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Variational Principle for the Seiberg–Witten Equations

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Abstract. Originally, the Seiberg–Witten equations were described to be dual to the Yang–Mills equation. The aim of this article is to present a Variational Principle for the SW-equation and some of their analytical properties, including the Palais-Smale Condition.

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1. Introduction

In november of 1994, Edward Witten gave a lecture at MIT about $N = 2$ Supersymmetric Quantum Field Theory and the ideas concerning the S-duality developed in a joint work with Seiberg in [15]. In order to please the mathematicians in the audience, he applied the new ideas to the Yang–Mills Theory to show them a new pair of 1^{st}-order PDE, named SW-monopole equation, and conjectured that the SW-theory is dual to the Yang–Mills theory; the duality being at the quantum level. A necessary condition for the duality is the equality of the expectation values of both theories. In topology, this means that for a fixed 4-manifold X there is a formula, conjectured by Witten in [20], where its Seiberg–Witten invariants are equal to the Donaldson invariants up to the factor $2^{2+\frac{1}{4}(7\chi(X)+11\tau(X))}$. After 10 years, it is believed that the conjecture is true, some tour the force has been done in [11] to prove it, but in its generality it is still an open question. This new pair of equations has a simpler analytical nature than the Yang–Mills equations. Even though the open question, and the fact that the physical meaning of the Seiberg–Witten equations ($\mathcal{SW}_\alpha$-eq.) is yet to be discovered, the mathematical usefulness of the equations is rather deep and efficient to understand one of the most basic phenomenon of differential topology in four dimension, namely, the existence of non-equivalent differential smooth structures on the same underlying topological manifold. It has not been efficient enough to solve either the smooth