Some results of analyticity and regularity for second order difference equations

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In

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the concept of maximal ℓ^p-regularity $(1 \leq p \leq \infty)$ was introduced in an analogous way to the continuous case studied in

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Following Blunck, let $\mathbb{X}$ be a (complex) Banach space and $A \in \mathcal{B}(\mathbb{X})$. We consider the first order Cauchy problem

$$\left\{ \begin{array}{rcl} u_{n+1} & = & Au_n + f_n, n \in \mathbb{Z}^+ \\ u_0 & = & x \in \mathbb{X}. \end{array} \right. \tag{1}$$

We denote by $u(x, f)$ the solution of (1) with initial condition $u_0 = x \in \mathbb{X}$ and inhomogeneous term $f : \mathbb{Z}^+ \to \mathbb{X}$. Let $(\Delta v)_n = v_{n+1} - v_n$.

Maximal Regularity

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We say that $A \in \mathcal{B}(\mathbb{X})$ has maximal ℓ^p-Regularity if the operator $f \mapsto \Delta u(0, f)$ is well defined and continuous on $\ell^p(\mathbb{Z}^+, \mathbb{X})$.

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it was shown that every powerbounded operator $A \in \mathcal{B}(\mathbb{X})$ (that is, $\sup_{n \in \mathbb{Z}^+} \|A^n\| < \infty$) that has Maximal ℓ^p-regularity $(1 < p < \infty)$ is analytic in Ritt sense (that is, $n \mapsto n(A - I)A^n$ is bounded), which can be classified as follows:

Theorem - Blunck (2001)

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Let $A \in \mathcal{B}(\mathbb{X})$. The following conditions are equivalent:

a) A is powerbounded and analytic;

b) $t \mapsto e^{t(A-I)}$ is a bounded analytic semigroup and $\sigma(A) \subset \mathbb{D}(0,1) \cup \{1\}$;

c) $\{(\lambda - 1)R(\lambda, A); |\lambda| > 1\}$ is bounded;

d) A is powerbounded and $\{(\lambda - 1)R(\lambda, A); |\lambda| = 1, \lambda \neq 1\}$ is bounded.

Characterization of maximal ℓ^p-Regularity - Blunck (2001)

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Let $\mathbb{X}$ be an UMD space and let $A \in \mathcal{B}(\mathbb{X})$ be powerbounded and analytic. The following conditions are equivalent:

a) A has maximal ℓ^p-regularity ($p \in (1, \infty)$);
b) $\{(\lambda - 1)R(\lambda, A); |\lambda| = 1, \lambda \neq 1\}$ is R-bounded;
c) $\{A^n, n(A - I)A^n; n \in \mathbb{Z}^+\}$ is R-bounded.

Important fact

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c) $\{A^n, n(A - I)A^n; n \in \mathbb{Z}^+\}$ is R-bounded.

Important fact

A detailed study on the analyticity of the functions $\lambda \mapsto R(\lambda, A)$ and $\lambda \mapsto (\lambda - 1)R(\lambda, A)$ is of fundamental importance in the characterization of maximal regularity above.

O Estudo de equações de segunda ordem

In

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it was studied maximal ℓ^p-regularity of the equation

$$\begin{cases} \Delta^2 u_n - (I - T) &= f_n, n \in \mathbb{Z}^+ \\ u_0 &= 0 \\ \Delta u_0 &= 0, \end{cases} \tag{2}$$

where $T \in \mathcal{B}(\mathbb{X})$ is powerbounded and analytic:

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Characterization of maximal ℓ^p-regularity - Cuevas (2007)

Let $\mathbb{X}$ be an UMD space, $p \in (1, \infty)$ and assume that $T \in \mathcal{B}(\mathbb{X})$ is powerbounded and analytic. The following conditions are equivalent:

a) T has maximal ℓ^p-regularity of order 2;

b) $\{(\lambda - 1)^2 R((\lambda - 1)^2, I - T); |\lambda| = 1, \lambda \neq 1\}$ is R-bounded.

Objective

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Our objective is to obtain analytical results for a special complex function related to the second-order equation

$$\left\{\begin{array}{rcl} u_{n+2} & = & Bu_{n+1} + Au_n + f_n, \, n \in \mathbb{Z}^+ \\ u_0 & = & x \in \mathbb{X} \\ \Delta u_0 & = & y \in \mathbb{X}, \end{array}\right. \tag{3}$$

where $A, B \in \mathcal{B}(\mathbb{X})$, and use them to study maximal ℓ^p-regularity for equation (3).

Remark

If $B = 2I$ and $A = -T$, the equation (3) becomes (2).

We denote by $u(x, y, f)$ the solution of (3) with initial conditions $u_0 = x$ and $\Delta u_0 = y$ and inhomogeneous term f.

(A, B)-Sine Family

We denote by $u(x, y, f)$ the solution of (3) with initial conditions $u_0 = x$ and $\Delta u_0 = y$ and inhomogeneous term f.

(A, B)-Sine Family

(A, B)-**sine family** $(S(n))_{n\in\mathbb{Z}^+} \subset \mathcal{B}(\mathbb{X})$: $S(n)x = u_n(0, x, 0)$.
(A, B) is the generator of $n \mapsto S(n)$.
(A, B)-**cosine family** $(C(n))_{n\in\mathbb{Z}^+} \subset \mathcal{B}(\mathbb{X})$:
$C(n) = S(n+1)x + S(n)(B+I)x$.

In order to obtain a integral representation of these families, we consider the function $\lambda \mapsto H(\lambda)^{-1}$, where $H(\lambda) = z^2 - zB - A$ and the sets $\rho(A, B) = \{z \in \mathbb{C}; H(\lambda)^{-1} \in \mathcal{B}(\mathbb{X})\}$ and $\sigma(A, B) = \mathbb{C} \setminus \rho(A, B)$.

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Lema

$\sigma(A, B)$ is compact. Moreover, $\lambda \mapsto H(\lambda)^{-1}$ is analytic in $\rho(A, B)$ and there exists $R > 0$ such that
$\lambda \mapsto \lambda H(\lambda)^{-1} \in H^\infty(\mathbb{C} \setminus \overline{\mathbb{D}(0, R)})$.

Integral representation

Integral representation

- $S(n) = \dfrac{1}{2\pi i} \displaystyle\int_\Gamma z^n H(z)^{-1} dz;$
- $C(n) = \dfrac{1}{2\pi i} \displaystyle\int_\Gamma z^n H(z)^{-1} [(z+1)I - B] dz,$

where $\Gamma \subset \rho(A, B)$ is a path around $z_0 = 0$ and $\sigma(A, B)$.

Parameter Variation Formula

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- $C(n) = \dfrac{1}{2\pi i}\displaystyle\int_\Gamma z^n H(z)^{-1}[(z+1)I - B] dz,$

where $\Gamma \subset \rho(A, B)$ is a path around $z_0 = 0$ and $\sigma(A, B)$.

Parameter Variation Formula

Let $A, B \in \mathcal{B}(\mathbb{X})$. The unique solution for (3) is given by

$$u_n(x, y, f) = C(n)x + S(n)y + (S * f)_{n-1}, n \in \mathbb{Z}^+ \setminus \{0\},$$

where $(S * f)_n = \displaystyle\sum_{k=0}^{n} S(n-k) f_k.$

The regions $\sum(c, \theta)$ e $\mathcal{D}(c, \eta)$

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In order to obtain analyticity results for the family $n \mapsto S(n)$, we consider the following regions:

- $\sum(c,\theta) = \{z \in \mathbb{C}; 0 < |arg(z-c)| < \theta\}$, where $\theta \in (0,\pi]$ and $c > 0$ (sector with center c and angle θ);
- $\mathcal{D}(c,\eta) = \{z \in \mathbb{C}; Re(z) - c > -\eta^2 Im(z)^2\}$ (parabolic sector with center $c > 0$).

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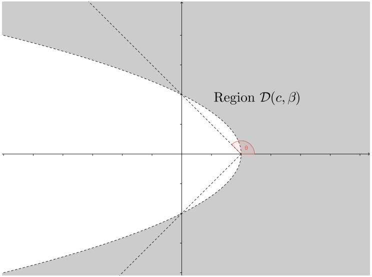

Proposition - Analyticity of (A, B)-Sine family (version 1)

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Assume that $\{z \in \mathbb{C}; Re(z) \geq c, z \neq c\} \subset \rho(A, B)$ and $\|(\lambda - c)H(\lambda)^{-1}\|_{\mathcal{B}(\mathbb{X})} \leq M$ on $\{z \in \mathbb{C}; Re(z) \geq c, z \neq c\}$ for some $c > 0$ and some $M > 0$. Then, there exist $\theta \in (\frac{\pi}{2}, \pi)$ and $L > 0$ such that $H(\cdot)^{-1}$ admits an analytic extension on $\sum(c, \theta)$ and

$$\|(\lambda - c)H(\lambda)^{-1}\|_{\mathcal{B}(\mathbb{X})} \leq L$$

for all $\lambda \in \sum(c, \theta)$.

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$$M_\delta = max\left\{\delta^2 + M[1 + \|A\|_{\mathcal{B}(\mathbb{X})}], M\delta + M[1 + \|A\|_{\mathcal{B}(\mathbb{X})}]\right\},$$

$H(\cdot)^{-1}$ admits an analytic extension on $\mathcal{D}(c, \beta) \cap \{z \in \mathbb{C}; |Im(z)| \leq \delta\}$. In addition, there exists $L = L(\delta, \beta(\delta)) > 0$ such that

$$\|(\lambda - c)^2 H(\lambda)^{-1}\|_{\mathcal{B}(\mathbb{X})} \leq L$$

for all $\lambda \in \mathcal{D}(c, \beta) \cap \{z \in \mathbb{C}; |Im(z)| \leq \delta\}$.

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R-Boundedness

A set $\tau \subset \mathcal{B}(\mathbb{X})$ is R-bounded if there exists $C > 0$ such that for all $n \in \mathbb{Z}^+$, $T_1, \cdots, T_n \in \tau$ and $x_1, \cdots x_n \in \mathbb{X}$,

$$\int_0^1 \left\| \sum_{k=0}^{n} r_k(t) T_k x_k \right\| dt \leq C \int_0^1 \left\| \sum_{k=0}^{n} r_k(t) x_k \right\| dt,$$

where $(r_k)_{k \in \mathbb{Z}^+}$ is a sequence of random functions $r_k : [0,1] \to \{1, -1\}$.

Some facts about R-boundedness

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- R-boundeness $\Rightarrow$ boundedness. The reciprocal is true in Hilbert spaces;
- If τ is R-bounded, then $\overline{\tau}$ is R-bounded;
- If τ is R-bounded, then the absolute convex hull of τ is R-bounded.

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$$\left\{(\lambda - c)H(\lambda)^{-1}; \lambda \in \sum(c, \gamma)\right\}$$

is R-bounded for some $\gamma \in (\frac{\pi}{2}, \pi)$.

Proposition - R-Analyticity of (A, B)-Sine family (version 2)

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Assume that $\{z \in \mathbb{C}; Re(z) \geq c, z \neq c\} \subset \rho(A, B)$ and $||(\lambda - c)^2 H(\lambda)^{-1}||_{\mathcal{B}(\mathbb{X})} \leq M$ on $\{z \in \mathbb{C}; Re(z) = c, z \neq c\}$ for some $c > 0$ and $M > 0$. If $\mathcal{U} = \left\{(\lambda - c)^2 H(\lambda)^{-1}; Re(\lambda) = c, \lambda \neq c\right\}$ is R-bounded, then, for each $\delta > 0$, the set

$$\left\{(\lambda - c)^2 H(\lambda)^{-1}; \lambda \in \mathcal{D}(c, \gamma), |Im(z)| \leq \delta\right\}$$

is R-bounded for all $\gamma \in \left(0, \frac{1}{N_\delta}\right)$, where
$N_\delta = max\{\delta^2 + R(\mathcal{U})[||A||_{\mathcal{B}(\mathbb{X})} + 1], \delta R(\mathcal{U}) + R(\mathcal{U})[||A||_{\mathcal{B}(\mathbb{X})} + 1]\}$.

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UMD Space

A Banach space $\mathbb{X}$ is an UMD space if the Hilbert Transform $H : S(\mathbb{R}, \mathbb{X}) \to S(\mathbb{R}, \mathbb{X})$ given by

$$Hf(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{f(s)}{s-t} ds$$

admits a bounded extension on $L^p(\mathbb{R}, \mathbb{X})$ for some $p \in (1, \infty)$.

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We say that (A, B) has maximal ℓ^p-regularity of order $m \in \{1, 2\}$ if the operator $f \mapsto \Delta^m u(0, 0, f)$ is well defined and continuous on $\ell^p(\mathbb{Z}^+, \mathbb{X})$.

Remark

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Remark

Using Parameter Variation Formula, the problem of maximal ℓ^p-regularity of order m of (A, B) is equivalent to show that the operator $f \mapsto \Delta^m S * f$ belongs to $\mathcal{B}(\ell^p(\mathbb{Z}^+, \mathbb{X}))$.

Theorem

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Let $A, B \in \mathcal{B}(\mathbb{X})$, $p \in [1, \infty]$ and suppose that $S : \mathbb{Z}^+ \to \mathcal{B}(\mathbb{X})$ is bounded. If (A, B) has maximal ℓ^p-regularity of order $m \in \{1, 2\}$, then:

a) $n||\Delta^m S(n)x||_{\mathbb{X}} \leq L||x||_{\mathbb{X}}$ for some $L > 0$ and for all $x \in \mathbb{X}$ and $\{\lambda \in \mathbb{C}; |\lambda| \geq 1, \lambda \neq 1\} \subseteq \rho(A, B)$;

b) if $p \in (1, \infty)$, there exists $M > 0$ such that $\left|\left|(\lambda - 1)^m H(\lambda)^{-1}\right|\right|_{\mathcal{B}(\mathbb{X})} \leq M$, for all $|\lambda| > 1$.

Characterization of maximal ℓ^p-regularity of order 1

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Let $\mathbb{X}$ be an UMD space, $A, B \in \mathcal{B}(\mathbb{X})$, $p \in (1, \infty)$ and assume that $S : \mathbb{Z}^+ \to \mathcal{B}(\mathbb{X})$ is bounded. In addition, suppose that exists $M > 0$ such that $\left|\left|(\lambda - 1)H(\lambda)^{-1}\right|\right|_{\mathcal{B}(\mathbb{X})} \leq M$, for all $|\lambda| > 1$. The following conditions are equivalent:

a) (A, B) has maximal ℓ^p-regularity of order 1;
b) $\{z \in \mathbb{C}; |z| = 1, z \neq 1\} \subset \rho(A, B)$ and the set $\{(\lambda - 1)H(\lambda)^{-1}; |\lambda| = 1, \lambda \neq 1\}$ is R-bounded.
c) $\{S(n), \Delta S(n); n \in \mathbb{Z}^+\}$ is R-bounded.

The study of maximal regularity of order 1 is more complicated. In the special case $B = 2I$, we have the following results:

Characterization of maximal ℓ^p-regularity of order 2 of $(A, 2I)$

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Let $\mathbb{X}$ be an UMD space, $A \in \mathcal{B}(\mathbb{X})$, $p \in (1, \infty)$ and assume that $S : \mathbb{Z}^+ \to \mathcal{B}(\mathbb{X})$ is bounded. In addition, suppose that exists $M > 0$ such that $||(\lambda - 1)^2 H(\lambda)^{-1}||_{\mathcal{B}(\mathbb{X})} \leq M$, for all $|\lambda| > 1$ and

$$\sup \left\{ ||(\lambda - 1)^2 H(\lambda)^{-1}||_{\mathcal{B}(\mathbb{X})} ; z = 1 + \eta i, \eta \in [-r, 0) \cup (0, r] \right\} \leq \frac{2}{1 + ||A||_{\mathcal{B}(\mathbb{X})}} \quad (4)$$

for some $r > 0$. The following conditions are equivalent:

a) $(A, 2I)$ has maximal ℓ^p-regularity;

b) $\{z \in \mathbb{C}; |z| = 1, z \neq 1\} \subset \rho(A, B)$ and the set $\{(\lambda - 1)^2 H(\lambda)^{-1}; |\lambda| = 1, \lambda \neq 1\}$ is R-bounded.

Sufficient Conditions for maximal regularity of order 2 of $(A, 2I)$

Let $\mathbb{X}$ be an UMD space, $A \in \mathcal{B}(\mathbb{X})$, $p \in (1, \infty)$ and assume that $S : \mathbb{Z}^+ \to \mathcal{B}(\mathbb{X})$ is bounded. If $\mathcal{R}(\{S(n); n \geq n_0\}) \leq \dfrac{1}{8(1 + ||A||_{\mathcal{B}(\mathbb{X})})}$ for some $n_0 \in \mathbb{Z}^+$ and $n \mapsto n\Delta^2 S(n)$ is bounded, then $(A, 2I)$ has maximal ℓ^p-regularity of order 2 for all $p \in (1, \infty)$.

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A linear operator $T \in \mathcal{B}(L^q)$ is a subpositive contraction if there exists a contraction S such that $|T(f)| = S(|f|)$.

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Example

Every integral operator $T(f)(x) = \int_\Omega K(x,y)f(y)dy$, with $\sup |K(x,y)| \leq 1$, is a subpositive contraction.

It is well known that if $T \in \mathcal{B}(\mathbb{X})$ is powerbounded and analytic, then $||(\lambda - 1)R(\lambda, T)|| \leq M$ for all $\lambda \in \sum(1, \theta)$, $\theta \in (\frac{\pi}{2}, \pi)$.

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Theorem

If $T \in \mathcal{B}(L^q)$, $q \in (1, \infty)$, is a subpositive analytic contration, with angle $\theta \in (\frac{\pi}{2}, \pi)$ and $\dfrac{e^{2it}}{be^{it} + a} \in \sum(1, \theta)$, then the pair (aT, bT) has maximal ℓ^p-regularity of order 1, for all $p \in (1, \infty)$.

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MUITO OBRIGADO!!!