Virtually Fibering Random Right-Angled Coxeter Groups

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Abstract: A group K *virtually algebraically fibers* if there is a finite index subgroup K' admitting a surjective homomorphism $K' \to \mathbb{Z}$ with finitely generated kernel. This notion arises from topology: a 3-manifold M is virtually a surface bundle over a circle precisely when the fundamental group of M virtually algebraically fibers, a well known result of Stallings. ses.

A *Right-Angled Coxeter group* (RACG) K is a group given by a presentation of the form

$$\langle x_1, x_2, \dots x_n \mid x_i^2, [x_i, x_j]^{\sigma_{ij}} \ : 1 \le i < j \le n \rangle$$

where $\sigma_{ij} \in \{0, 1\}$ for each $1 \le i < j \le n$. One can encode this information with a graph Γ_K whose vertices are the generators $x_1, \dots, x_n$ and $x_i \sim x_j$ if and only if $\sigma_{ij} = 1$. Conversely given a graph G on n vertices, we will denote the corresponding RACG by $K(G)$. Recently, Jankiewicz, Norin, and Wise developed a framework to show virtual fibering of a RACG using Betsvina-Brady Morse theory and ultimately translated the virtual fibering problem for K into a combinatorial game on the graph Γ_K. The method was successful on many special cases and also allowed them to construct examples where Betsvina-Brady cannot be applied to find a virtual algbraic fibering.

A natural question to consider is whether this approach is successful for a 'generic' RACG, i.e., given a probability measure μ_n on the set of RACG's of rank at most n, is it true that a.a.s. as $n \to \infty$, a group sampled from μ_n virtually algebraically fibers. This question is also considered in the work of Jankiewicz, Norin and Wise. Specifically they consider sampling Γ_K from the Erdős-Renyi random graph model $\mathcal{G}(n, p)$ and they prove the following result:

Theorem (Jankiewicz-Norin-Wise)**.** *Assume that*

$$\frac{(2 \log n)^{\frac{1}{2}} + \omega(n)}{n^{\frac{1}{2}}} \le p < 1 - \omega(n^{-2}),$$

and let G be sampled from $\mathcal{G}(n, p)$. Then, asymptotically almost surely, the associated Right-Angled Coxeter group $K(G)$ virtually algebraically fibers.

We extend this result to the smallest possible range of p, in fact we prove a hitting time type result. Namely we show that as soon as Γ_K has minimum degree 2 then a.a.s. K virtually algebraically fibers.

Theorem. *Let* $G_0, G_1, \ldots, G_{\binom{n}{2}}$ *denote the random graph graph process on* n *vertices where* $G_{i+1} = G_i \cup \{e_i\}$ *and* e_i *is picked uniformly at random from the non-edges of* G_i. *Let* $T = \min_t \{t \ : \delta(G_t) = 2\}$, *then a.a.s. the random graph process is such that* $K(G_m)$ *virtually algebraically fibers if and only if* $T \leq m < \binom{n}{2}$. *In particular for any* p *satisfying*

$$\frac{\log n + \log\log n + \omega(n)}{n} \leq p < 1 - \omega(n^{-2})$$

and G $\mathcal{G}(n, p)$, *the random Right-Angled Coxeter group* $K(G)$ *virtually algebraically fibers a.a.s.*

This is joint work with R. Glebov and I. Karpas.