(Semilinear) Parabolic Equations with Unbounded Attractors

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Outline

1. 1-D dissipative reaction-diffusion equations
2. Non-dissipative equations with unbounded attractors
3. Additional results

Outline

Scalar reaction-diffusion equations

$$\begin{cases} u_t = u_{xx} + f(x,u,u_x)\ ,\ \ x \in [0,\pi] \\ u_x(t,0) = u_x(t,\pi) = 0 \\ u(0,x) = u_0(x), \quad f \in C^2, \end{cases}$$

- Defines a (local) C^1-semiflow $(t, u_0) \mapsto u(t,\cdot) \in X$.
- By Sobolev embedding

$$X = H^2([0,\pi]) \cap \{u_x(0), u_x(\pi) = 0\} \subset C^1.$$

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- f is dissipative: fixed large ball in X absorbing any solution $u(t)$, for $t > t_0(u_0)$.
- Explicit sufficient conditions are, for instance,

$$f(x,u,0).u < 0, \text{ for } |u| \text{ large enough},$$

and

$$|f(x,u,p)| \leq c(1+|p|^\gamma),$$

with $c > 0$ and $0 \leq \gamma < 2$, uniformly for x and u in compact sets: Amann 85.

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Global attractor

- By dissipativeness, the semiflow possesses a global attractor $\mathcal{A}$.
- $\mathcal{A} \subset X$ is the maximal compact invariant subset.
- General references on global attractors:
  - Evolution equations: Hale 88, Babin-Vishik 92, Hale-Magalhães-Oliva 02.
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- The semiflow has a gradient structure due to the existence of a Lyapunov function V of the form

$$V(u) = \int_0^{\pi} h(x, u, u_x)dx.$$

- V is strictly decreasing along nonequilibrium solutions $u = u(t, \cdot)$.
- Any trajectory $u(t)$, $t \geq 0$, which is bounded in X converges to some steady state solution as $t \to \infty$.
  - Matano 78, Matano 88, Zelenyak 68.
- $\mathcal{A}$ is gradient-like and with an associated Morse decomposition.

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where E denotes the set of equilibria and

$$\mathcal{H} := \cup_{v,w \in E} C(v,w)$$

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- Hyperbolicity of all the equilibria guarantees finiteness and nondegeneracy for E, then the global attractor is given as

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The nodal properties of solutions play an important role on the analysis of the heteroclinics set $\mathcal{H}$.

- For any $\tilde{u} \in X$ let $z(\tilde{u})$, the zero number of $\tilde{u}$, denote the number of strict sign changes of

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- The PDE information is contained in the connecting orbits $C(v, w)$. The main goal is to determine precisely which equilibria are connected, using only information on E.

- Consider the IVP

$$u_{xx} + f(x, u, u_x) = 0$$
$$u(0) = a, \ \ u_x(0) = 0.$$

The manifold of solutions

$$M_f = \{(x, u, v) \in \mathbb{R}^3 : u = u(x; a), v = u_x(x; a)\}$$

is a smooth bidimensional manifold determining the dynamics on the attractor.

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- The exact criterion, based on the permutation σ, for connections was obtained: Fiedler-Rocha 96, Wolfrum 01.

- The global attractor is given by $\mathcal{A} = E \cup \mathcal{H}$ where the heteroclinic set $\mathcal{H}$ is completely determined by σ:

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Non-dissipative reaction-diffusion equations

- **fast non-dissipative equation**: singularities may develop after finite time.
- **slowly non-dissipative equation**: longtime existence without dissipativity features trajectories which escape to infinity in infinite time (grow-up).
  - Non-dissipative nonlinearities with slow growth.
  - An elementary but instructive example:

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Slowly non-dissipative example

Consider the linear equation

$$u_t = u_{xx} + bu, \quad b > 0$$

for $0 < x < \pi$ with Neumann boundary conditions.

- Any solution $u(t)$ either converges to zero or goes to infinity as $t \to \infty$, being attracted to the $([\sqrt{b}] + 1)$-dimensional eigenspace E_+.
- It is natural to define the attractor as the invariant subspace E_+, i.e., an unbounded set.
- Unbounded attractors of evolution equations: Chepyzhov-Goritskii 92.

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- A positive linear growth for f is sufficient to ensure grow-up with no blow-up

$$f(x, u, u_x) = bu + g(x, u, u_x)$$

$$b > 0, \quad g : [0, \pi] \times \mathbb{R}^2 \to \mathbb{R}^2 \text{ is a bounded } C^2 \text{ function}$$

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- Existence of unbounded global solution: no compactness for the attractor.
- Unbounded global attractor: nonempty minimal set which is positively invariant and attracts all bounded subsets.
- Gradient structure (Lyapunov functional): any bounded solution converges forwards in time to some (bounded) equilibrium.
- Any normalized unbounded solution converges to an eigenvalue $\varphi_k(x)$ of $-\partial_{xx} - bI$,

$$\lim_{t\to\infty} \frac{u(t,\cdot)}{\|u(t,\cdot)\|} = \pm\varphi_k(\cdot), \text{ in the } L^2\text{-norm.}$$

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Poincaré Projection

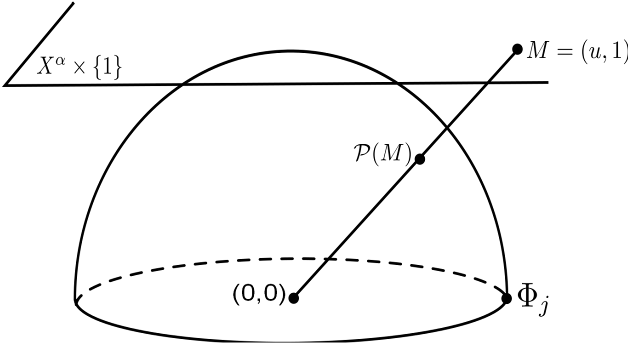

Figure: Projection of $M = (u, 1) \in X^\alpha \times \{1\}$.

- $\mathcal{H} = \{(\chi, z) \in X^\alpha \times \mathbb{R} | \langle \chi, \chi \rangle_\alpha^2 + z^2 = 1, z \geq 0\}$.

Equilibria at infinity

- $\Phi_j^{\pm}$ are equilibrium points on the *sphere at infinity*.

- We thus define objects $\Phi_j^{\pm,\infty}$ at infinity as

$$\mathcal{P}(\Phi_j^{\pm,\infty}) = \Phi_j^{\pm}, \text{ for } j = 0, 1, ..., [\sqrt{b}],$$

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The unbounded global attractor

Theorem (P.-Rocha, J. Dyn. Diff. Eq. (2016))

The unbounded global attractor $\mathcal{A}$ related to the SND problem is composed by the set of equilibria and their heteroclinic connections,

$$\mathcal{A} = E^b \cup E^\infty \cup \{\textit{heteroclinic connections}\}.$$

Moreover, there is a permutation σ of the equilibria providing the criteria to describe the heteroclinics set.

- Asymptotic analysis for $g = g(u)$: Ben-Gal (2010)
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Unbounded global attractor related to σ

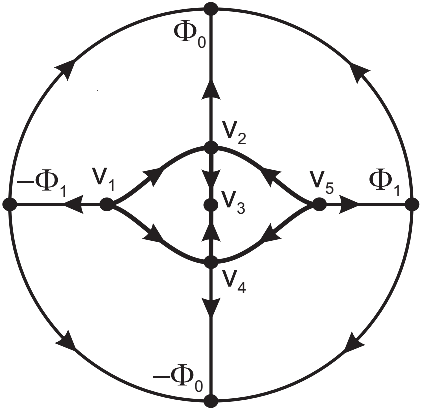

Figure: Unbounded global attractor with permutation $\sigma = \{5, 2, 3, 4, 1\}$.

Outline

Periodic Slowly Non-dissipative Problem

Theorem (P. , SIAM J. Math. Anal. (2016))

The non-compact global attractor $\mathcal{A}$ related to the periodic SND problem decomposes into equilibria, equilibria at infinity, periodic orbits, frozen waves at infinity, and heteroclinics,

$$\mathcal{A} = \mathcal{E}^b \cup \mathcal{R}^b \cup \mathcal{E}^\infty \cup \mathcal{R}^\infty \cup \{\textit{heteroclinic connections}\}.$$

Moreover, the heteroclinics set can be described using only information on nodal properties.

Stability of unbounded attractors

Theorem (Carvalho-P., Proc. Royal Soc. of Edinb. A (2017))

Unbounded attractors of slowly non-dissipative equations are stable under small autonomous $b = b_\epsilon$ and non-autonomous perturbations $b = b(t)$.

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- Two-parameter Chafee-Infante equation

$$u_t = u_{xx} + bu - \epsilon u^3.$$

- Consider the one-parameter family of dissipative PDEs, as the parameter $\epsilon \to 0$

$$u_t = u_{xx} + bu + g(u) - \epsilon u^{2n+1}.$$

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The compact global attractors

$$\mathcal{A}_\epsilon = \mathcal{E}_\epsilon^u \cup \mathcal{E}_\epsilon^b \cup \mathcal{H}$$

converge to the unbounded attractor $\mathcal{A}$ as $\epsilon \to 0$, in compact sets $K \subset X^\alpha$. However, each nonconstant equilibrium solution $\phi_j^{\pm\epsilon} \in \mathcal{E}_\epsilon^u$, does not converge to the equilibrium at infinity $\Phi_j^\pm \in \mathcal{A}$, as $\epsilon \to 0$.

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Theorem (Bruschi,Carvalho-P., Indiana U. Math. J. (2018))

The compact global attractors

$$\mathcal{A}_\epsilon = \mathcal{E}_\epsilon^u \cup \mathcal{E}_\epsilon^b \cup \mathcal{H}$$

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Quasilinear equations with unbounded attractors

- Lappicy-P. (2018): construct explicitly unbounded attractors of quasilinear parabolic equations

$$u_t = a(x, u, u_x)u_{xx} + bu + f(x, u, u_x)$$

  - Take linearizations around the N bounded equilibria $\mathcal{E}$;
  - $\mathcal{E}^\infty = \{\Phi^{\pm}_{i,k} : 1 \leq i \leq N, 0 \leq k \leq N_i\}$ set at infinity;
  - N_i is the number of unstable directions for the equilibrium $e_i \in \mathcal{E}$;
  - The attractor also decomposes into equilibria $\mathcal{E} \cup \mathcal{E}^\infty$ and heteroclinics;
  - N Chafee-Infante-like structures at the sphere at infinity.

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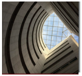

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Workshop for Women in Differential Equations

UFABC - Santo André, Brazil, July 25-27
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