BOUNDARY VALUE PROBLEMS FOR THE 2ND-ORDER SEIBERG-WITTEN EQUATIONS

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It is shown that the nonhomogeneous Dirichlet and Neuman problems for the 2nd-order Seiberg-Witten equation on a compact 4-manifold X admit a regular solution once the nonhomogeneous Palais-Smale condition $\mathcal{H}$ is satisfied. The approach consists in applying the elliptic techniques to the variational setting of the Seiberg-Witten equation. The gauge invariance of the functional allows to restrict the problem to the Coulomb subspace $\mathcal{C}_\alpha^{\mathfrak{C}}$ of configuration space. The coercivity of the $\mathcal{SW}_\alpha$-functional, when restricted into the Coulomb subspace, imply the existence of a weak solution. The regularity then follows from the boundedness of L^∞-norms of spinor solutions and the gauge fixing lemma.

1. Introduction

Let X be a compact smooth 4-manifold with nonempty boundary. In our context, the Seiberg-Witten equations are the 2nd-order Euler-Lagrange equation of the functional defined in Definition 2.3. When the boundary is empty, their variational aspects were first studied in [3] and the topological ones in [1]. Thus, the main aim here is to obtain the existence of a solution to the nonhomogeneous equations whenever $\partial X \neq \emptyset$. The nonemptiness of the boundary inflicts boundary conditions on the problem. Classically, this sort of problem is classified according to its boundary conditions in *Dirichlet problem* ($\mathfrak{D}$) or *Neumann problem* ($\mathcal{N}$).

Originally, the Seiberg-Witten equations were described in [8] as a pair of 1st-order PDE. The solutions of these equations were known as $\mathcal{SW}_\alpha$-monopoles, and their main achievement were to shed light on the understanding of the 4-dimensional differential topology, since new smooth invariants were defined by the topology of their moduli space of solutions (moduli gauge group). In the same article, Witten introduced a variational formulation for the equations and showed that its stable critical points turn out to be exactly the $\mathcal{SW}_\alpha$-monopoles. The variational aspects of the $\mathcal{SW}_\alpha$-equations were first explored in [3], where they proved that the functional satisfies the Palais-Smale condition and the solutions of the Euler-Lagrange (2nd-order) equations share the same important analytical properties as the $\mathcal{SW}_\alpha$-monopoles. Therefore, it is natural to ask if the equations fit into a Morse-Bott-Smale theory, where the lower number of critical points

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