The Einstein-Seiberg-Witten Equation on Four-Manifolds

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1 Introduction

Although the physical meaning of the Seiberg-Witten equations (SW-eq.) is yet to be discovered, the mathematical meaning is rather deep and highly efficient to understand the most basic phenomenon of differential topology in four dimension, namely, the existence of non-equivalent differential smooth structures on the same underlying topological manifold. The Seiberg-Witten equations arised through the ideas of duality described in Witten [2]. It is conjectured that the Seiberg-Witten equations are dual to Yang-Mills equations (YM-eq.). The duality is at the quantum level, since one of its necessary condition is the equality of the expectations values for dual theories. In topology, this means that fixed a 4-manifold its Seiberg-Witten invariants are equal to Donaldson invariants. A good reference for SW-eq. is [7]
One of the main features of Seiberg-Witten theory on a 4-manifold X is the obstruction imposed by the geometry of X on the SW-invaraints. In [1], Witten proved that the Seiberg-Witten invariants of X are trivial whenever X admits a riemannian metric with non-negative scalar curvature ($k_g \geq 0$). A consequence of the same argument is the non-existence of solutions to the Seiberg-Witten equations in $\mathbb{R}^4$, contrarialy to the YM-eq. which solutions in $\mathbb{R}^4$ are the *instantons*. In case $k_g < 0$ the SW-eq. may have solutions. So, the SW-eq. may have local solutions on a 4-manifold if locally $k_g < 0$. Until now, there is a result of non-existence to the SW-eq. given by the obstruction k_g [?]. The theory lacks a analytical theorem of existence, the strongest result known is Taubes's theorem in [?] where he showed that if X is sympletic and α is the canonical class of X, then the Seiberg-Witten invariant $SW_\alpha \neq 0$. Considering the importance played by the metric on the SW theory, its a interesting question to understand the coupling among the SW equation with the Einstein equation. However, the SW-equation were not obtained by variational methods. In fact, the SW-equation are 1^{st}-order equation while the Euler-Lagrange are usually 2^{nd}-order equations. Therefore, the coupling is performed through a more general set of SW-eq. obtained as Euler-Lagrange equation of a functional first used by Witten [1]. In fact, Witten used the functional to prove that only a finite number of moduli spaces of SW-theory are non-trivial.
The coupling among Yang-Mills equation and Einstein equation are physicaly much more interesting, it is also extremely hard to handle its analytical prop-

Subject: **Re: jas99**
Date: Thu, 8 Mar 2001 15:05:40 -0300 (BRT)
From: Joao C A Barata <jbarata@fma.if.usp.br>
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On Thu, 8 Mar 2001, Celso M. Doria wrote:

>
> *Caro Joao,*
>
> *Tudo bem !*
> *Gostaria de saber sobre as Atas da Escola JASwieca de 1999, ja*
> *foi publicado ? Gostaria de saber numero pag e etc*
> *do trabalho que enviei. Dados para o relatorio CAPES. E claro, gostria*
> *de ter uma copia tambem.*
>
> *Um abraco,*
>
> *Celso*
>

Caro Celso,
você já deveria ter recebido há vários meses uma cópia de um exemplar
enviado pela SBF. Lamento que isso não tenha ocorrido. Vou checar com eles
para ver o que aconteceu.

Quanto aos dados, são os seguintes:
The Einstein-Seiberg-Witten Equation on Four Manifolds
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Abraços,
JCABarata.

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