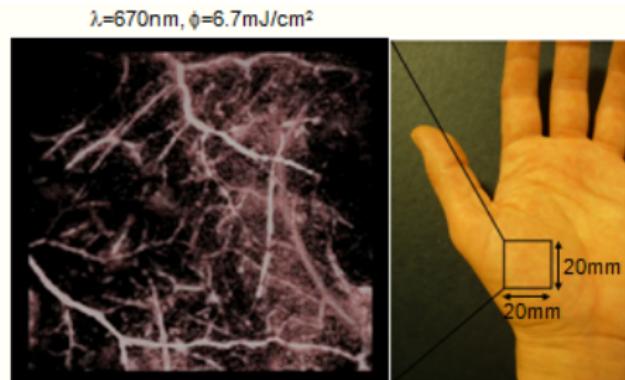
$$\frac{\partial p}{\partial t} \Big|_{t=0} = 0$$

Boundary value Problem (t running backwards from T to 0)

$$\left(c^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) p = 0$$

$$p(\mathbf{r}, t)|_{t=T} = 0$$

$$p(\mathbf{r}, t)|_{\partial\Omega} = p^{\text{obs}}(\mathbf{r}_s, t)$$



20mm x 20mm x 6mm
dx=dy=250μm

PhotoAcoustic Tomography

Heterogeneous Sound Speed

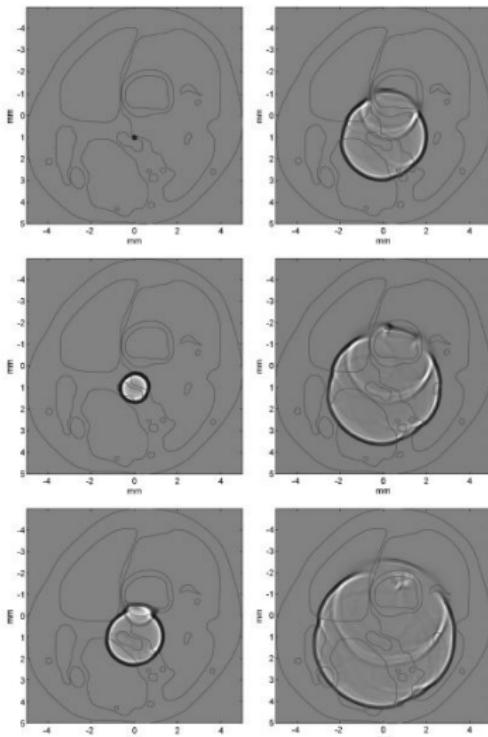


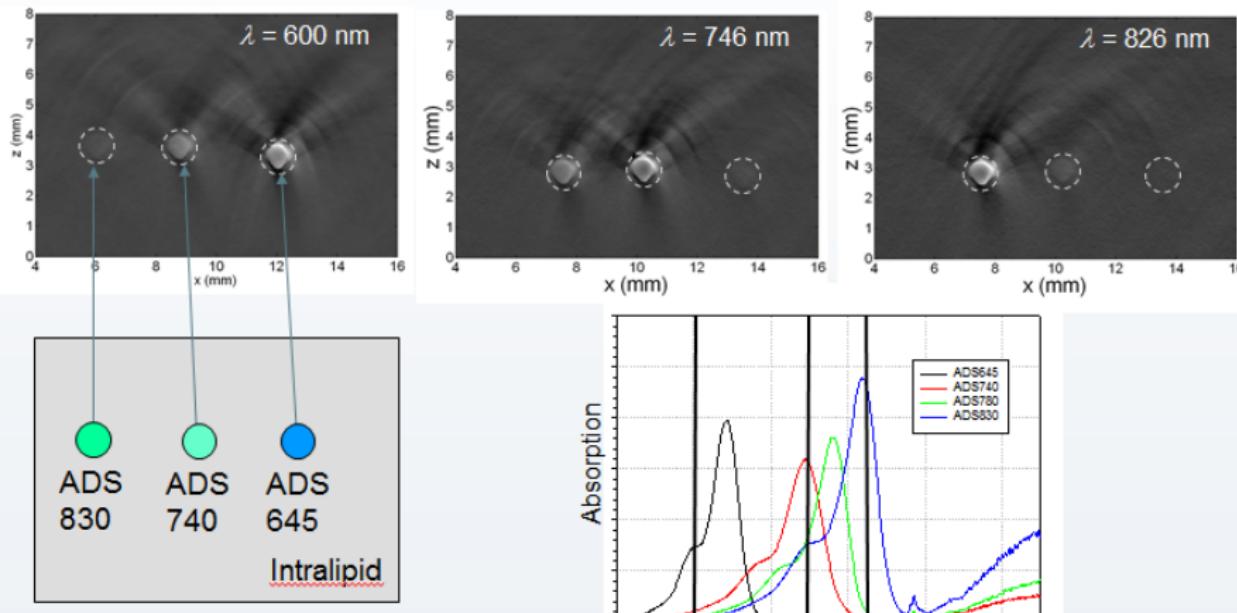
FIG. 10. Propagation of acoustic waves from a small, circular, photoacoustic source, through a heterogeneous medium with tissue-like properties (see Fig. 9). The frames are snapshots of the acoustic field at intervals of $0.21 \mu\text{s}$ following the optical pulse. The boundaries between the regions with different acoustic properties are superimposed, to show how the wave fronts are distorted by the heterogeneities.

PhotoAcoustic Tomography

Spectroscopic PAT

Spectroscopic PAT

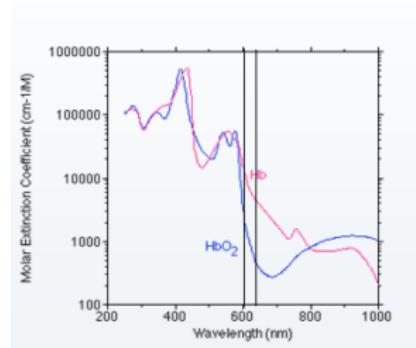
- PAT's strength lies in spectroscopy using images at multiple wavelengths



PhotoAcoustic Tomography

Spectroscopic PAT

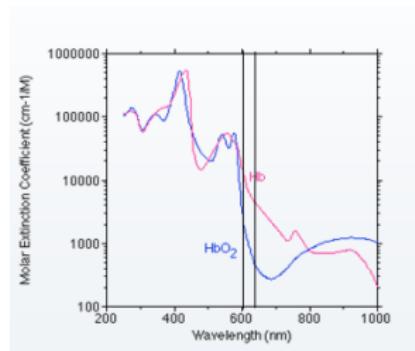
- absorption at different wavelengths gives spectral images



PhotoAcoustic Tomography

Spectroscopic PAT

- absorption at different wavelengths gives spectral images
- but fluence is also different at different wavelengths

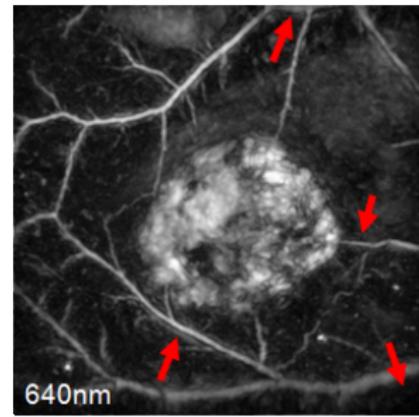
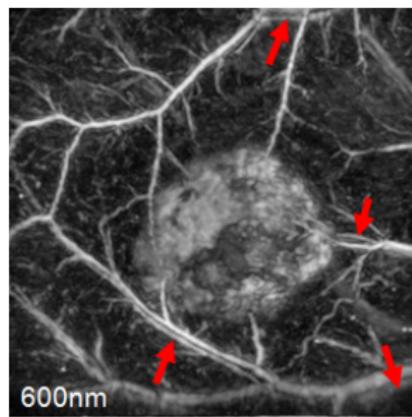
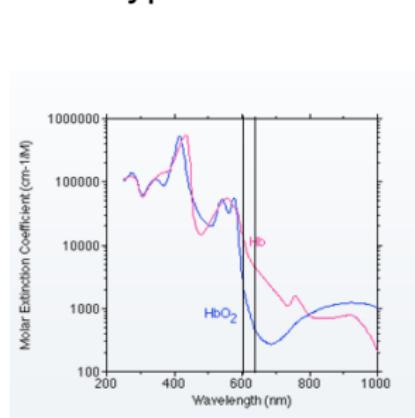


PhotoAcoustic Tomography

Spectroscopic PAT

- absorption at different wavelengths gives spectral images
- but fluence is also different at different wavelengths

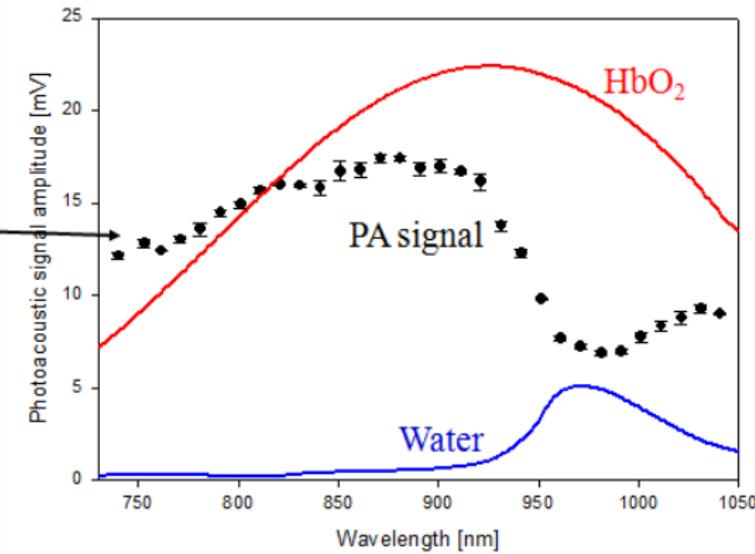
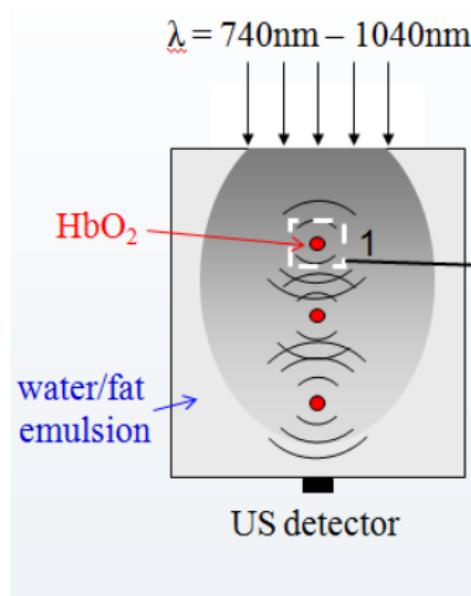
tumour type LS174T



PhotoAcoustic Tomography

Spectral Distortion

Spectral Distortion

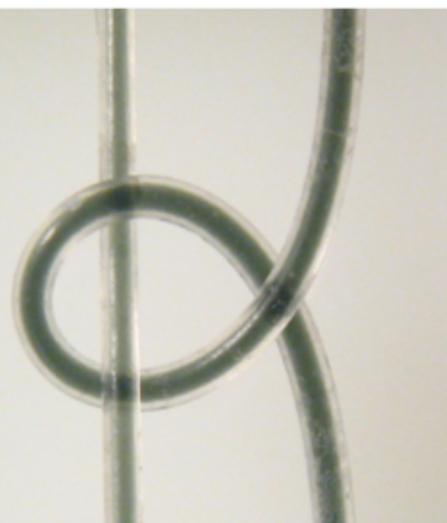
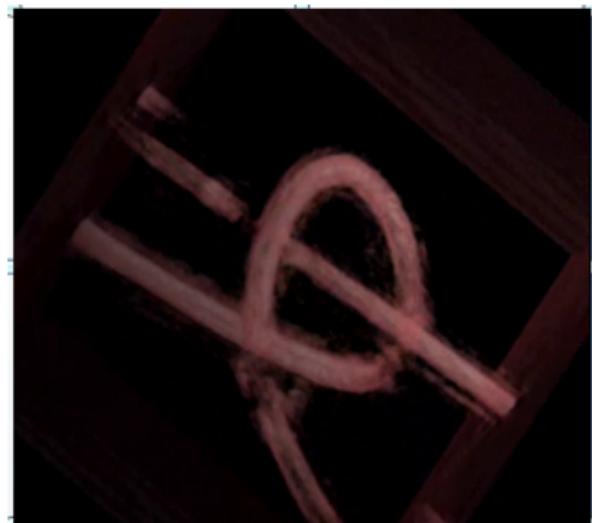


Spectrum corrupted by wavelength dependence of fluence

PhotoAcoustic Tomography

Structural Distortion

Structural Distortion



- Structural distortion due to non-uniform internal light fluence
- Structural distortion at each wavelength = spectral distortion at each point

Outline

- 1 Introduction
- 2 PhotoAcoustics
- 3 PhotoAcoustic Forward Model
- 4 Quantitative PhotoAcoustic Tomography
- 5 Summary
- 6 Acknowledgements

PhotoAcoustic Forward Model

Second order wave equation for homogeneous media

$$\left(\nabla^2 - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right) p(\mathbf{r}, t) = 0, \quad p(\mathbf{r}, 0) = p_0(\mathbf{r}), \quad \frac{\partial}{\partial t} p(\mathbf{r}, 0) = 0 \quad (1)$$

$$p(\mathbf{r}, t) = \frac{1}{c_0^2} \int \left(g(\mathbf{r}, t | \mathbf{r}_0, t_0) \frac{\partial p_0(\mathbf{r})}{\partial t_0} - p_0(\mathbf{r}) \frac{\partial g(\mathbf{r}, t | \mathbf{r}_0, t_0)}{\partial t_0} \right) d\mathbf{r}_0 \quad (2)$$

$$g(\mathbf{r}, t | \mathbf{r}_0, t_0) = \frac{c_0^2}{(2\pi)^3} \int \frac{\sin(c_0 kt)}{c_0 k} e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_0)} d\mathbf{k} \quad k = |\mathbf{k}| \quad (3)$$

$$p(\mathbf{r}, t) = \frac{1}{(2\pi)^3} \int \int p_0(\mathbf{r}) \cos(c_0 kt) e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}_0)} d\mathbf{k} d\mathbf{r}_0 \quad (4)$$

$$= \mathcal{F}^{-1} \{ \mathcal{F} \{ p_0(\mathbf{r}) \} \cos(c_0 kt) \} \quad (5)$$

Simple numerical algorithm using wave propagator $\cos(c_0 kt)$
(Cox and Beard 2005)

PhotoAcoustic Forward Model

Linear Lossless Acoustic Equations

$$p(\mathbf{r}, t) = \mathcal{F}^{-1} \{ \mathcal{F} \{ p_0(\mathbf{r}) \} \cos(c_0 kt) \} \quad (6)$$

- Cannot input a time-varying pressure, so no use for time-reversal imaging
- FFT \Rightarrow periodic boundary conditions (wave wrapping)
- Instead: solve equivalent first-order system

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_0} \nabla p \quad \text{linearised momentum conservation} \quad (7)$$

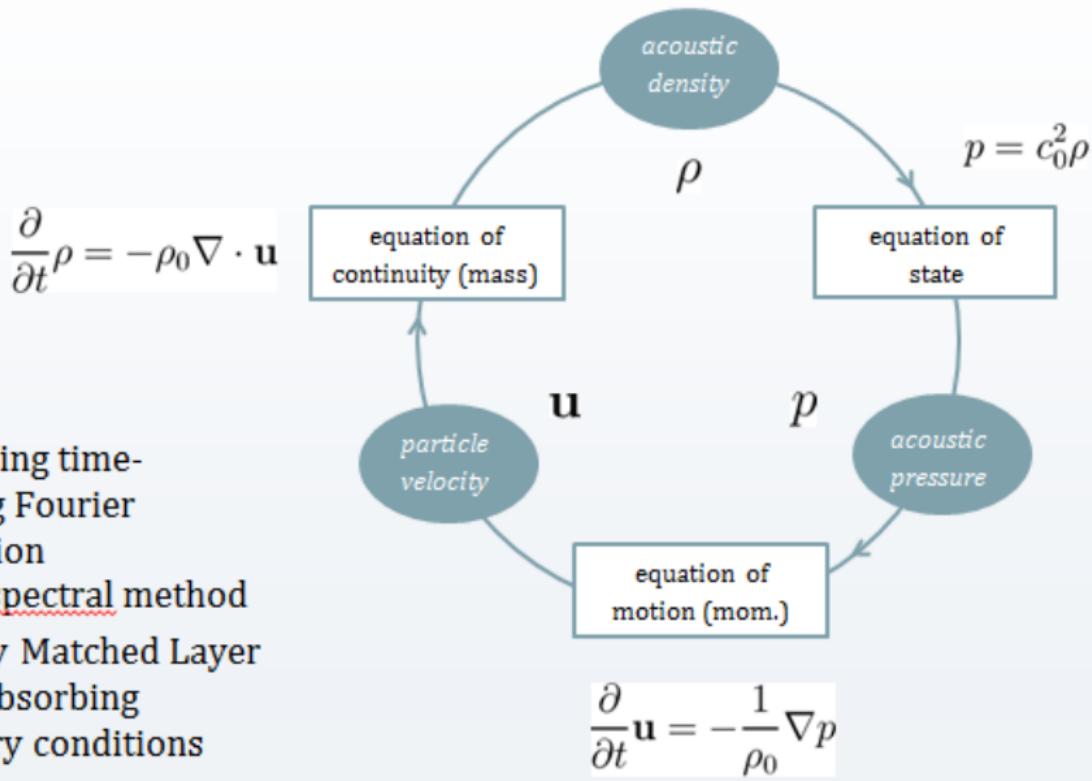
$$\frac{\partial \rho}{\partial t} = -\rho_0 \nabla \cdot \mathbf{u} \quad \text{linearised mass conservation} \quad (8)$$

$$p = c_P^2 \rho \quad \text{linearised equation of state} \quad (9)$$

p acoustic pressure, \mathbf{u} particle velocity, ρ acoustic density

PhotoAcoustic Forward Model

k-Space Acoustic Propagation Model



PhotoAcoustic Forward Model

Modelling Acoustic Absorption

- Photoacoustic waves may contain frequencies much higher than conventional ultrasound imaging (tens of MHz)
- Acoustic absorption in soft tissue over ranges of interest (1-50 MHz or so) typically takes the form

$$\alpha = \alpha_0 \omega^y, \quad 1 \leq y \leq 1.5$$

- What wave equations account for absorption like this?
- Can they be time-reversed to correct for absorption during image reconstruction?

PhotoAcoustic Forward Model

Modelling Absorption & Dispersion

- Using Kramers-Kronig relations to find a purely dispersive term.
- Incorporating these into the acoustic equation of state gives

$$p = c_0^2 \left(\underbrace{1}_{\text{adiabatic}} + \tau \underbrace{\frac{\partial}{\partial t} (-\nabla^2)^{y/2-1}}_{\text{absorption only}} + \eta \underbrace{(-\nabla^2)^{(y+1)/2-1}}_{\text{dispersion only}} \right) \rho$$

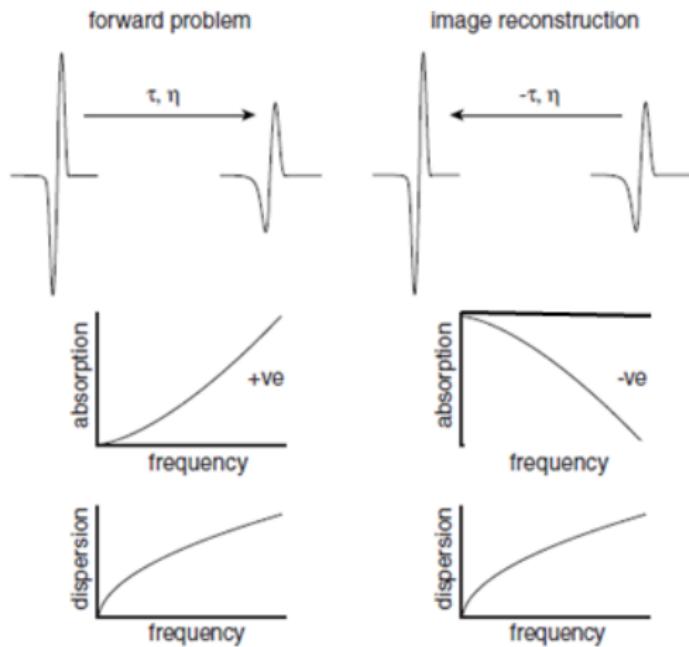
- Separate dispersion and absorption terms: time reversal is then possible

$$p = c_0^2 \left(1 - \tau \frac{\partial}{\partial t} (-\nabla^2)^{y/2-1} + \eta (-\nabla^2)^{(y+1)/2-1} \right) \rho$$

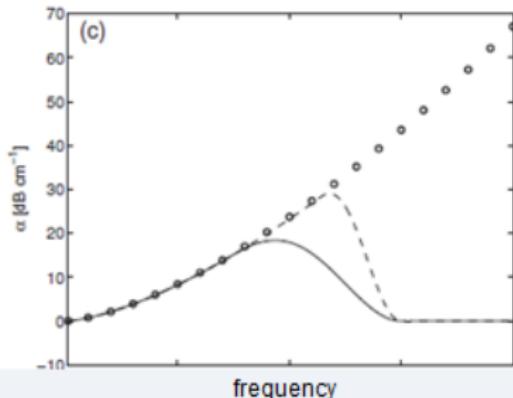
PhotoAcoustic Forward Model

Absorption & Dispersion in Time Reversal Imaging

- Absorbing wave equation: no time symmetry
- Absorption → amplification, dispersion unchanged

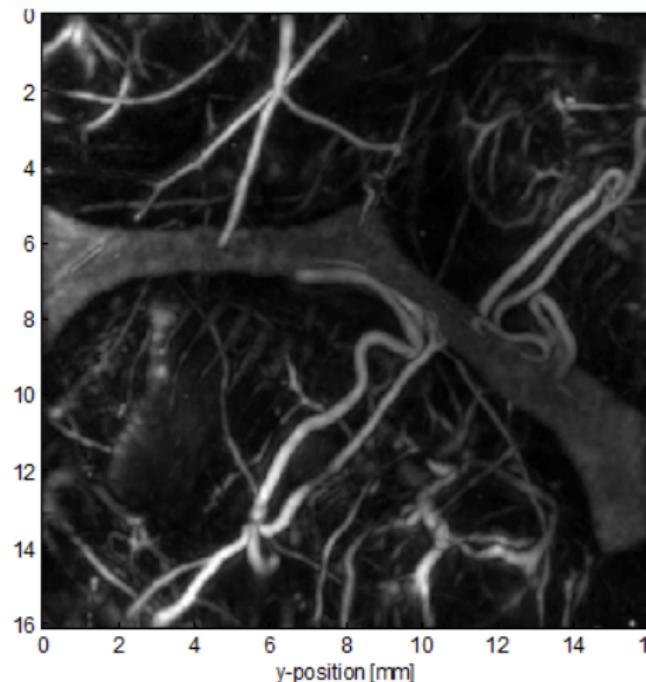


- avoid amplifying noise: regularisation

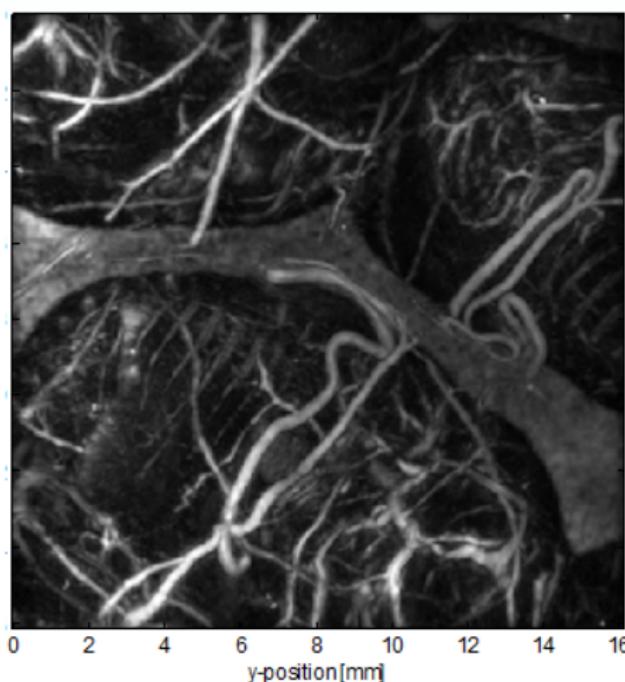


PhotoAcoustic Forward Model

Time Reversal PAT with Absorption Correction



no absorption correction



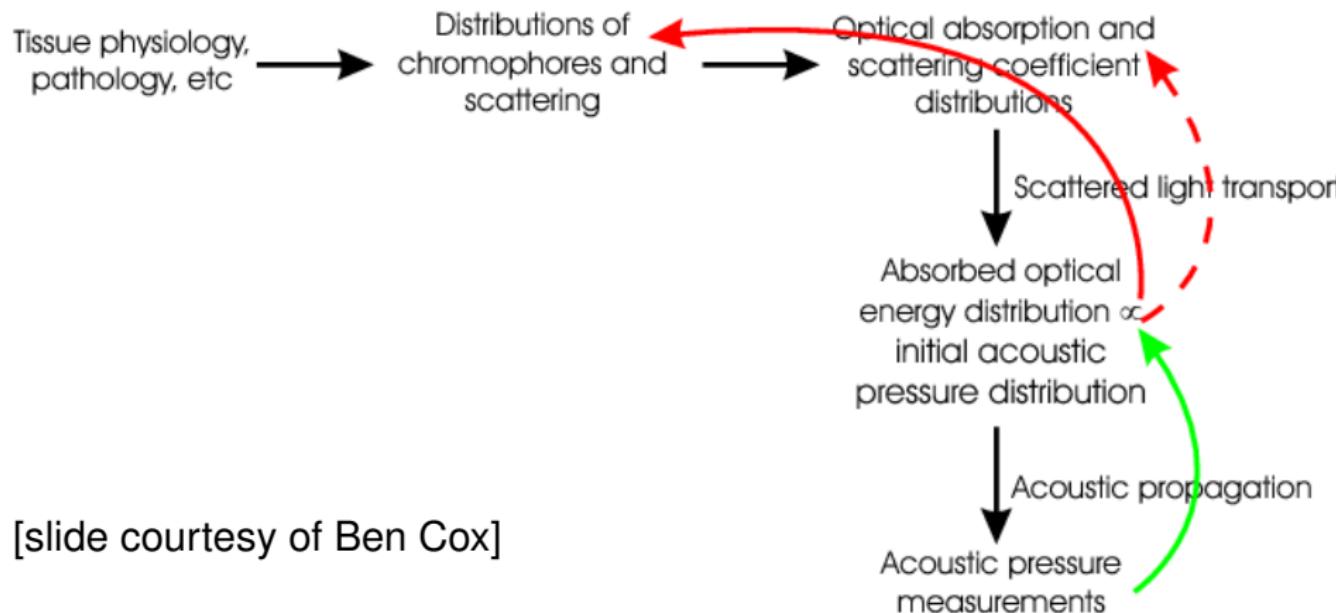
with absorption correction

Outline

- 1 Introduction
- 2 PhotoAcoustics
- 3 PhotoAcoustic Forward Model
- 4 Quantitative PhotoAcoustic Tomography
- 5 Summary
- 6 Acknowledgements

PhotoAcoustic Tomography

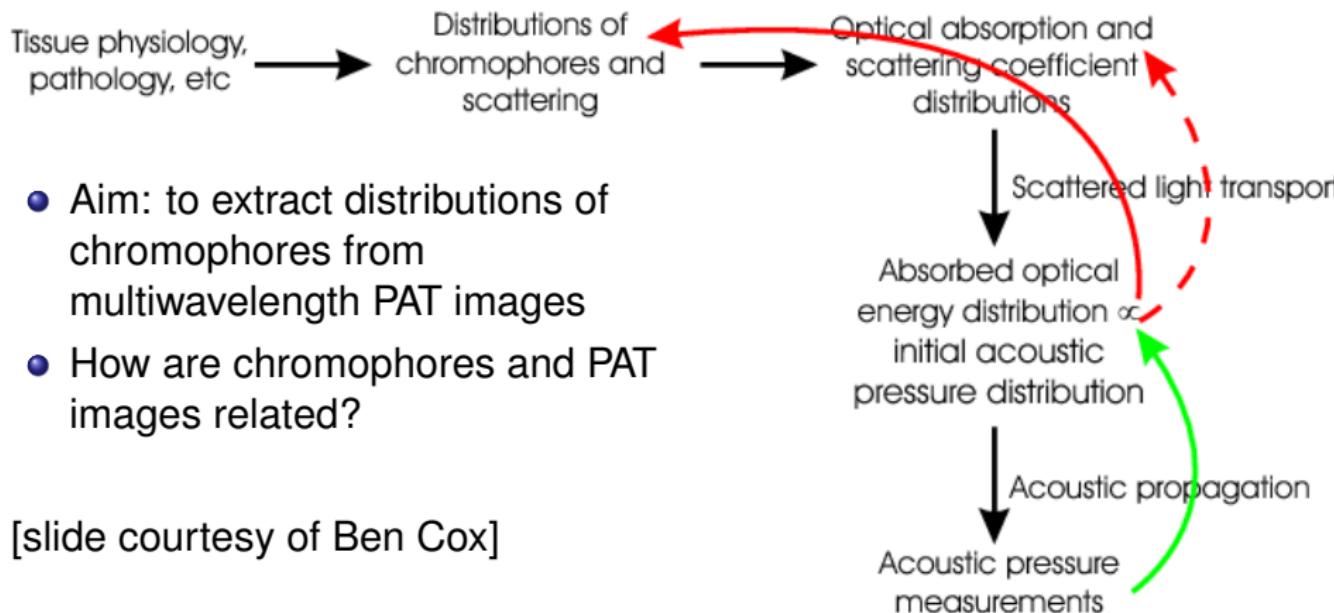
Quantitative PhotoAcoustic Tomography



[slide courtesy of Ben Cox]

PhotoAcoustic Tomography

Quantitative PhotoAcoustic Tomography



[slide courtesy of Ben Cox]

Quantitative PhotoAcoustic Tomography

Outline

- The Optical Inverse Problem
- Fluence Models
- Parameter Estimation

Modelling in Optical Tomography

Radiative Transfer Equation (RTE)

The radiative transfer equation is an integro-differential equation expressing the conservation of energy which takes the following time-independent form as required in PAT

$$(\hat{\mathbf{s}} \cdot \nabla + \mu_a(\mathbf{r}) + \mu_s(\mathbf{r})) \phi(\mathbf{r}, \hat{\mathbf{s}}) = \mu_s \int_{S^{n-1}} \Theta(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \phi(\mathbf{r}, \hat{\mathbf{s}}') d\hat{\mathbf{s}}' + q(\mathbf{r}, \hat{\mathbf{s}}) \quad (10)$$

where $\phi(\mathbf{r}, \hat{\mathbf{s}}, t)$ is the radiance, $\Theta(\hat{\mathbf{s}}, \hat{\mathbf{s}}')$ is the scattering phase function,

Wave effects, polarisation, radiative processes, inelastic scattering, and reactions (such as ionisation) are all neglected in this model.

By writing a variational form of equation(10) it can be discretised using the finite element method (Tarpainen2005). When the radiance ϕ , source q or phase function Θ depend strongly on the direction $\hat{\mathbf{s}}$ it is necessary to discretise finely in angle $\hat{\mathbf{s}}$, and the model can become computationally intensive.

Modelling in Optical Tomography

Physical Models of Light Propagation

The Radiative Transfer Equation (RTE) is a natural description of light considered as photons. It represents a balance equation where photons in a constant refractive index medium, in the absence of scattering, are propagated along rays $\mathbf{I} := \mathbf{r}_0 + l\hat{\mathbf{s}}$

$$\hat{\mathbf{s}} \cdot \nabla U + \mu_a U = 0 \quad \equiv \quad \mathcal{T}_{\mu_a} U = 0 \quad (11)$$

whose solution

$$U = U_0 \exp \left[- \int_I \mu_a(\mathbf{r}_0 + l\hat{\mathbf{s}}) dl \right] \quad (12)$$

is the basis for the definition of the *Ray Transform*

$$g_{\hat{\mathbf{s}}}(p) := -\ln \left[\frac{U}{U_0} \right] = \int_{-\infty}^{\infty} \mu_a(p\hat{\mathbf{s}}_{\perp} + l\hat{\mathbf{s}}) dl \quad \equiv \quad g_{\hat{\mathbf{s}}} = \mathcal{R}_{\hat{\mathbf{s}}} \mu_a \quad (13)$$

Modelling in Optical Tomography

The Radiative Transfer Equation

In the presence of scattering, and with source terms q , eq.(11) becomes

$$[\mathcal{T}_{\mu_{\text{tr}}} - \mu_s \mathcal{S}] U = q \quad (14)$$

where $\mu_{\text{tr}} = \mu_s + \mu_a$ is the attenuation coefficient and \mathcal{S} is the scattering operator, which is local (non propagating).

A series solution for eq.(14) can be formally written as

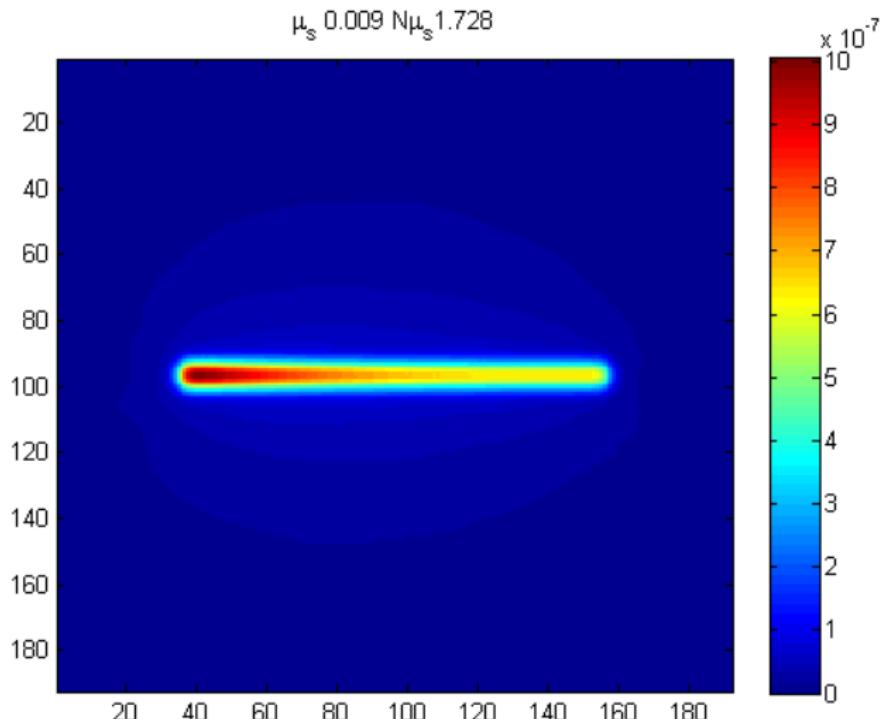
$$U = \left[\mathcal{T}_{\mu_{\text{tr}}}^{-1} + \mathcal{T}_{\mu_{\text{tr}}}^{-1} \mu_s \mathcal{S} \mathcal{T}_{\mu_{\text{tr}}}^{-1} + \dots \left(\mathcal{T}_{\mu_{\text{tr}}}^{-1} \mu_s \mathcal{S} \right)^k \mathcal{T}_{\mu_{\text{tr}}}^{-1} \dots \right] q \quad (15)$$

This is the **method of successive approximation** (Sobolev 1963). The first term may be found from the Ray Transform, giving an alternative equation for the *collided flux*

$$[\mathcal{T}_{\mu_{\text{tr}}} - \mu_s \mathcal{S}] U_{\text{collided}} = \mu_s \mathcal{S} \underbrace{\mathcal{T}_{\mu_{\text{tr}}}^{-1} q}_{\text{uncollided}} \quad (16)$$

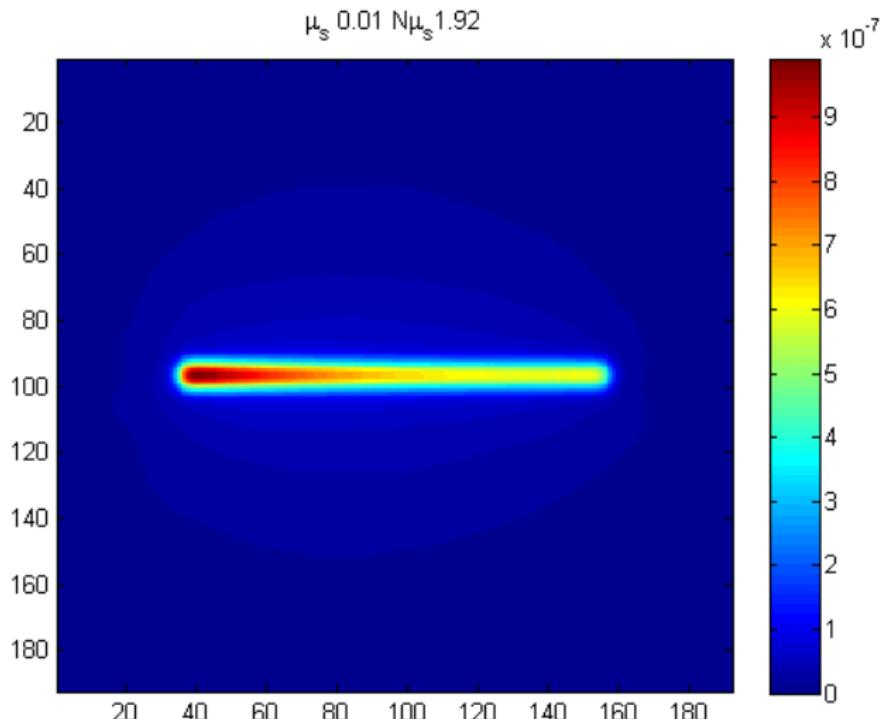
Modelling in Optical Tomography

RTE solutions



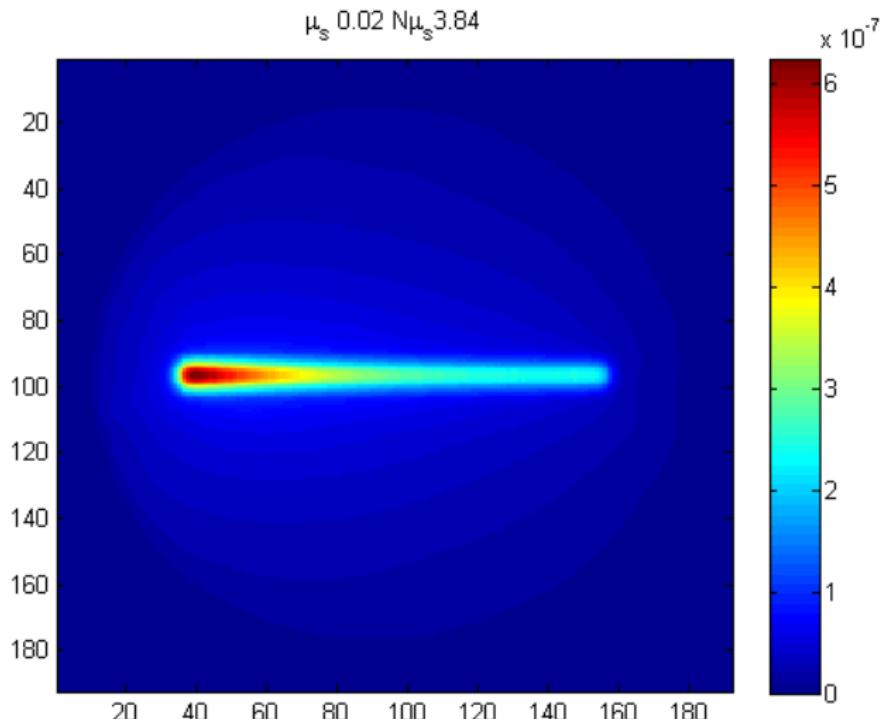
Modelling in Optical Tomography

RTE solutions



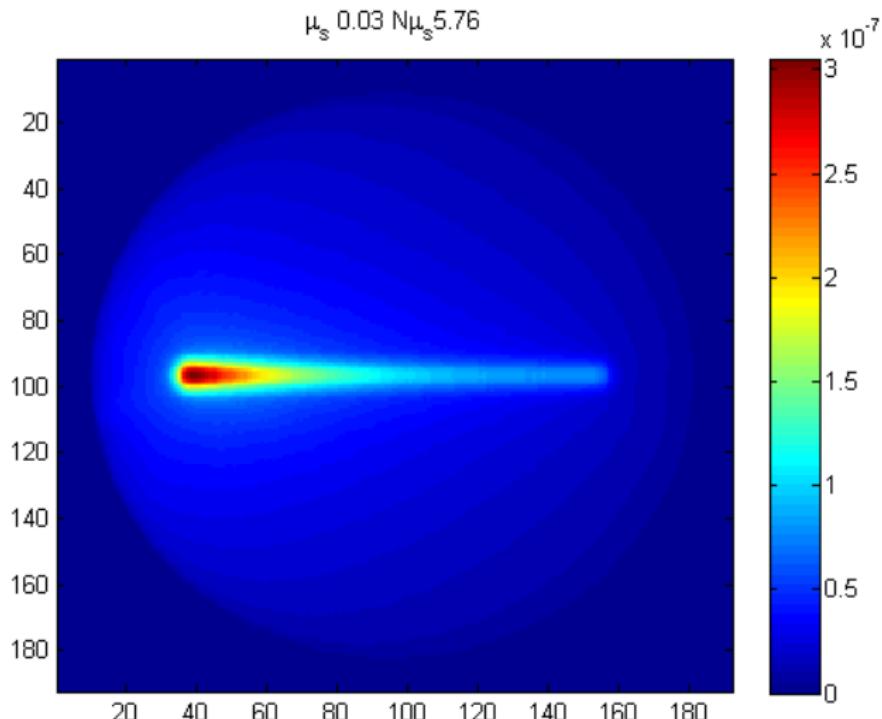
Modelling in Optical Tomography

RTE solutions



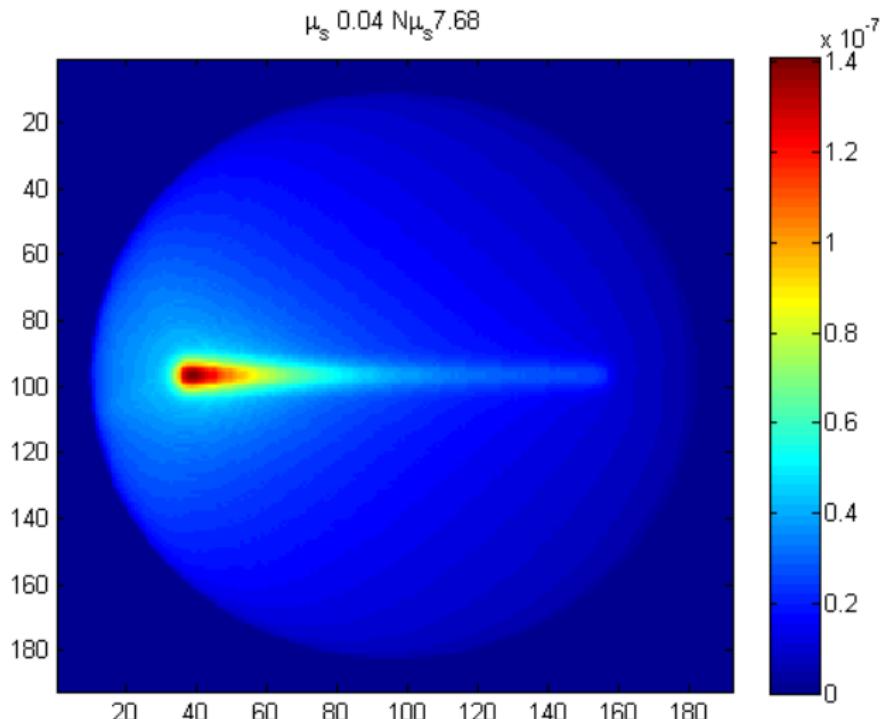
Modelling in Optical Tomography

RTE solutions



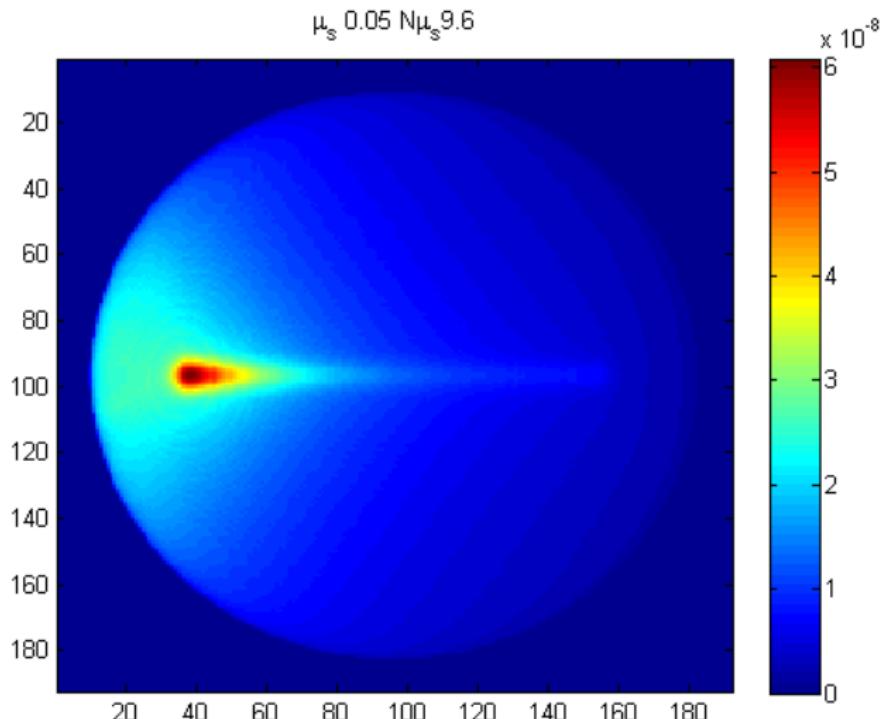
Modelling in Optical Tomography

RTE solutions



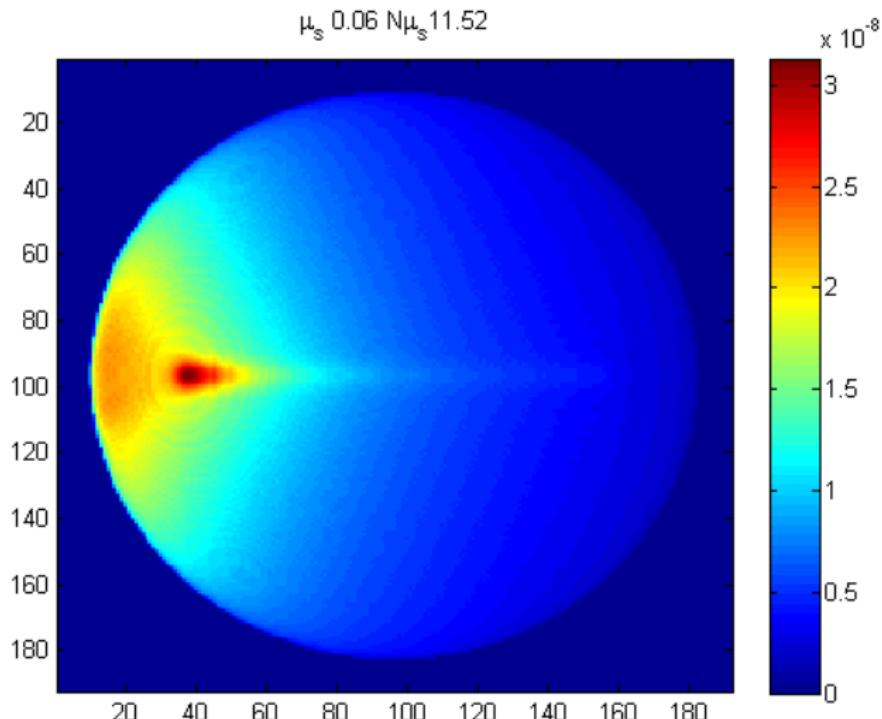
Modelling in Optical Tomography

RTE solutions



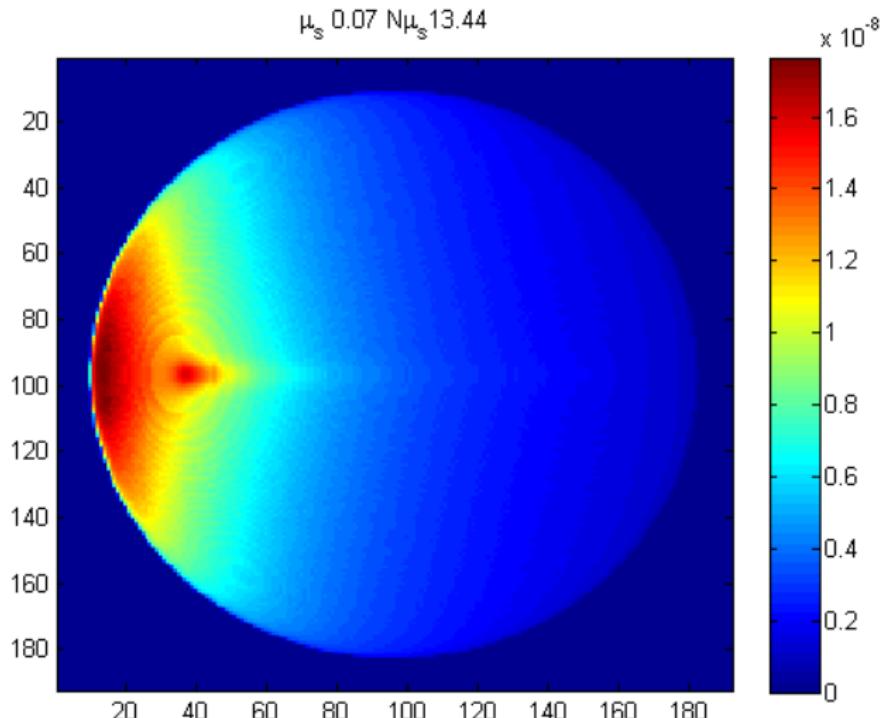
Modelling in Optical Tomography

RTE solutions



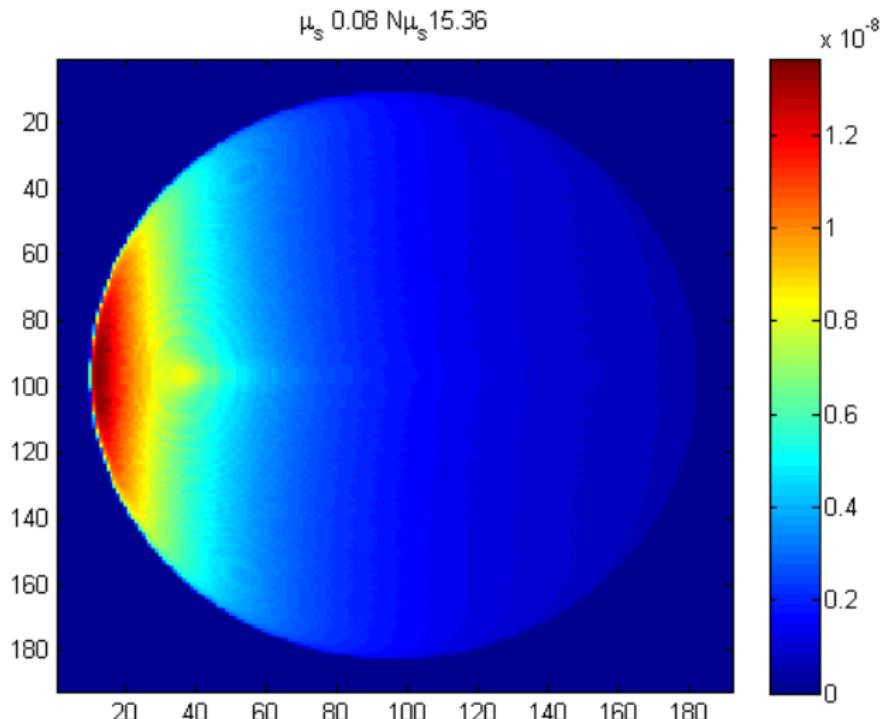
Modelling in Optical Tomography

RTE solutions



Modelling in Optical Tomography

RTE solutions



Modelling in Optical Tomography

Diffusion Approximation

In the Diffusion pproximation (DA), the radiance is approximated by first order spherical harmonics only ($\hat{\mathbf{s}} \equiv [Y_{1,-1}, Y_{1,0}, Y_{1,1}]$), giving

$$\phi(\mathbf{r}, \hat{\mathbf{s}}) \approx \frac{1}{4\pi} \Phi(\mathbf{r}) + \frac{3}{4\pi} \hat{\mathbf{s}} \cdot J(\mathbf{r}) \quad (17)$$

where $\Phi(\mathbf{r})$ and $J(\mathbf{r})$ are the photon density and current defined as

$$\Phi(\mathbf{r}) = \int_{S^{n-1}} \phi(\mathbf{r}, \hat{\mathbf{s}}) d\hat{\mathbf{s}} \quad (18)$$

$$J(\mathbf{r}) = \int_{S^{n-1}} \hat{\mathbf{s}} \phi(\mathbf{r}, \hat{\mathbf{s}}) d\hat{\mathbf{s}}. \quad (19)$$

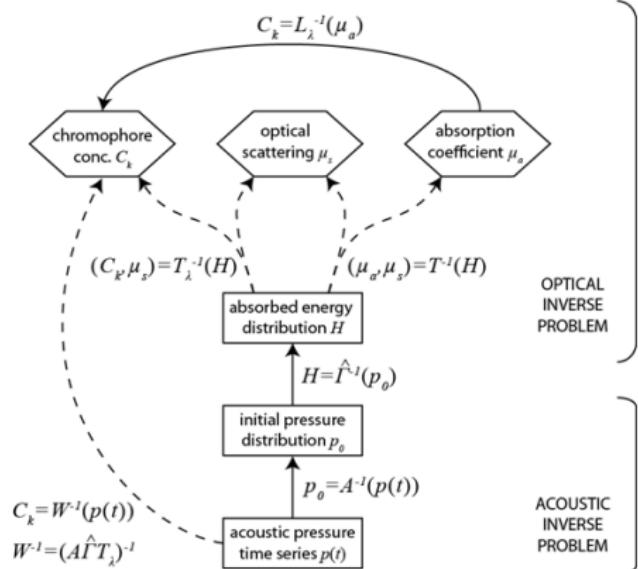
Inserting the approximation (17) into equation (10) results in a second order PDE in the photon density

$$-\nabla \cdot D \nabla \Phi(\mathbf{r}) + \mu_a \Phi(\mathbf{r}) = q_0(\mathbf{r}) \equiv \mathcal{D}\Phi = q_0, \quad (20)$$

with $D = \frac{1}{\mu_a + (1-g)\mu_s}$. Equation(20) and its associated frequency and time domain versions, including the Telegraph Equation, are the most commonly used in DOI.

Quantitative PhotoAcoustic Tomography

Overview



The inverse problems in QPAT. Solid lines indicate linear operators and dot-dash lines those that are inherently nonlinear. The acoustic pressure time series $p(t)$ are the measured data and the chromophore concentrations C_k are the unknowns. The concentrations may be obtained step by step : linear acoustic inversion A^{-1} , thermoelastic scaling $\hat{\Gamma}^{-1}$ nonlinear optical inversion for the optical coefficients T^{-1} and finally L_λ^{-1} a linear spectroscopic inversion of the absorption spectra to recover the chromophore concentrations.

Quantitative PhotoAcoustic Tomography

PAT images and chromophores

- PAT images \propto absorbed energy distribution $h(\mathbf{r}, \lambda)$
- $p_0(\mathbf{r}, \lambda)$ is related to absorption coefficient $\mu_a(\mathbf{r}, \lambda)$ via the fluence, $\Phi(\mathbf{r}, \lambda)$ and the *Grüneisen parameter*:

$$p_0(\mathbf{r}, \lambda) = \Gamma H(\mathbf{r}, \lambda) = \Gamma \mu_a(\mathbf{r}, \lambda) \Phi(\mathbf{r}, \lambda)$$

- μ_a is related to chromophores concentrations $C^{(k)}$ via specific absorption coefficients ϵ_k :

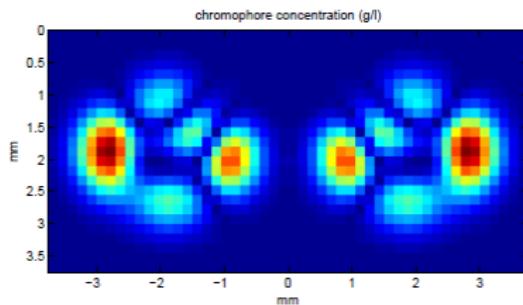
$$\mu_a(\mathbf{r}, \lambda) = \sum_{k=1}^K \epsilon_k(\lambda) C^{(k)}(\mathbf{r})$$

Inverse problem is non-linear but well-posed. Solve using diffusion or transport methods

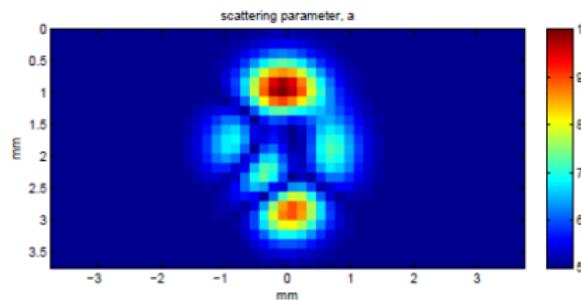
Quantitative PhotoAcoustic Tomography

MultiSpectral QPAT

chromophore concentration $C(x)$

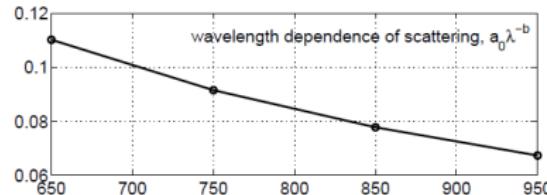
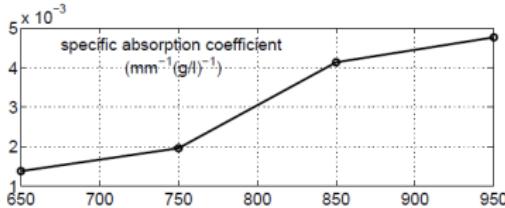


scattering parameter $a(x)$



- C range: 5-15 g/l HbO₂
- $\mu'_s = aa_0\lambda(nm)^{-b} \text{ mm}^{-1}$, $a_0 = 500$, $b = 1.3$, a range: 5-10

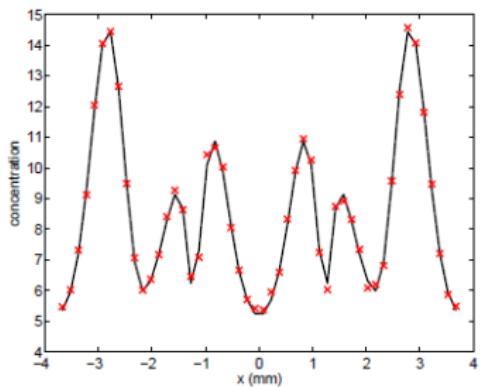
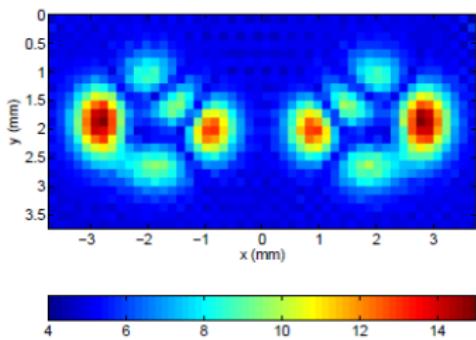
Wavelength Dependence



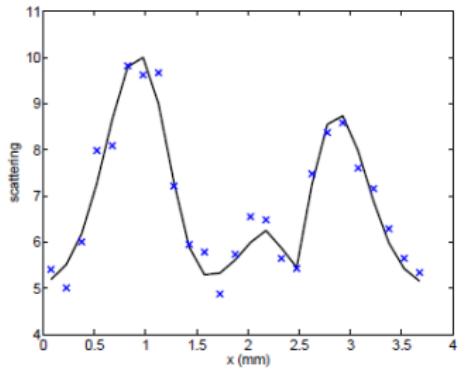
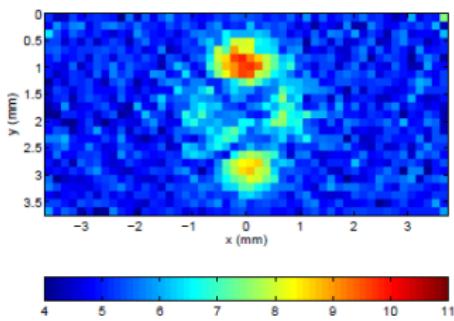
Quantitative PhotoAcoustic Tomography

MultiSpectral QPAT reconstructions

Reconstructed
Concentration



Reconstructed
Scattering 'a'



Quantitative PhotoAcoustic Tomography

Inverse Problem

Find the absorption and scattering coefficients μ_a, μ_s given the absorbed energy density image

$$h(\mathbf{r}) = \frac{p_0(\mathbf{r})}{\Gamma} = \mu_a(\mathbf{r})\Phi(\mu_a(\mathbf{r}), \mu_s(\mathbf{r}))$$

when the fluence Φ is unknown.

Strategy used here : fit a model of light transport to the reconstructed data

$$\{\hat{\mu}_a, \hat{\mu}_s\} = \arg \min_{\mu_a, \mu_s} \left[\mathcal{E} := \|h^{\text{obs}} - F(\mu_a, \mu_s)\|^2 + R(\mu_a, \mu_s) \right]$$

where $F(\mu_a, \mu'_s) = \mu_a \Phi((\mu_a, \mu'_s))$ is the *forward model* of optical energy absorption, and R is a regularisation term.

In this talk, the regularisation term is *Total Variation*.

Quantitative PhotoAcoustic Tomography

Linearisation

Discretise parameters into a suitable basis

$$\mu_a(\mathbf{r}) = \sum_j \mu_{aj} u_j(\mathbf{r}), \quad \mu_s(\mathbf{r}) = \sum_j \mu_{sj} u_j(\mathbf{r})$$

The functional gradient vectors are given by

$$\begin{aligned} g_a &= \frac{\partial \mathcal{E}}{\partial \mu_{aj}} = - \sum_{m,k} (h_k^m - \mu_{ak} \Phi_k^m) JA_{kj}^m + \frac{\partial R}{\partial \mu_{aj}} \\ g_s &= \frac{\partial \mathcal{E}}{\partial \mu_{sj}} = - \sum_{m,k} (h_k^m - \mu_{ak} \Phi_k^m) JS_{kj}^m + \frac{\partial R}{\partial \mu_{sj}} \end{aligned}$$

with the absorption and scattering Jacobians respectively

$$JA_{kj}^m = \Phi_k^m \delta_{kj} + \mu_{ak} \frac{\partial \Phi_k^m}{\partial \mu_{aj}}, \quad JS_{kj}^m = \mu_{ak} \frac{\partial \Phi_k^m}{\partial \mu_{sj}}. \quad (21)$$

Quantitative PhotoAcoustic Tomography

Gauss-Newton Approach

By combining the absorption and scattering Jacobians for every illumination into a single Jacobian matrix, $J \in \mathbb{R}^{MK \times 2K}$

$$J = \left[\begin{array}{c|c} JA^1 & JS^1 \\ \vdots & \\ JA^M & JS^M \end{array} \right],$$

the Hessian, $H \in \mathbb{R}^{2K \times 2K}$, may be approximated by $H \approx J^T J$. The update to the absorption and scattering coefficients, $[\delta\mu_{ak}, \delta\mu_{sk}]^T$, can then be calculated by a Newton step according to

$$\begin{bmatrix} \delta\mu_{ak} \\ \delta\mu_{sk} \end{bmatrix} = -H^{-1} \begin{bmatrix} g_a \\ g_s \end{bmatrix}.$$

Quantitative PhotoAcoustic Tomography

Construction of Jacobians

For the Radiative Transfer Equation the following equations were used to directly calculate the Jacobians JA and JS column by column

$$(\hat{\mathbf{s}} \cdot \nabla + \mu_{ak} + \mu_{sk}) \frac{\partial \phi_k^m(\hat{\mathbf{s}})}{\partial \mu_{aj}} - \mu_{sk} \int_{S^{n-1}} \Theta(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\partial \phi_k^m(\hat{\mathbf{s}}')}{\partial \mu_{aj}} d\hat{\mathbf{s}}' = -\delta_{kj} \phi_k^m(\hat{\mathbf{s}})$$
$$(\hat{\mathbf{s}} \cdot \nabla + \mu_{ak} + \mu_{sk}) \frac{\partial \phi_k^m(\hat{\mathbf{s}})}{\partial \mu_{sj}} - \mu_{sk} \int_{S^{n-1}} \Theta(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \frac{\partial \phi_k^m(\hat{\mathbf{s}}')}{\partial \mu_{sj}} d\hat{\mathbf{s}}' =$$
$$\delta_{kj} \int_{S^{n-1}} \Theta(\hat{\mathbf{s}}, \hat{\mathbf{s}}') \phi_k^m(\hat{\mathbf{s}}') d\hat{\mathbf{s}}' - \delta_{kj} \phi_k^m(\hat{\mathbf{s}})$$

Quantitative PhotoAcoustic Tomography

Construction of Jacobians

For the diffusion approximation

$$(\mu_{ak} - \nabla \cdot D_k \nabla) \frac{\partial \Phi_k^m}{\partial \mu_{aj}} = -\delta_{kj} \Phi_k^m$$

$$(\mu_{ak} - \nabla \cdot D_k \nabla) \frac{\partial \Phi_k^m}{\partial D_j} = \nabla \cdot (\delta_{kj} \nabla \Phi_k^m)$$

$\partial \Phi_k^m / \partial \mu_{sj}$ is then obtained from $\partial \Phi_k^m / \partial D_j$ using the relation
 $\partial D / \partial \mu_s = -3D^2(1 - g)$.

Quantitative PhotoAcoustic Tomography

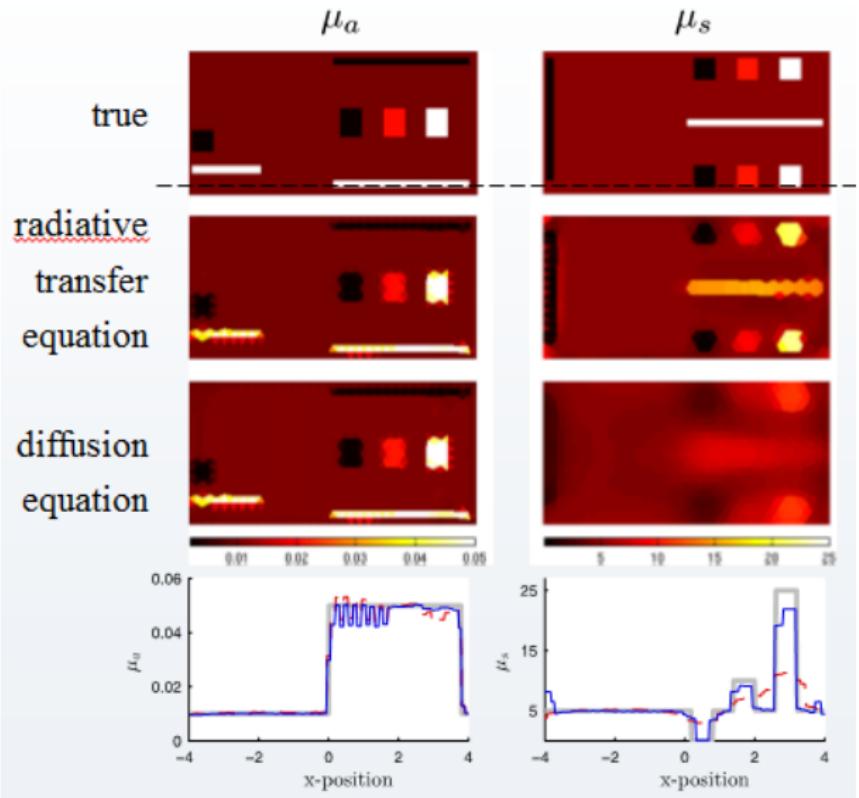
RTE-based Inversions

- Fixed-point iteration, known scattering (Yao, Sun, Jiang 2009)
- Use separated unscattered, singly-scattered and multiply scattered components (Bal, Jollivet, Jugnon 2010): can't do in practice.
- Gauss-Newton inversions with TV regularization (C., Tarvainen, Arridge, 2011; Tarvainen, Cox, Kaipio, A. 2012)
- Gradient-based inversions with Tikhonov reg. (Saratoon, Tarvainen, Cox, A., *submitted*)

Quantitative PhotoAcoustic Tomography

RTE-based Inversions (Gauss-Newton)

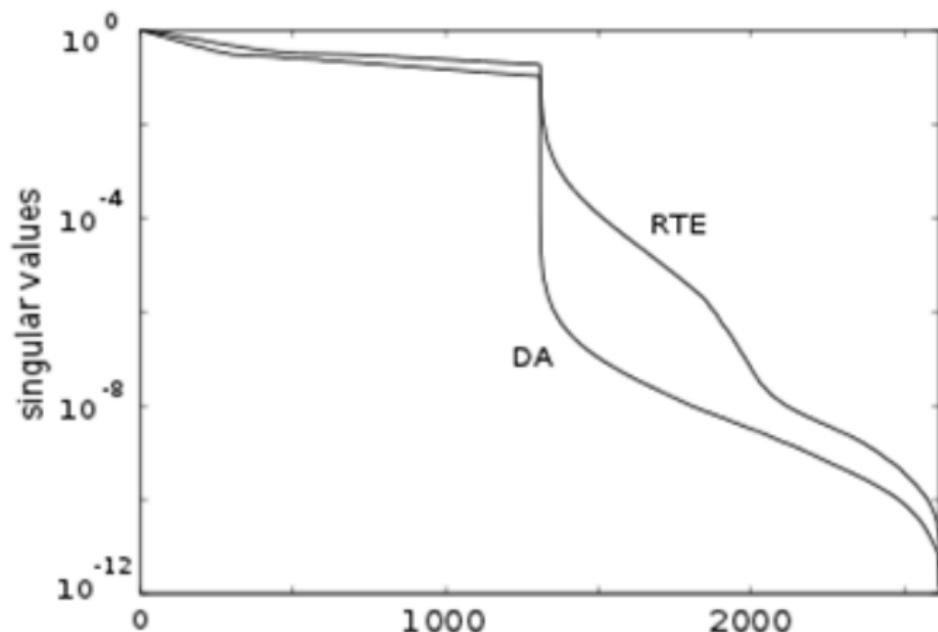
Using 4 images from
4 illumination
directions
(Tarvainen, Cox.,
Kaipio, A. 2012)



Quantitative PhotoAcoustic Tomography

SVD comparison

SVD of Hessian reveals different information content

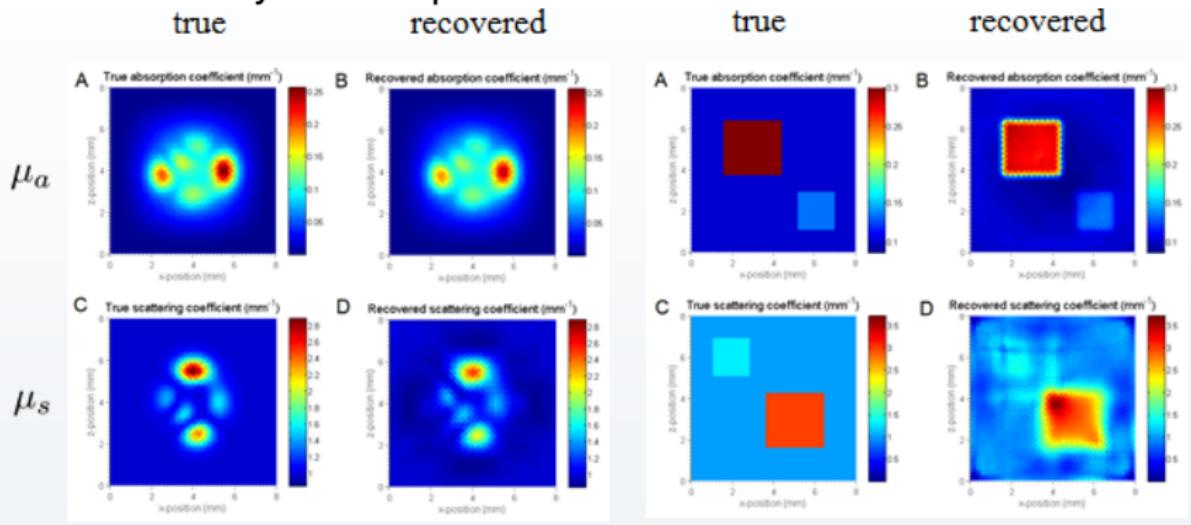


Quantitative PhotoAcoustic Tomography

Matrix Free method

Explicit construction of Jacobians is too expensive \Rightarrow use matrix free method based on adjoint fields

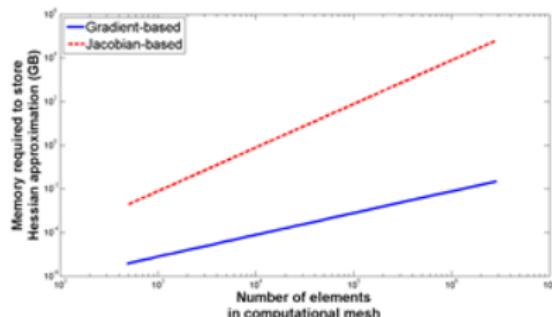
Limited memory BFGS optimisation



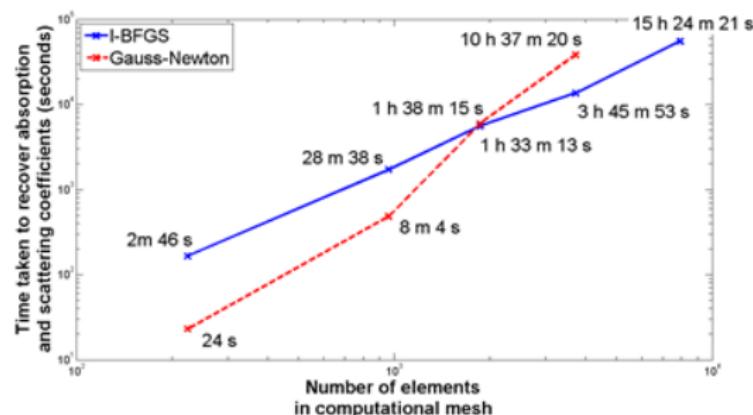
Using 4 images from 4 illumination directions, Tikhonov regularisation
(Saratoon, Tarvainen, Cox, A., submitted)

Quantitative PhotoAcoustic Tomography

Gauss-Newton vs. Gradient-Based



- Jacobian-based methods appear not to be feasible for 3D problems
- Gradient-based methods might be.



(Saratoon, Tarvainen, C., Arridge, submitted)

Outline

- 1 Introduction
- 2 PhotoAcoustics
- 3 PhotoAcoustic Forward Model
- 4 Quantitative PhotoAcoustic Tomography
- 5 Summary
- 6 Acknowledgements

Summary

- Photoacoustic imaging: great potential as biomedical imaging method
- Importance of spectroscopic aspect of photoacoustics sometimes overlooked (as it is not present in thermoacoustics?)
- Much progress made in quantitative photoacoustics recently
- Linearized approaches probably not sufficient in practice
- Complete 3D problem is of large scale
- Accuracy of light models in ballistic regime (close to surface)
- Questions about Grüneisen parameter remain. (Under DA not possible to recover all of Γ , μ_a , D with one wavelength, but with multiple wavelengths it could be (Bal, Ren 2011, Ren, Bal 2011).)

Quantitative PhotoAcoustic Tomography

Discussion

- Multi-Wavelength vs Single Wavelength Inversions
- Nonlinearity
- PA Generation Efficiency

Outline

- 1 Introduction
- 2 PhotoAcoustics
- 3 PhotoAcoustic Forward Model
- 4 Quantitative PhotoAcoustic Tomography
- 5 Summary
- 6 Acknowledgements

Acknowledgements

- **Collaborators :**
 - *UCL:* P.Beard, M.Betcke, T.Betcke, Y.Kurylev, B.Cox, J.Laufer, T.Saratoon, B.Treeby,
 - *Kuopio:* J. Kaipio, V. Kolehmainan, T. Tarvainen, M. Vaukhonen,
- **Funding**
 - This work was supported by EPSRC grant EP/E034950/1 and by the Academy of Finland (projects 122499, 119270 and 213476)
 - *Other funding :* MRC, Wellcome Trust, CEC Framework, Royal Society