Phenomenological-Geometrical Approach for Superconductivity

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The understanding of superconductivity goes through many interesting phenomena, the planar structure in High T_c cuprates, the coexistence between two order states (for instance the superconducting phase with a magnetic order), the presence of two gaps and the formation of configurations like stripes (interplay between spin and charge). In this paper we proposed a geometrical-phenomenological model to discuss some of the mentioned phenomena, using the formalism of Eintein-Cartan geometry and obtaining the ground state equations. We showed that these two ground state equations are enough to solve in the limi $\kappa = \frac{1}{\sqrt{2}}$ all the four equations of motion of our model. Also a detailed discussion of the limits of the ground state equations are considered.

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I. INTRODUCTION

The Ginzburg-Landau (GL) model[20] explained satisfactorily many of the phenomenological features of superconductivity, its form it is based on the theory of second order phase transitions, the principle of minimal coupling to electromagnetism and the presence of macroscopic wave function that acts as an order parameter. Moreover, this model also describes a mechanism for the spontaneous symmetry breaking (SSB) of gauge theories and with this we explain succesfully the Meissner effect.

The SSB idea inspired physicists who used it to describe the mechanism of mass generation developed in high energy physics. This idea is also used in grand unified theories (GUTs) which attempt to describe physics beyond the Standar Model. Within the GUTs, there is a $N = 2$, Supersymmetric Yang-Mills theory that is equivalent to the Donaldson theory[25]. In this context, E. Witten proposes a new way to calculate many of the results of Donaldson theory by using the Seiberg-Witten functional whose dual equations turn to be the ground state equations used in many gauge theories with SSB mechanism including the Ginzburg-Landau model.

Because we needed to explain the Meissner effect in condensed matter physics we now are able to use the SSB mechanism in high energy physics, we believe that there is a feedback coming from the Seiberg-Witten equations that can help us to understand the formation of the superconducting phase at High T_c in condensed matter physics.

A hundred years have passed since the discovery of superconductivity and we still finding new types of superconductors whose properties cannot be explained by its microscopical theory (BCS theory)[3] nor by its phenomenological counterpart the GL model[20]. For instance, the planar structure in High T_c cuprates, the presence of superconductivity in coexistence with another order state, the appereance of two gaps and the inhomogeneities of them, the formation of new geometrical electronic structures as spin and charge density waves (SDW-CDW), all of them are very interesting topics that goes beyond the conventional theories (BCS and GL). In fact, the interplay between charge and spin together with the superconducting phase do not only coexist but seems to reciprocally stabilize[1,4,19,22]. We will address this coexistence by introducing a minimal coupling of the charge and spin degrees of freedom (electromagnetic potential $\vec{A}$ and spin connection $\vec{\omega}^{ab}$) with a two component spinorial order parameter. The spin connection not only allow us to coupled the order parameter with spin degrees of freedom but also introduces a local symmetry of rotations. This rotational symmetry seems to be important for the understanding of the pseudogap phase of High T_c superconductors[8]. The presence of the spin connection will turn out to be one of the key ingredients for the introduction of a geometrical background of the Einstein-Cartan type.

Besides the introduction of the two component order parameter Ψ_α and the spin connection $\vec{\omega}^{ab}$ we also introduce another field that we called the spin correlation $e_i^a(x)$, its name is because it introduce position dependence in the Pauli matrices (spin operators) through the relation $e_i^a(x)\sigma_a = \sigma_i(x)$. The introduction of all these new degrees of freedom have physical motivation from the point of view of condensed matter and superconductivity, however at this point we can also analyze our new fields as they were coming from a gravitational theory (remember we were highly inspired with the Seiberg-Witten functional). Therefore if we treat the spin connection $\vec{\omega}^{ab}$ as the gauge field of Lorentz rotations and the spin correlation $e_i^a(x)$ as the vielbein which introduce the symmetry of general coordinate transformations, we ended up with a genuine framework to study superconductivity geometrically. This framework turns out to be the geometry of Einstein-Cartan[11,12,17,23,24].

When we use the Einstein-Cartan theory for the study of superconductivity, we need to give an interpretation for new physical quantities like torsion and curvature from the condensed matter perspective. One of the most common intepretations is related with the gauge theory of defects in state solid physics[10,13,15,16,18] where the effects of torsion and curvature can be viewed as the insertions of the densities of dislocations and disclinations in the atoms arrangements of the material.

Finally the purpose of this paper is to propose a free energy to study superconductivity from a geometric perspective considering a spinorial order parameter minimally coupled with a spin-charge framework. Moreover we will use the first order formalism to introduce a geometric framework of the Einstein-Cartan type, this mechanism is convenient because is closed related with theory of defects in solids. All geometrical quantities will be motivated in ideas coming from the understanding of High T_c superconductors.

This paper is divided as follows, in section II we propose a free energy functional respecting the main ideas of the Ginzburg-Landu model but with a geometrical structure and then we calculate the four equations of motion. Section III is devoted to the introduction of a generelization of the Lichnerowicz-Weitzenböck formula for spaces with torsion, applying this formula we obtain the ground state equations for a specific value of κ. Then we analyze the ground state equations in section IV from the perspective of condensed matter physics and study its diferent limits. Finally we present the conclusions of our proposed model in section V.

II. THE GINZBURG-LANDAU-LIKE FREE ENERGY AND THE EQUATIONS OF MOTION

In this section, we propose a phenomenological model that considers charge and spin interactions minimally coupled with a two order spinorial parameter Ψ_α whose ground state contains both spin-vortices and magnetic-vortices.

Taking into account the theory of second order phase transitions[20], spin-charge density background[4] and the principle of minimal coupling of gauge theories we proposed the following free energy in analogy with Ginzburg-Landau model[20].

$$F = \int d^3xe\left\{\frac{1}{2m}(|\vec{D}\Psi|^2 - gR|\Psi|^2 - \epsilon^{ijl}\frac{T^k_{ij}}{2}[\Psi^\dagger\sigma_k(D_l\Psi) + (D_l\Psi)^\dagger\sigma_k\Psi]) - \vec{\alpha}.(\Psi^\dagger\vec{\sigma}\Psi) + \frac{\beta}{2m}|\Psi|^4 + \frac{\vec{h}^2}{8\pi} + \frac{|\vec{\alpha}|^2}{2\beta}\right\}, \quad (1)$$

where, $\vec{\alpha}$ is a constant vector, the covariant derivative is given by

$$D_i\Psi_\alpha = [\frac{\hbar}{i}\delta^\beta_\alpha\partial_i - \frac{\hbar g}{2}\omega^{ab}_i(\Sigma_{ab})^\beta_\alpha - \frac{\hbar q}{c}\delta^\beta_\alpha A_i]\Psi_\beta, \quad (2)$$

in the above expression, A_i and ω^{ab}_i are the electromagnetic and spin connection responsible for the local $U(1)$ and $SU(2)$ symmetries respectively. Also e is the determinant of local field e^a_i which allow us to introduce the notion spin correlations through the relation

$$e^a_i(x) = \left\langle 0\left|\frac{1}{2}\{\sigma_i(x),\sigma^a\}\right|0\right\rangle \quad (3)$$

σ_a are the constant Pauli matrices. The quantities R and T^k_{ij} are related to the field strengths of the $SU(2)$ symmetry together with the spin correlations and $\vec{h}$ is the local magnetic field coming from the $U(1)$ symmetry. See the appendix (A).
This model (1), have many parameters $\hbar$, q, c, m, β, g we will perform a scale transformation to reduce the number of these. To achieve this we will perform a scale transformation (reduce units), see the appendix (B). After the scale transformation we ended up with the following free energy

$$F = \int d^3xe\left\{|\vec{D}\Psi|^2 - gR|\Psi|^2 - \hat{\alpha}.(\Psi^\dagger\vec{\sigma}\Psi) - \frac{1}{2}\epsilon^{ijl}T^k_{ij}[\Psi^\dagger\sigma_k(D_l\Psi) + (D_l\Psi)^\dagger\sigma_k\Psi] + \frac{1}{2}|\Psi|^4 + \kappa^2\vec{h}^2 + \frac{1}{2}\right\}, \quad (4)$$

This free energy have four independent fields, the order parameter $\Psi_\alpha(x)$ which will obey a Ginzburg-Landau-like equation, the electromagnetic field $A_i(x)$ together with the spin connection $\omega^{ab}_i(x)$ wich describe the intertwined interaction of magnetism and spin minimally coupled with the order parameter.

We also introduce the vielbein field $e^a_i(x)$ which, together with the spin connection $\omega^{ab}_i(x)$, allow us to introduce a geometric framework to the study of the free energy[17,24]. The approach we used here is well-known in gravitational theories as the first order formalism for an Einstein-Cartan space, moreover it have shown to be useful also in defect theory of materials[10,16,18], this last view we will adopt here as is more properly for non relativistic systems. Now using the variational principle we obtain

$$\left(\vec{\nabla}.\vec{D} - gR - \widehat{\alpha}.\vec{\sigma} - T^l D_l - \nabla_l T^l + |\Psi|^2\right)\Psi = 0, \quad (5)$$

$$\partial_i(eF^{ij}) = \frac{e}{2\kappa^2}\left(\Psi^\dagger T^j \Psi - j^j\right), \quad (6)$$

$$g|\Psi|^2\left(R_{ij} - \frac{1}{2}g_{ij}R\right) + \frac{1}{2}\nabla_k J^k_{ji} + T^k_{jm}J^m_{ik} = \Theta_{ij}, \quad (7)$$

$$\left(T^i_{ba} - e^i_{[b}\partial_{a]}\right)|\Psi|^2 + \Psi^\dagger T^i \Sigma_{ab}\Psi + \Psi^\dagger \Sigma_{ab} T^i \Psi = K^i_{ab}, \quad (8)$$

where

$$T^l = \sigma_k \frac{1}{2}\epsilon^{ijl}T^k_{ij}, \quad (9)$$

$$j^j = \frac{1}{2}\left[\Psi^\dagger(D^j\Psi) + (D^j\Psi)^\dagger\Psi\right], \quad (10)$$

$$J^{ij}_k = \epsilon^{ijl}\left[\Psi^\dagger\sigma_k(D_l\Psi) + (D_l\Psi)^\dagger\sigma_k\Psi\right], \quad (11)$$

$$K^i_{ab} = \frac{1}{2}\left[\Psi^\dagger\Sigma_{ab}(D^i\Psi) + (D^i\Psi)^\dagger\Sigma_{ab}\Psi\right] + \frac{1}{4}J^i_{ab}, \quad (12)$$

The energy momentum tensor of our free energy (1) is given by

$$\Theta_{ij} = (D_i\Psi)^\dagger(D_j\Psi) - \hat{\alpha}_i(\Psi^\dagger\sigma_j\Psi) + \kappa^2 h_i h_j - \frac{1}{2}g_{ij}\left[(\vec{D}\Psi)^\dagger.(\vec{D}\Psi) - \hat{\alpha}.(\Psi^\dagger\vec{\sigma}\Psi) + \frac{1}{2}|\Psi|^4 + \kappa^2\vec{h}^2 + \frac{1}{2}\right], \quad (13)$$

Besides the complexity of the equations of motion and its mixing, we will show in the next section that thanks to the Lichnerowicz-Weitzenböck formula these four equations of motion are not that complicated as first appears. In fact, chossing a properly value of κ can reduce these four equations of motion to only two first order differential equations.

III. THE LICHNEROWICZ-WEITZENBÖCK FORMULA WITH TORSION AND THE NEW EQUATIONS OF MOTION

The Lichnerowicz-Weitezenböck formula[7] is often used when we want to obtain the ground state equations that minimize the energy of our system. Although this formula is mainly related when we have spinors on a pseudo Riemannian manifolds, its form and idea is always used when we study topological configurations in gauge theories as kinks, vortices or monopoles.
Usually this formula is applied in spaces with symmetric affine conection (zero torsion), but here in this section we will formulated a generalization for spaces with torsion, this formula takes the following form:

$$\int d^3xe\left(|\vec{\sigma}.\vec{D}\Psi|^2 + \Psi^\dagger\vec{\sigma}\Psi.\vec{h}\right) = \int d^3xe\left[|\vec{D}\Psi|^2 - gR|\Psi|^2 - \Psi^\dagger T^i(D_i\Psi) - (D_i\Psi)\dagger T^i\Psi\right]. \quad (14)$$

The above formula (14) is obtained by neglecting the surface terms and using the metricity condition

$$\nabla_i g_{jk}(x) = 0, \quad (15)$$

this condition is naturally defined in pseudo Riemannian spaces and means that we preserve the distances (scalar product), and angles, its definition in the language of the first order formalism is given by condition for the vielbein:

$$\nabla_i e^a_j(x) = 0, \quad (16)$$

which allow us to obtain the Fock-Ivanenko condition

$$\nabla_i \sigma_j(x) = 0, \quad (17)$$

a detailed derivation of the Fock-Ivanenko condition can be seen in[21].

Now we applied the formula (14) into the free energy (4) and complete some squares, then the free energy becomes

$$F = \int d^3xe \left[|\vec{\sigma}.\vec{D}\Psi|^2 + \frac{1}{2}\left(\vec{h} + \Psi^\dagger\vec{\sigma}\Psi - \hat{\alpha}\right)^2 + \left(\kappa^2 - \frac{1}{2}\right)\vec{h}^2 + \vec{h}.\hat{\alpha}\right]. \quad (18)$$

This mechanism to obtain the ground state equations was first developed by Bogomolnyi[5] and in this particular model for $\kappa = \frac{1}{\sqrt{2}}$ those are

$$(\vec{\sigma}.\vec{D})\Psi = 0, \quad (19)$$
$$\vec{h} = \hat{\alpha} - \Psi^\dagger\vec{\sigma}\Psi. \quad (20)$$

It is not easily to see that these two first order differential equations satisfy all four equations of motion (5-8), in order to understand how this happens we obtain again the equations of motion for $\kappa = \frac{1}{\sqrt{2}}$ using the free energy expression given by (18) rather than (4).

$$\vec{\sigma}.\vec{\nabla}(\vec{\sigma}.\vec{D})\Psi + (\vec{h} - \hat{\alpha}).\vec{\sigma}\Psi + \Psi|\Psi|^2 = 0, \quad (21)$$
$$e(\vec{\sigma}.\vec{D}\Psi)\sigma^k\Psi + \frac{1}{2}\partial_j[e\epsilon^{ijk}(h_i + \Psi^\dagger\sigma_i\Psi - \hat{\alpha}_i)] = 0, \quad (22)$$
$$\frac{1}{2}G_iG_j + (D_{ij}\Psi)^\dagger(D^k_k\Psi) + (D^k_k\Psi)^\dagger(D_{ij}\Psi) = \theta_{ij}, \quad (23)$$
$$\Psi^\dagger\Sigma_{ab}\sigma^i(\vec{\sigma}.\vec{D}\Psi) + (\vec{\sigma}.\vec{D}\Psi)^\dagger\Sigma_{ab}\Psi = 0, \quad (24)$$

where

$$D_{ij} = \sigma_iD_j \quad (25)$$
$$G_i = h_i - \Psi^\dagger\sigma_i\Psi - \hat{\alpha}_i \quad (26)$$
$$\theta_{ij} = \frac{1}{2}\left(|D^k_k\Psi|^2 + \frac{1}{2}\vec{G}^2\right). \quad (27)$$

It is readily to see that these two first order differential equations (19, 20) are indeed solutions of the four equations of motion (5-8), (21-24). This limit of course was obtained for an specific value of $\kappa = \frac{1}{\sqrt{2}}$, however the validity of the ground state equations can go beyond that specific value[6].

IV. ANALYSIS OF THE GROUND STATE EQUATIONS

In this section we will study the the Seiberg-Witten equations[25] (ground states) using the the tools of for a Riemann-Cartan geometry in the first order formalism. The equations (19, 20) have the following form:

$$e^i_a\sigma^a(\delta_{\alpha\beta}\frac{1}{i}\partial_i - \frac{1}{2}\omega^{ab}_i(\Sigma_{ab})_{\alpha\beta} - A_i\delta_{\alpha\beta})\Psi_\beta = 0, \quad (28)$$
$$\hat{\alpha}_i - \Psi^\dagger e^a_i\sigma_a\Psi = h_i, \quad (29)$$

The equation (19), looks like a massless Dirac equation minimally coupled to the electromagnetic field and spin connection, however our framework is not relativistic so we must be careful with the analogies. Now, we will discuss the diferent limits of the ground state equations.

A. Interaction free and no spin correlations

This limit is obtained by taking

$$\omega^{ab}_i = 0, \quad (30)$$
$$A_i = 0, \quad (31)$$
$$e^a_i = \delta^a_i, \quad (32)$$

and is characterized by planar confinent[9] perpendicular to given coordinate that exponentially decreases the value of the order parameter Ψ_α. The solutions in this case for the ground state equations have the following form

$$\Psi \propto e^{-q_3}e^{i(k_1x_1+k_2x_2)}, \quad (33)$$
$$\hat{\alpha}_i \propto e^{-2q_3}\sigma_i. \quad (34)$$

In this limit the constant vector $\hat{\alpha}_i$ can be interpreted as the spin operators rescricted to the plane perpendicular to the coordinate q_3, because it is proportional to the Pauli matrices with a factor e^{-2q_3}. It is important to mention that some high T_c superconductors, specifically the cuprates, present layers of CuO_2 that are weakly coupled, therefore they show a planar structure[14]. The idea to simulate this planar behaviour by introducing a two component order paramter Ψ_α have been already studied[9] and it occurs because the order parameter obeys a Dirac-like equation in three dimensions.

B. The Abrikosov limit

This is the limit where we obtained the Abrikosv vortices[2], no spin correlation neither spin connection are consider

$$\begin{aligned} A_i &\neq 0, & (35)\\ \omega_i^{ab} &= 0, & (36)\\ e_i^a &= \delta_i^a, & (37)\end{aligned}$$

Considering a two component spinor as our order parameter

$$\Psi_\alpha = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \qquad (38)$$

then the ground state equations in its matrix language reads

$$\begin{aligned} D_3\psi_1 + (D_1 - iD_2)\,\psi_2, && (39)\\ (D_1 + iD_2)\psi_1 - D_3\psi_2 = 0, && (40)\\ h_i = \alpha_i - \Psi^\dagger\sigma_i\Psi. && (41)\end{aligned}$$

The above equations are strongly coupled to each other, however if we consider a weak electromagnetic interaction, then we can assume a planar behavior similar to the free interaction case. The ground state equations now are

$$\begin{aligned} (D_1 - iD_2)\,\psi_2, && (42)\\ (D_1 + iD_2)\psi_1 = 0, && (43)\\ h_3 = \alpha_3 - \Psi^\dagger\sigma_3\Psi, && (44)\end{aligned}$$

these equations are the Abrikosov ground state conditions and contains vortices solutions. Vortices are carried by ψ_1 and anti-vortices by ψ_2, the spinorial order parameter Ψ_α carries both in a single representation.

If we consider the spin connection ($\vec{\omega}^{ab}$) interaction instead of the electromagnetic one, we would have obtained another type of topological defect. In fact the homotopy group related with the local symmetry of this gauge field ($SU(2)$) let us to work with monopoles and no vortices. One of the reason for not having vortices but monopoles is because the group of rotations is non Abelian. Nevertheless, the condition of confinment of the order parameter restrict the ground state equations to a plane, in this case the group of rotations is Abelian and type of topological defect is of spin vortices.

V. CONCLUSION

We have proposed a phenomenological model for studying superconductivity in a geometrical fashion and relate many of the geometrical fetures with the problems currently discussed in superconductivity. Even though the model have many new degrees of freedom (curvature and torsion) we discussed the viability to interpret those as densities of defects (disclinations and dislocations). Moreover, the free energy of the model obeys the Abrikosov-like ground state equations that turn out to be the Seiberg-Witten equations. We have demonstrated that these two equations are totally enough to solve the four equations of motion coming from the free energy and describe topological defects such as magnetic and spin vortices.

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APPENDIX A: SOME ASPECTS OF THE $SU(2)$-$U(1)$ GAUGE MODEL WITH SPIN CORRELATIONS

This model is based on the ideas of the Einstein-Cartan geometry, where the spin correlation $e_i^a(x)$ is identified as the tetrad or vielbein which more generically is related to the general coordinate transformation as follows

$$e_i^a(x) = \frac{\partial x_i}{\partial x_a}, \qquad (A1)$$

in our presented model (1) this will introduce the notion of local spins through $e_i^a(x)\sigma_a = \sigma_i(x)$ and no necessarily should be related to any general coordinate transformation in this sense here we differ a little bit with the approach of Einstein-Cartan geometry. The geometry of Einstein-Cartan describes a gravitational pseudo Riemannian space with torsion[17,23,24], but also can be interpreted as the material space of solids where curvature and torsion are the density of disclinations and dislocations respectively[13,15,16,18].

The field strengths of the $SU(2)$-$U(1)$ gauge model with spin correlations are given by

$$\begin{aligned} R_{ij}^{ab} &= \partial_i\omega_j^{ab} - \partial_j\omega_i^{ab} + g(\omega_{ic}^a\omega_j^{cb} - \omega_{jc}^a\omega_i^{cb}), & (A2)\\ T_{ij}^a &= \partial_i e_j^a - \partial_j e_i^a + g(\omega_{ic}^a e_j^c - \omega_{jc}^a e_i^c), & (A3)\\ F_{ij} &= \partial_i A_j - \partial_j A_i, & (A4)\end{aligned}$$

the above equation means that the local magnetic field is given by $h_i = 1/2\epsilon_{ijk}F^{jk}$.

Using the tetrad we construct the metric

$$g_{ij}(x) = \eta_{ab}e_i^a(x)e_j^b(x), \qquad (A5)$$

where,

$$\eta_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (A6)$$

so we can use both the metric and/or the tetrad to raise, lower and contract indices. The scalar of curvature and the torsion in the equation (1) are contracted expressions of the field strengths A2 and A3

$$R = e^i_b e^j_a R^{ab}_{ij} \tag{A7}$$
$$T^k_{ij} = e^k_a T^a_{ij}, \tag{A8}$$

APPENDIX B: THE SCALE TRANSFORMATION OF THE FREE ENERGY

To reduce the number of parameters of the free energy (1) we make a scale transformation

$$\Psi = \sqrt{\frac{|\vec{\alpha}|}{\beta}}\Psi', \tag{B1}$$

$$x^i = \xi x'^i, \tag{B2}$$

$$\xi^2 = \frac{\hbar^2}{2m|\vec{\alpha}|}, \tag{B3}$$

$$\phi_0 = \frac{2\pi\hbar c}{q}, \tag{B4}$$

$$D_i = \frac{\hbar}{\xi} D'_i, \tag{B5}$$

$$A_i = \frac{\phi_0}{2\pi\xi} A'_i, \tag{B6}$$

$$\omega^{ab}_i = \frac{1}{\xi}\omega^{ab}_i, \tag{B7}$$

$$\kappa = \frac{mc}{\hbar q}\sqrt{\frac{\beta}{2\pi}}, \tag{B8}$$

the above expressions allow us to rewrite the free energy (1) in the form of (4), where the free energy is normalized to a factor

$$F = \frac{\beta}{|\vec{\alpha}|^2} F, \tag{B9}$$

therefore we ended up with only two parameters g and κ, this scaling was first developed by A. Abrikosov in his seminal work[2].

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1 Yoichi A, Kouji S, Seiki K, and A. N. Lavrov. Electrical resistivity anisotropy from self-organized one dimensionality in high-temperature superconductors. *Phys. Rev. Lett.*, 88:137005, Mar 2002.

2 A. A. Abrikosov. *Soviet Physics JETP*, 5:1174, 1957.

3 John Bardeen, L.N. Cooper, and J.R. Schrieffer. Theory of superconductivity. *Phys.Rev.*, 108:1175–1204, 1957.

4 E. Berg, E. Fradkin, S. A. Kivelson, and J. M. Tranquada. Striped superconductors: how spin, charge and superconducting orders intertwine in the cuprates. *New Journal of Physics*, 11(11):115004, 2009.

5 E. B. Bogomolny. Stability of Classical Solutions. *Sov. J. Nucl. Phys.*, 24:449, 1976. [Yad.Fiz.24:861-870,1976].

6 E H Brandt. The flux-line lattice in superconductors. *Reports on Progress in Physics*, 58(11):1465, 1995.

7 T. Branson and O. Hijazi. Bochner-Weitzenboeck formulas associated with the Rarita-Schwinger operator. *ArXiv High Energy Physics - Theory e-prints*, October 2001.

8 R. Daou, J. Chang, David L., Olivier C., F. Laliberte, N. Doiron-Leyraud, B. J. Ramshaw, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer. Broken rotational symmetry in the pseudogap phase of a high-tc superconductor. *NATURE*, 463:519, 2010.

9 M. M. Doria, A. R. de C. Romaguera, and F. M. Peeters. The ground states of the two-component order parameter superconductor. *EPL (Europhysics Letters)*, 92(1):17004, 2010.

10 D.G.B. Edelen and D.C. Lagoudas. *Gauge theory and defects in solids*. Mechanics and physics of discrete systems. North-Holland, 1988.

11 Friedrich W. Hehl and Yuri N. Obukhov. Elie cartan's torsion in geometry and in field theory, an essay. November 2007.

12 Friedrich W. Hehl, Paul von der Heyde, G. David Kerlick, and James M. Nester. General relativity with spin and torsion: Foundations and prospects. *Rev. Mod. Phys.*, 48:393–416, Jul 1976.

13 A. Kadić and D.G.B. Edelen. *A gauge theory of dislocations and disclinations*. Lecture notes in physics. Springer-Verlag, 1983.

14 M. A. Kastner, R. J. Birgeneau, G. Shirane, and Y. Endoh. Magnetic, transport, and optical properties of monolayer copper oxides. *Rev. Mod. Phys.*, 70:897–928, Jul 1998.

15 M. O. Katanaev. Introduction to the geometric theory of defects, 2005.

16 M.O Katanaev and I.V Volovich. Theory of defects in solids and three-dimensional gravity. *Annals of Physics*, 216(1):1 – 28, 1992.

17 T.W.B. Kibble. Lorentz invariance and the gravitational field. *J.Math.Phys.*, 2:212–221, 1961.

18 H. Kleinert. *Gauge fields in condensed matter*. Number v. 2 in Gauge Fields in Condensed Matter. World Scientific, 1989.

19 G. Koutroulakis, M. D. Stewart, V. F. Mitrović, M. Hor-

vatić, C. Berthier, G. Lapertot, and J. Flouquet. Field evolution of coexisting superconducting and magnetic orders in cecoin$_5$. *Phys. Rev. Lett.*, 104:087001, Feb 2010.
[20] Evgeny M. Lifschitz Lev D. Landau. *Electrodynamics of Continuous Media*. 1984.
[21] A. A. Vargas-Paredes M. M. Doria and J. A. Helayel-Neto. A ground state condition for the superconducting state in presence of a spin-charged background. *Submited*, 2011.
[22] Manuel Nú nez Regueiro. The universal behaviour of superconducting quantum critical points. private communication, 2011.
[23] V.D. Sabbata and M. Gasperini. *Introduction to gravitation.* World Scientific, 1985.
[24] R. Utiyama and T. Fukuyama. Gravitational field as a generalized gauge field. *Prog. Theor. Phys.*, 45:612–627, 1971.
[25] Edward Witten. Monopoles and four manifolds. *Math. Res. Lett.*, 1:769–796, 1994.