Global existence of strong solutions for the flow of magnetic fluids in exterior three-dimensional domains

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Resumo: We discuss the main ideas involved in proving a recent result (Oliveira, J.C., Acta Appl. Math. (2018)) of global existence of strong solutions for a system of nonlinearly coupled PDEs that model the flow of magnetic fluids in exterior three-dimensional domains.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\ \mathbf{u} - \eta\Delta\mathbf{u} + \nabla p = \mu_0\,\mathbf{M}\cdot\nabla\mathbf{H} + \frac{\mu_0}{2}\,curl\,(\mathbf{M}\times\mathbf{H}) \quad \text{in } Q \tag{1}$$

$$\text{div}\,\mathbf{u} = 0 \quad \text{in } Q \tag{2}$$

$$\frac{\partial \mathbf{M}}{\partial t} + (\mathbf{u}\cdot\nabla)\ \mathbf{M} + \frac{1}{\delta}\ (\mathbf{M} - \chi_0\,\mathbf{H}) = \frac{1}{2}\,[curl\mathbf{u}]\times\mathbf{M} - \beta_0\,\mathbf{M}\times(\mathbf{M}\times\mathbf{H}) \quad \text{in } Q \tag{3}$$

$$\text{div}\ (\mathbf{H}+\mathbf{M}) = F \quad \text{in } Q \tag{4}$$

$$curl\mathbf{H} = 0 \quad \text{in } Q \tag{5}$$

$$\mathbf{u}(0) = \mathbf{u}_0,\ \nabla\cdot\mathbf{u}_0 = 0,\ \mathbf{M}(0) = \mathbf{M}_0 \quad \text{in } \Omega \tag{6}$$

$$\mathbf{u} = 0,\ \mathbf{H}\cdot\mathbf{n} = -\mathbf{M}\cdot\mathbf{n} \text{ in } \Sigma \tag{7}$$

where $\Omega \subset \mathbb{R}^3$ is an exterior domain, simply-connected with regular boundary, $Q = \Omega\times(0,\infty)$, $\Sigma = \partial\Omega\times(0,\infty)$, and $\mathbf{n}$ denotes the unit vector normal to the boundary and pointing outwards.