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Instability of reducible critical points of the Seiberg–Witten functional

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ABSTRACT

The Euler–Lagrange equations for the variational approach to the Seiberg–Witten equations admit reducible solutions. In this context, the instability of the reducible solutions is achieved by assuming the existence of a parallel spinor or the negativeness of a Perelman–Yamabe type of invariant defined for a $spin^c$-structure.

1. Introduction

Let (M, g) be a closed riemannian four manifold with scalar curvature k_g. By considering the least eigenvalue λ_g of the operator $\triangle_g + \frac{k_g}{4}$, where $\triangle_g = d^*d$ is the Laplace–Beltrami operator associated to g, Perelman introduced in [1,2] the smooth invariant

$$\bar{\lambda}(M) = sup_{g\in\mathcal{M}}\lambda_g[vol(M, g)]^{1/2} \tag{1}$$

where $\mathcal{M}$ is the space of C^∞-metrics on M. Let $[g]$ be the conformal class of $g \in \mathcal{M}$, Kobayashi [3] and Schoen [4] independently introduced the smooth manifold invariant

$$\mathcal{Y}(M) = \sup_{[g]} \inf_{g} \frac{\int_M k_g dv_g}{vol(M, g)^{1/2}}. \tag{2}$$

Assuming $\mathcal{Y}(M) < 0$, Akutagawa–Ishida–Le Brun proved [5] the equality $\bar{\lambda}(M) = \mathcal{Y}(M)$. A similar quantity turns up by measuring the instability of reducible critical points of the Seiberg–Witten functional, though in this case it depends on a $spin^c$-structure on M. There exist smooth 4-manifolds admitting a $spin^c$ structure $\mathfrak{c}$ such that the Seiberg–Witten invariant $SW(\mathfrak{c}) \neq 0$. These $spin^c$ structures are named basic classes and they are in the realm of the 4-dim differential topology. The space $Spin^c(M)$ of $spin^c$ structures on M might be identified with

$$\{\mathfrak{c} = \alpha_{\mathfrak{c}} + \beta_{\mathfrak{c}} \in H^2(X, \mathbb{Z}) \oplus H^1(X, \mathbb{Z}_2) \mid w_2(X) = \alpha \ mod \ 2\}. \tag{3}$$

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