Dynamics of stochastic Bratteli diagrams

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Resumo: In 2000, Killeen and Taylor defined a stochastic adding machine by the following way. Write an integer n in base 2, add 1 to n and assume that the carry is added with probability p and it is not added with probability $1-p$. With this, Killeen and Taylor obtained a coutable Markov chain whose transition operator has spectrum equal to the Julia set of a quadratic map.

In this work, we will consider the stochastic adding machine associated to others bases of numeration, like the Fibonacci base, and we also define the stochastic Vershik map on Bratteli diagrams. We will show that the spectrum of their transition operators is connected to Julia sets in C^d, with $d \geq 2$. Particularly, if we have a stationary ordered Bratteli diagram, where its representation matrix is given by $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{N})$ with $abc > 0$, then the point spectrum of the transition operator S associated to its stochastic Vershik map is connected to the Julia set E defined by $E := \{z \in \mathbb{C} : (g_n(z))_{n\geq 1} \text{ is bounded}\}$ where $g : \mathbb{C}^2 \to \mathbb{C}^2$ is the map defined by $g(x,y) = \left(\frac{1}{p}x^a y^b - \left(\frac{1}{p} - 1\right), \frac{1}{p}x^c y^d - \left(\frac{1}{p} - 1\right)\right)$