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A Fibre-Bundle Treatment to a Class of Extended Gauge Models

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Abstract

We adopt the analytic approach to connections on a general vector bundle to discuss the independence of vector gauge potentials introduced simultaneously in association with a single simple group.

The natural geometrical structures underlying the mathematical formulation of Yang-Mills theories are the so-called fibre bundles or, more specifically, principal firbre bundles, whose base manifold is the D-dimensional spacetime on which the theories are built up and whose fibres are given by the gauge group itself. The fibre bundles describe how the internal space of a system with internal symmetry group twists around at different spacetime points. The principal bundle carries the information on the topological structure of the Yang-Mills and eventual Higgs fields of physical theories[1].

Our purpose in this letter is to adopt the fibre bundle approach to reassess a class of extended gauge models[2], characterised essentially by the introduction of a family of vector gauge potentials that share a common transformation law under a given compact and simple gauge group G. It is perhaps worthwhile to mention that a geometrical framework has been set up for these models which is based on a Kaluza-Klein approach with spontaneous compactification[3] of a higher-dimensional matter-gravity coupled theory[4]. Nevertheless, since the fibre bundles are the natural mathematical setting to describe the gauge theories, we believe it behooves us to present a description of the extended models of Ref. [2] in terms of connections on an arbitrary vector bundle.

We shall adopt the analytic approach to discuss the connections[5]. Let us take a D-dimensional Minkowski space as the base manifold M^D and denote by E a smooth vector bundle over M^D with fibre F_x at the point x of M^D.

A smooth section of the bundle E is a smooth function s from M^D to E such that $\pi \cdot s = 1$, where π stands for the projection of the bundle E onto M^D. $C^\infty(M^D)$ denotes the set of smooth functions defined on M^D, whereas $C^\infty(E)$ refers to the vector space of smooth sections of E. We now have the elements at our disposal to define a connection on the vector bundle