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The Homotopy Type of Seiberg-Witten Configuration Space

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ABSTRACT: Let X be a closed smooth 4-manifold. In the Theory of the Seiberg-Witten Equations, the configuration space is $\mathcal{A}_\alpha \times_{\mathcal{G}_\alpha} \Gamma(S^+_\alpha)$, where $\mathcal{A}_\alpha$ is defined as the space of u_1-connections on a complex line bundle over X, $\Gamma(S^+_\alpha)$ is the space of sections of the positive complex spinor bundle over X and $\mathcal{G}_\alpha$ is the gauge group. It is shown that $\mathcal{A}_\alpha \times_{\mathcal{G}_\alpha} \Gamma(S^+_\alpha)$ has the same homotopic type of the Jacobian Torus

$$T^{b_1(X)} = \frac{H^1(X,\mathbb{R})}{H^1(X,\mathbb{Z})},$$

where $b_1(X) = dim_{\mathbb{R}} H^1(X,\mathbb{R})$.

Key words: connections, Gauge fields, 4-manifolds

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1. Introduction

Although the physical meaning of the Seiberg-Witten equations ($\mathcal{SW}_\alpha$-eq.) is yet to be discovered, the mathematical meaning is rather deep and efficient to understand one of the most basic phenomenon of differential topology in four dimension, namely, the existence of non-equivalent differential smooth structures on the same underlying topological manifold. The Seiberg-Witten equations arose through the ideas of duality described in Witten [12]. It is conjectured that the Seiberg-Witten equations are dual to Yang-Mills equations; the duality being at the quantum level. A necessary condition is the equality of the expectation values for the dual theories. In topology, this means that fixed a 4-manifold its Seiberg-Witten invariants are equal to Donaldson invariants. A basic reference for $\mathcal{SW}_\alpha$-eq. is [2].

Let (X,g) represent a fixed riemannian structure on X. Originally, the $\mathcal{SW}_\alpha$-equations were 1^{st}-order differential equations and their solutions (A,ϕ) satisfying

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