E-mail sobre Invariants de Witten-Seidberg.

,FROM: GHEORGHE CRACIUN GCRACIUN@MATH.OHIO-STATE.EDU.
,SUBJECT: MUTATII DE ULTIMA ORA
,TO: BURGHELE@MATH.OHIO-STATE.EDU
,DATE: MON, 7 NOV 1994 10:07:05 -0500 (EST)
,X-MAILER: ELM [VERSION 2.4 PL23]
,MIME-VERSION: 1.0
.. , SUBJECT: THE END OF DONALDSON THEORY
.. , STATUS: R
.. ,
.. , IT'S OFFICIAL, AND COMING OUT OF THE MOUTH OF TAUBES HIMSELF
.. , (THOUGH PARAPHRASED): DONALDSON THEORY IS OVER.
.. , IN HIS TALK TODAY ENTITLED WITTEN'S MAGICAL EQUATION, TAUBES
.. , OUTLINED HOW AN EQUATION PROPOSED BY WITTEN AS AN AFTERTHOUGHT
.. , AT THE END OF HIS RECENT MIT TALK ON DONALDSON THEORY AND
.. , N=2 SUPERSYMMETRIC QFT, WHERE HE CONJECTURED IT SHOULD GIVE
.. , THE DONALDSON INVARIATNS, ESSENTIALLY TRIVIALIZES ALL OF THE WORK
.. , DONE IN DONALDSON THEORY, FROM THE START TO THE RECENT
.. , KRONHEIMER-MROWKA STUFF TO A WHOLE SLEW OF PROBLEMS SOLVED IN THE LAST
.. , COUPLE OF WEEKS, INCLUDING THE THOM CONJECTURE ON CP2.
.. ,
.. , AS ALL OF YOU UNDERSTAND THIS STUFF BETTER THAN ME, EXCUSE ME FOR THE
.. , GOOFS AND I'LL BE SPARING ON DETAIL -IT'LL ALL BE COMPLETELY REVISED
.. , AND DIFFERENT WITHIN A WEEK ANYWAY.
.. , IN ANY CASE, WITTEN GAVE AN EQUATION IN TERMS OF VERY CLASSICAL
.. , INFORMATION (BASICALLY JUST SPINORS AND LINE BUNDLES), WHICH
.. , GIVES RISE TO A NEW SET OF INVARIANTS, WHICH HE CONJECTURES AGREE
.. , WITH THE DONALDSON ONES. SO FAR THIS HAS NOT BEEN PROVEN GENERALLY
.. , BUT IS KNOWN TO AGREE FOR ALL KNOWN FOUR-MANIFOLDS.
.. , IN ANY CASE, REGARDLESS OF THE RELATION TO DONALDSON INVARIATNS,
.. , THESE WITTEN INVARIANTS GIVE TRIVIAL PROOFS (SEVERAL OF WHICH
.. , HE OUTLINED IN THE TALK) OF THE MAJOR THEOREMS OF DONALDSON THEORY,
.. , WITH, AS TAUBES SAID, ABOUT 1/1000 OF THE LENGTH.
.. ,
.. , THE SETUP IS YOU HAVE X A COMPACT 4D ORIENTED SURFACE WITH L A
.. , COMPLEX LINE BUNDLE, C1(L) MOD 2 = W2(TX) - THE CHERN CLASS OF L
.. , AGREES WITH THE STIEFELWHITNEY CLAS OF TX MOD 2.
.. , IF X IS SPIN (W2=0) CONSIDER ALSO THE SPINOR BUNDLE, OTHERWISE
.. , WE TAKE A SPIN-C BUNDLE - PRINCIPAL FIBER BUNDLE
.. , FOR (SPIN 4 X S1)/Z2 INSTEAD OF SP(4)=SU2 X SU2.
.. , WE THEN TENSOR THIS BUNDLE WITH THE SQUARE ROOT OF L, AND
.. , TAKE THIS AS THE AUXILLIARY BUNDLE FOR OUR CONSTRUCTION.
.. , THE DATA ARE A CONNECTION A ON L, AND A SECTION PSI OF
.. , THE SELF-DUAL PART OF THE AUXILLIARY BUNDLE.
.. , THERE ARE TWO EQUATIONS (ONE IS THAT PSI BE
.. , A HARMONIC SPINOR (THE DIRAC EQUATION FOR PSI, DPSI=0).
.. , THE SECOND IS THAT THE SELF-DUAL PROJECTION OF THE CURVATURE OF
.. , A BE GIVEN IN COORDINATES A,B AS (F*F)(A,B)=-1/2)PSI,E A E E PSI,
.. , WITH E A AND E E BEING CLIFFORD MULTIPLICATION ACTING ON OUR
.. , SECTION PSI.. OK I DON'T UNDERSTAND THIS EQUATION SO EXCUSE
.. , ME FOR THE VAGUENESS..
.. ,
.. , OUR MODULI SPACE WILL BE PAIRS A, PSI SOLVING THESE EQUATIONS
.. , MOD THE ACTION OF AUT L ON A. (NOTE THAT OUR BUNDLES AND ALL
.. , OUR FIXED, AND EVERYTHING IS JUST IN TERMS OF SPINORS AND LINE

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>> > FROMO EAEZ'UCRNATH.UCR.EDU (JCHN EAEZ)
>> > SUEJECTC THIS WEEK'S FINDS 44 = SPECIAL EDITIONO THE END OF DONALDSON
>>THEORY
>> > MESSAGE=IDO ]3SKEF1SC71'CALAXY.UCR.EDU,
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>> > APPROVEDO GREG KUPERBERG )SCI=MATH=RESEARCH'MATH.UILC.EDU,, MODERATOR FOR
>>SCI.MATH.RESEARCH
>> > LINESC 128
>> >
>> > THIS WEEK'S FINDS IN MATHEMATICAL PHYSICS (WEEK 44)
>> > JCHN EAEZ
>> >
>> > I GOT SOME NEWS TODAY FROM ALLEN KNUTSON, CONFIRMED BY MY COLLEAGUE
>> > YAT=SUN FOON, WHICH SHOULD BE OF GREAT INTEREST.  BRIEFLY, IT APPEARS
>> > THAT WITTEN HAS COME UP WITH A NEW WAY OF DOING DONALDSON THEORY THAT IS
>> > FAR EASIER THAN ANY PREVIOUSLY KNOWN.  ACCORDING TO TALEES, MANY OF THE
>> > MAIN THEOREMS IN DONALDSON THEORY SHOULD NOW HAVE PROOFS THAT ARE 1/1000TH
>> > AS LONG=
>> >
>> > I SUPPOSE TO FIND THIS EXCITING ONE MUST ALREADY HAVE SOME IDEA OF WHAT
>> > DONALDSON THEORY IS.  BRIEFLY, DONALDSON THEORY IS A THEORY BORN IN THE
>> > 1980S THAT REVOLUTIONIZED THE STUDY OF SMOOTH 4=DIMENSIONAL MANIFOLDS BY
>> > USING AN IDEA FROM PHYSICS, NAMELY, THE SELF=DUAL YANG=MILLS EQUATIONS.
>> > THE YANG=MILLS EQUATIONS DESCRIBE MOST OF THE FORCES WE KNOW AND LOVE
>> > (NOT GRAVITY), BUT ONLY IN 4 DIMENSIONS CAN ONE GET SOLUTIONS OF THEM OF
>> > A SPECIAL KIND, KNOWN AS SELF=DUAL SOLUTIONS.  (IN PHYSICS THESE
>> > SELF=DUAL SOLUTIONS ARE KNOWN AS INSTANTONS, AND THEY WERE USED BY 'T
>> > HOOFT TO SOLVE A PROBLEM PLAGUING PARTICLE PHYSICS, CALLED THE U(1)
>> > PUZZLE.)
>> >
>> > MATHEMATICALLY, 4=DIMENSIONAL MANIFOLDS ARE VERY DIFFERENT FROM
>> > MANIFOLDS OF ANY OTHER DIMENSION=  FOR EXAMPLE, ONE CAN ASK WHETHER R N
>> > ADMITS ANY SMOOTH STRUCTURE OTHER THAN THE USUAL ONE.  (TECHNICALLY, A
>> > SMOOTH STRUCTURE FOR A MANIFOLD IS A MAXIMAL SET OF COORDINATE CHARTS
>> > COVERING THE MANIFOLD WHICH HAVE SMOOTH TRANSITION FUNCTIONS.  LOOSELY,
>> > IT'S A DEFINITION OF WHAT COUNTS AS A SMOOTH FUNCTION.)  THE ANSWER IS
>> > NO = EXCEPT IF N = 4, WHERE THERE ARE UNCOUNTABLY MANY SMOOTH STRUCTURES=
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,, , STRUCTURE
,, ,
,, , (ESSENTIALLY THE EULER CHARACTERISTIC OF A LINE BUNDLE OVER THE
,, , MODULI SPACE)
,, , AND TO PROVE THE RIEMANN THEOREM. THE PROOFS AS TAKEN SAID ARE
,, , EMBARASSINGLY
,, , EASY AND PROBABLY NOBODY WILL CLAIM ANY CREDIT FOR THEM (IT IS TRUE -
,, , HE
,, ,
,, , PROVED THE EXISTENCE, THE COMPACTNESS AND THE ORIENTABILITY THEOREM
,, , IN THE
,, ,
,, , LECTURE TODAY). MOREOVER THE CORRESPONDING INVARIANTS ARE VERY EASY
,, , TO COMPUTE
,, , AND USING THEM ONE CAN SOLVE ALL THE PROBLEMS IN DONALDSON THEORY. IT
,, , TOOK HIM
,, , TEN MINUTES TO PROOF DONALDSON THEOREMS 1,2 AND 3 USING THESE
,, , INVARIANTS. HE
,, ,
,, , WAS VERY BITTER AND WAS JOKING ALL THE TIME THAT HE FEELS LIKE
,, , STEALING CANDY
,, , FROM THE KIDS. ANYWAY, THE BOTTOM LINE IS THAT THE STUFF WORKS AND
,, , WORKS WITH
,, , MIRACULOUS EASINESS. THERE ARE TWO TEAMS WORKING ON THE STUFF RIGHT
,, , NOW
,, ,
,, , TAUBES, DONALDSON AND FINTUSHEL AND STERN ARE BUILDING THE FOUNDATIONS
,, , IN THE
,, , NEW LANGUAGE. THEY HAVE ALREADY PROVEN ALL THE THEOREMS FOR THE NEW
,, , INVARIANTS,
,, , INCLUDING THE BLOW-UP FORMULAS. ALSO TAUBES-FINTUSHEL-STERN HAVE JUST
,, , FINISHED
,, , THE PROOF OF THE EQUIVALENCE THEOREM - THESE NEW INVARIANTS ARE
,, , INDEED
,, ,
,, , EQUIVALENT TO THE DONALDSON POLYNOMIALS. THE OTHER TEAM IS
,, , KRONHEIMER-MROWKA.
,, , THEY HAVE CONCENTRATED ON THE APPLICATIONS. THEY HAVE REPROVED THE
,, , KRONHEIMER-
,, , MROWKA THEOREM (IT TAKES TEN LINES), HAVE PROVEN THAT THE ONLY BASIC
,, , CLASS
,, ,
,, , FOR AN ALGEBRAIC SURFACE IS THE CANONICAL CLASS AND HAVE ATTACKED ALL
,, , THE
,, , STANDARD CONJECTURES OF THE THEORY. SO FAR, THEY HAVE PROVEN THE THIN
,, ,
,, , CONJECTURE ON $E \otimes E \to F$, THE 11/8 CONJECTURE AND RIGHT
,, , NOW ARE ATTACKING THE FINTUSHEL-STERN CONJECTURE ABOUT THE RATIONAL
,, , BLOW-UPS.
,, ,
,, , ENJOY,
,, ,
,, , GEORGE
,, ,
,, ,