Robotica (2013) volume 31, pp. 61–70. © Cambridge University Press 2012
doi:10.1017/S0263574712000057

Symmetry and invariants of kinematic chains and parallel manipulators

Roberto Simoni†*, Celso Melchiades Doria‡ and Daniel Martins§

†*Centro de Engenharia da Mobilidade – Campus Joinville*
‡*Departamento de Matemática – Campus Trindade*
§*Departamento de Engenharia Mecânica – Campus Trindade, Universidade Federal de Santa Catarina – 88040-900 – Florianópolis, Santa Catarina, Brasil*

(Accepted February 6, 2012. First published online: March 21, 2012)

SUMMARY

This paper presents applications of group theory tools to simplify the analysis of kinematic chains associated with mechanisms and parallel manipulators. For the purpose of this analysis, a kinematic chain is described by its properties, i.e. degrees-of-control, connectivity and redundancy matrices. In number synthesis, kinematic chains are represented by graphs, and thus the symmetry of a kinematic chain is the same as the symmetry of its graph. We present a formal definition of symmetry in kinematic chains based on the automorphism group of its associated graph. The symmetry group of the graph is associated with the graph symmetry. By using the group structure induced by the symmetry of the kinematic chain, we prove that degrees-of-control, connectivity and redundancy are invariants by the action of the automorphism group of the graph. Consequently, it is shown that it is possible to reduce the size of these matrices and thus reduce the complexity of the kinematic analysis of mechanisms and parallel manipulators in early stages of mechanisms design.

KEYWORDS: Kinematic chain; Parallel manipulators; Graph symmetry; Automorphism group; Actions; Orbits.

1. Introduction

Mathematical models are commonly difficult to handle in a general setting. Symmetry in mathematical models is useful to simplify the understanding of a model and to determine the patterns for which the model is appropriate. Thus, it is a common strategy to study cases of symmetry in order to learn more about a model. In nature, there are different types of mathematical models and also different types of symmetries, but thanks to the symmetry concept, many models are now reasonably well understood. In our setting, the mathematical model associates a kinematic chain with a graph. Graphs are extensively used in the literature of mechanisms and machine to describe kinematic chains.[3,35,50] Belfiore and Di Benedetto,[3] Liberati and Belfiore[27] and Martins and Carboni[30] discuss how the topological structure of a kinematic chain of a parallel manipulator can be described quite extensively by degrees-of-control, connectivity and redundancy matrices. These matrices are square symmetric with dimension $n \times n$, where n is the number of links of the kinematic chain. One aim of this paper is to develop a method to reduce the size of the degrees-of-control, connectivity and redundancy matrices of a kinematic chain associated with kinematic chains of mechanisms and parallel manipulators. In this context, the graph symmetry plays an important role because it provides a group structure that fits our purposes.

The graph of a kinematic chain is a graph on which the vertices represent the links and the edges represent the joints of the kinematic chain.[35] Hence, in early stages of mechanisms design, such as number synthesis,[35,50] the analysis of a kinematic chain is reduced to the analysis of its graph, and thus in this paper the term graph will be synonymous with kinematic chain.

In order to achieve our aims, we investigate symmetries and invariants by the action of the automorphism group of the graph representing kinematic chains of mechanisms and parallel manipulators. Symmetries of graphs are related to automorphisms.[10,37] By exploring these symmetries it is possible to reduce the matricial representation of important properties to the kinematic analysis of kinematic chains.

The main result of this study was to prove that the degrees-of-freedom (DoF), connectivity and redundancy matrices are all invariants by the action of the automorphism group of the graph. This invariance is the main tool used to reduce the size of the matrices. It is shown that the matrix size is reduced from $n \times n$ to $o \times n$, where n is the number of links and o is the number of orbits by the action of the automorphism group of the graph. Higher graph symmetry means a smaller number of orbits o, as will be clearly shown through examples.

The group theory has been used by some authors in the context of analyzing kinematic chains. Tsai[50] uses the symmetry group of a kinematic chain to identify when two kinematic chains are identical (isomorphism problem). Tuttle[51] uses group theory to identify all distinct bases of a kinematic chain enumeration process. Simoni *et al.*[44,45] have applied the group theory tools in the enumeration of kinematic chains, mechanisms and parallel manipulators.

The group theory has several applications in mechanisms and robotics such as to characterize all equivalent ways a modular robots can be constructed or assembled from its components,[8,9] assembly planning[18,29] and for positioning robots or robotic end effectors.[36,52]

* Corresponding author. E-mail: roberto.emc@gmail.com