1ª LISTA DE EXERCÍCIOS - DISCIPLINA MTM 5163: CÁLCULO C

1. Esboce o TRAÇO da curva $\gamma : \mathbb{R} \to \mathbb{R}^2$, onde γ é definida por:

   (a) $\gamma(t) = (1, t)$;
   (b) $\gamma(t) = (2t - 1, t + 2)$;
   (c) $\gamma(t) = (t, t^3)$;
   (d) $\gamma(t) = (t^2, t)$;
   (e) $\gamma(t) = (t^2, t^4)$;
   (f) $\gamma(t) = (\cos t, 2\sin t)$;
   (g) $\gamma(t) = (\sin t, \sin t)$;
   (h) $\gamma(t) = (\sin t, \sin^2(t))$;
   (i) $\gamma(t) = (e^t \cos t, e^t \sin t)$, para $t > 0$;
   (j) $\gamma(t) = (t - \sin t, 1 - \cos t)$, para $t > 0$.

2. Esboce o TRAÇO da curva $\gamma : \mathbb{R} \to \mathbb{R}^3$, onde γ é definida por:

   (a) $\gamma(t) = (1, t, 1)$;
   (b) $\gamma(t) = (1, 1, t)$;
   (c) $\gamma(t) = (t, t, 1)$;
   (d) $\gamma(t) = (\cos t, \sin t, 2)$;
   (e) $\gamma(t) = (t, t, 1 + \sin t)$;
   (f) $\gamma(t) = (1, 1, \frac{1}{t})$, para $t > 0$;
   (g) $\gamma(t) = (\cos t, \sin t, e^{-t})$, para $t > 0$;
   (h) $\gamma(t) = (1 + \sin t, 1 + \sin t, \cos t)$, para $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$.

3. Sejam $\gamma(t) = (t, \sin t, 2)$ e $\lambda(t) = (3, t, t^2)$. Calcule:

   (a) $\gamma(t) \cdot \lambda(t)$;
   (b) $e^{-t}\gamma(t)$;
   (c) $\gamma(t) - 2\lambda(t)$;
   (d) $\gamma(t) \wedge \lambda(t)$;

4. Sejam $\gamma(t) = (\sin t, \cos t, t)$ e $\lambda(t) = (\sin t, \cos t, 1)$. Calcule:

   (a) $\gamma(t) \cdot \lambda(t)$;
   (b) $e^{-t}\gamma(t)$;
   (c) $\gamma(t) - 2\lambda(t)$;
   (d) $\gamma(t) \wedge \lambda(t)$;

5. Calcule $\lim_{t \to t_0} \gamma(t)$, onde:

   (a) $\gamma(t) = \left(\frac{\sqrt{t}-1}{t-1}, t^2, \frac{t-1}{t} \right)$ e $t_0 = 1$;
   (b) $\gamma(t) = \left(\frac{\tan 3t}{t}, \frac{e^{2t}-1}{t}, t^2 \right)$ e $t_0 = 0$;
   (c) $\gamma(t) = \left(\frac{t^3-8}{t^2-4}, \frac{\cos\left(\frac{\pi}{t}\right)}{t-2}, 2t \right)$ e $t_0 = 1$.

6. Calcule γ' e γ'' nos seguintes casos:

   (a) $\gamma(t) = (3t^2, e^{-t}, \ln(t^2 + 1))$;
   (b) $\gamma(t) = (t^{\frac{2}{3}}, \cos(t^2), 3t)$;
   (c) $\gamma(t) = (\sin 5t, \cos 4t, -e^{-2t})$.

7. Calcule $\int_a^b \gamma(t)dt$, onde:

(a) $\gamma(t) = (t, e^t, 0)$, $a = 0$ e $b = 1$;

(b) $\gamma(t) = (\sin 3t, \frac{1}{1+t^2}, 1)$, $a = -1$ e $b = 1$;

(c) $\gamma(t) = (3, 2, 1)$, $a = 1$ e $b = 2$.

8. Sejam $\gamma(t) = (t, 1, e^t)$ e $\lambda(t) = (1, 1, 1)$. Calcule

(a) $\int_0^1 [\gamma(t) \cdot \lambda(t)]dt$;

(b) $\int_0^1 [\gamma(t) \wedge \lambda(t)]dt$;

9. Calcule o comprimento da curva dada e verifique quais delas são regulares.

(a) $\gamma(t) = (t\cos t, t\sin t)$, $t \in [0, 2\pi]$;

(b) $\gamma(t) = (2t - 1, t + 1)$, $t \in [1, 2]$;

(c) $\gamma(t) = (\cos t, \sin t, e^{-t})$, $t \in [0, \pi]$;

(d) $\gamma(t) = (e^{-t}\cos t, e^{-t}\sin t, e^{-t})$, $t \in [0, 1]$;

(e) $\gamma(t) = (t, \ln t)$, $t \in [1, e]$.

10. Seja γ uma curva definida por $\gamma(t) = (\ln t, \sqrt{1 - t^2}, t^2)$. Determine o domínio de γ e calcule $\gamma(\frac{3}{5})$.

11. Sejam $\gamma, \lambda, \beta : I \to \mathbb{R}^3$ três curvas definidas num intervalo $I \subseteq \mathbb{R}$. Verifique que:
(a) $\gamma(t) \wedge \lambda(t) = -\lambda(t) \wedge \gamma(t)$ e
(b) $\gamma(t) \cdot (\lambda(t) + \beta(t)) = \gamma(t) \cdot \lambda(t) + \gamma(t) \cdot \beta(t)$.

12. Seja $\gamma : I \to \mathbb{R}^2$ uma curva de classe C^2 e suponha que existe $\mu \in \mathbb{R}$ tal que $\gamma'(t) = \mu\gamma(t)$. Mostre que $\gamma(t) \wedge \gamma'(t)$ é constante em I.

13. Seja $\gamma : \mathbb{R} \to \mathbb{R}^3$ um curva de classe C^2. Mostre que se $\gamma(t) \wedge \gamma'(t)$ é constante em $\mathbb{R}$ então $\gamma(t) \wedge \gamma''(t) = 0$.

14. Dê exemplos de curvas γ e λ que possuam o mesmo traço, mas que tenham comprimentos de arcos distintos.

15. Verifique que as curvas abaixo estão parametrizadas pelo comprimento de arco.
(a) $\gamma(s) = (\cos s, \sin s)$, $s \geqslant 0$;
(b) $\gamma(s) = (r\cos(\frac{s}{r}), r\sin(\frac{s}{r}))$, $s \geqslant 0$ e $r > 0$ é um número real fixado;
(c) $\gamma(s) = (\frac{s}{\sqrt{5}}, \frac{2s}{\sqrt{5}})$, $s \geqslant 0$;