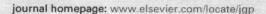


Contents lists available at ScienceDirect

## Journal of Geometry and Physics





# Instability of reducible critical points of the Seiberg–Witten functional



Celso Melchiades Doria\*

Universidade Federal de Santa Catarina, Campus Universitário, Trindade, Florianópolis-SC, CEP: 88.040-900, Brazil

#### ARTICLE INFO

Article history: Received 24 June 2013 Accepted 5 October 2015 Available online 22 October 2015

MSC:

58J05 58E50

58Z99

58Z05

Keywords: Seiberg-Witten Parallel spinor

#### ABSTRACT

The Euler-Lagrange equations for the variational approach to the Seiberg-Witten equations admit reducible solutions. In this context, the instability of the reducible solutions is achieved by assuming the existence of a parallel spinor or the negativeness of a Perelman-Yamabe type of invariant defined for a *spin*<sup>c</sup>-structure.

© 2015 Elsevier B.V. All rights reserved.

### 1. Introduction

Let (M,g) be a closed riemannian four manifold with scalar curvature  $k_g$ . By considering the least eigenvalue  $\lambda_g$  of the operator  $\Delta_g + \frac{k_g}{4}$ , where  $\Delta_g = d^*d$  is the Laplace–Beltrami operator associated to g, Perelman introduced in [1,2] the smooth invariant

$$\bar{\lambda}(M) = \sup_{g \in \mathcal{M}} \lambda_g [vol(M, g)]^{1/2} \tag{1}$$

where  $\mathfrak{M}$  is the space of  $C^{\infty}$ -metrics on M. Let [g] be the conformal class of  $g \in \mathfrak{M}$ , Kobayashi [3] and Schoen [4] independently introduced the smooth manifold invariant

$$\mathcal{Y}(M) = \sup_{[g]} \inf_{g} \frac{\int_{M} k_{g} dv_{g}}{vol(M, g)^{1/2}}.$$
 (2)

Assuming y(M) < 0, Akutagawa–Ishida–Le Brun proved [5] the equality  $\bar{\lambda}(M) = y(M)$ . A similar quantity turns up by measuring the instability of reducible critical points of the Seiberg–Witten functional, though in this case it depends on a  $spin^c$ -structure on M. There exist smooth 4-manifolds admitting a  $spin^c$  structure c such that the Seiberg–Witten invariant  $SW(c) \neq 0$ . These  $spin^c$  structures are named basic classes and they are in the realm of the 4-dim differential topology. The space  $Spin^c(M)$  of  $Spin^c$  structures on M might be identified with

$$\{c = \alpha_c + \beta_c \in H^2(X, \mathbb{Z}) \oplus H^1(X, \mathbb{Z}_2) \mid w_2(X) = \alpha \mod 2\}.$$
 (3)

http://dx.doi.org/10.1016/j.geomphys.2015.10.002 0393-0440/© 2015 Elsevier B.V. All rights reserved.

<sup>\*</sup> Tel.: +55 48 96128385. E-mail address: c.m.doria@ufsc.br.