UNIVERSIDADE FEDERAL DE SANTA CATARINA

DEPARTAMENTO DE MATEMÁTICA

S U M Á R I O

Requerente(s): **Prof. Antônio Carlos Gardel Leitão**

Título do Projeto: **Tópicos em Problemas Inversos IV**

Assunto: **Relatório Final de Projeto de Pesquisa.**

SERVIÇO PÚBLICO FEDERAL
MINISTÉRIO DA EDUCAÇÃO
UNIVERSIDADE FEDERAL DE SANTA CATARINA

SÍNTESE DO PROJETO DE PESQUISA

Situação: Relatório Final em aprovação

Número: 201805587

1. Título:

TOPICOS EM PROBLEMAS INVERSOS IV

2. Resumo:

O presente projeto visa dar continuidade `a pesquisa cientifica por mim desenvolvida (juntamente com varios colaboradores) nas areas de analise numerica e problemas inversos, mais especificamente: metodos iterativos para identificacao de parametros.

PARA MAIORES DETALHES, VER ARQUIVO PDF ANEXO.

Palavras-chave:

PROBLEMAS INVERSOS; IDENTIFICACAO DE PARAMETROS; METODOS ITERATIVOS;;ANALISE NUMERICA;

3. Coordenador:

Nome: Antonio Carlos Gardel Leitao
Departamento: MTM/CFM - DEPARTAMENTO DE MATEMÁTICA / MTM/CFM
Tipo: Professor
Regime de Trabalho: DE
Valor Mensal: Sem remuneração
Forma de Remuneração: Sem bolsa
Carga Horária Semanal: 20.00h

4. Entidades Participantes:

Financiadores:
Valor Total: R$ 0,00
Fundações:
Instituições não financiadoras envolvidas no projeto: IMPA; UFRJ
Tipo de Instrumento Contratual: Não será celebrado instrumento jurídico com a UFSC.

5. Período:

Previsão de Inicio: 01/05/2018
Início Efetivo: 01/05/2018
Duração: 36 Meses
Término: 01/05/2021
Aprovação: 07/05/2018

6. Área do Projeto:

Grupo de Pesquisa:

7. Comitê de Ética:

Não se aplica;

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SÍNTESE DO PROJETO DE PESQUISA

Situação: Relatório Final em aprovação

Número: 201805587

8. Equipe do Projeto:

CPF / Nome	Tipo	Período	Depto/Curso	Valor Mensal / Valor Total	Teto Excedid	Carga Hora. Semanal	Paad	Situação
Antonio Carlos Gardel Leitao 012.830.017-56	Professor Coordenador	01/05/2018 à 01/05/2021	MTM/CFM - DEPARTAMENTO DE MATEMÁTICA / MTM/CFM	R$ 0,00 / R$ 0,00		20.00h	Sim	Aprovado

Número total de participantes na equipe do projeto: 1

0 externos à UFSC (0,00%)

1 vinculados à UFSC (100,00%)

SERVIÇO PÚBLICO FEDERAL
MINISTÉRIO DA EDUCAÇÃO
UNIVERSIDADE FEDERAL DE SANTA CATARINA

SÍNTESE DO PROJETO DE PESQUISA

Situação: Relatório Final em aprovação

Número: 201805587

9. Financiamento:

Não se aplica.

10. Propriedade Intelectual:

Não se aplica.

11. Relatório Final:

Data efetiva de término: 01/05/2021

Tipo		Descrição
Produção bibliográfica	Artigos em periódicos internacional	2 artigos submetidos;

Receita total (inclui rendimento): R$ 0,00
Despesa realizada: R$ 0,00
Saldo: R$ 0,00

Atividades propostas para o projeto foram concluidas de forma satisfatória.

Descriçao detalhada dessas atividades pode ser encontradas no ARQUIVO PDF ANEXO: "21-04-15mtm-aleitao.pdf"

12. Movimentações:

Data	Responsável	Ação	Notificados	Comentários
13/04/2018 - 11:26h	Antonio Carlos Gardel Leitao	Enviou o projeto para aprovação	Cleverson Roberto da Luz	
13/04/2018 - 11:26h	Antonio Carlos Gardel Leitao	Criou o projeto		
16/04/2018 - 12:11h	Cleverson Roberto da Luz	Solicitou alterações	Antonio Carlos Gardel Leitao	
25/04/2018 - 19:54h	Antonio Carlos Gardel Leitao	Reenviou o projeto para aprovação	Cleverson Roberto da Luz	
01/05/2018 - 10:21h	Cleverson Roberto da Luz	Aprovou o projeto	Aldrovando Luis Azeredo Araujo	
07/05/2018 - 16:08h	Aldrovando Luis Azeredo Araujo	Aprovou o projeto	Antonio Carlos Gardel Leitao	
01/04/2021 - 06:00h		Prazo do projeto de pesquisa quase encerrado	Antonio Carlos Gardel Leitao, formulariopesquisa@contato.ufsc.br	
01/04/2021 - 10:13h	Antonio Carlos Gardel Leitao	Enviou relatório final para aprovação	Cleverson Roberto da Luz	

Relatório de atividades

Modalidade: Projeto de Pesquisa MTM-UFSC

Período de Execução: 01/Mai/2018 a 30/Abr/2021

Título: Tópicos em Problemas Inversos IV

Aprovado em ata no: 238

Coordenador: Antonio Leitão

1 Apresentação:

O presente projeto visou dar continuidade à pesquisa científica por mim desenvolvida (juntamente com vários colaboradores) nas áreas de **análise numérica** e **problemas inversos**, mais especificamente: **métodos iterativos para identificação de parâmetros**.

2 Objetivos propostos:

O foco das investigações científicas do projeto concluído foi voltado a análise dos denominados **problemas inversos e mal-postos**, e teve como objetivos principais:

i) Investigação de questões teóricas (tais como existência, estabilidade, regularidade, ...) relacionadas a modelos matemáticos específicos (ver, e.g., [27, 30, 31, 33, 34, 35]);

ii) Análise de métodos numéricos estáveis para obtenção de soluções aproximadas para problemas mal-postos (ver, e.g., [29, 28, 25, 26, 32].

3 Objetivos alcançados e bibliografia:

A seguir são listadas as publicações obtidas como conseqëncia das pesquisa realizada nesse projeto.

References

Artigos submetidos:

[1] F. Filippozzi, E. Hafemann, J. Rabelo, F. Margotti, A. Leitão. *Range-relaxed criteria for choosing the Lagrange multipliers in Levenberg Marquardt Kaczmarz type methods,* submitted

[2] J. Rabelo, Y. Saporito, A. Leitão. *On randomized Kaczmarz type methods for solving large scale systems of linear ill-posed equations,* submitted

Artigos aceitos para publicação:

[3] A. Leitão, F. Margotti, B. Svaiter. *Range-relaxed criteria for choosing the Lagrange multipliers in Levenberg-Marquardt method,* IMA Journal of Numerical Analysis (2021), to appear (doi:10.1093/imanum/draa050)

[4] F. Filippozzi, J. Rabelo, R.Boiger, A. Leitão. *Range-relaxed criteria for choosing the Lagrange multipliers in iterated-Tikhonov Kaczmarz type methods for solving systems of ill-posed operator equations,* Inverse Problems (2021), to appear (doi:10.1088/1361-6420/abc233)

Artigos publicados:

[5] J.A.M. Valle, A. Madureira, A. Leitão. *A Computational Approach for the inverse problem of neuronal conductances determination,* Journal of Computational Neuroscience 48 (2020), no. 3, 281-297 (doi:10.1007/s10827-020-00752-7)

[6] M.P. Machado, F. Margotti, A. Leitão. *On the choice of Lagrange multipliers in the iterated Tikhonov method for linear ill-posed equations in Banach spaces,* Inverse Problems in Science and Engineering 28 (2020), no. 6, 796-826 (doi:10.1080/17415977.2019.1662001)

[7] M.P. Machado, F. Margotti, A. Leitão. *On nonstationary iterated Tikhonov methods for ill-posed equation in Banach spaces,* In: New Trends in Parameter Identification for Mathematical Models, Ed.: B. Hofmann et al., Birkhäuser Basel, 2018, 175-193 (doi:10.1007/978-3-319-70824-9_10)

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Outras referências bibliográficas:

[11] B. Hofmann, A. Leitão, J.P. Zubelli. *New Trends in Parameter Identification for Mathematical Models,* Series Title: Trends in Mathematics, Birkhäuser Basel, 2018

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Florianópolis, 5 de Abril de 2021.

A. Leitão.

Journal of Computational Neuroscience
https://doi.org/10.1007/s10827-020-00752-7

A computational approach for the inverse problem of neuronal conductances determination

Jemy A. Mandujano Valle[1] · Alexandre L. Madureira[1,2] · Antonio Leitão[3]

Received: 10 October 2018 / Revised: 12 May 2020 / Accepted: 18 May 2020

Abstract

The derivation by Alan Hodgkin and Andrew Huxley of their famous neuronal conductance model relied on experimental data gathered using the squid giant axon. However, the experimental determination of conductances of neurons is difficult, in particular under the presence of spatial and temporal heterogeneities, and it is also reasonable to expect variations between species or even between different types of neurons of the same species.

We tackle the inverse problem of determining, given voltage data, conductances with non-uniform distribution in the simpler setting of a passive cable equation, both in a single or branched neurons. To do so, we consider the minimal error iteration, a computational technique used to solve inverse problems. We provide several numerical results showing that the method is able to provide reasonable approximations for the conductances, given enough information on the voltages, even for noisy data.

Keywords Cable equation · Conductances determination · Inverse problems · Iterative regularization methods

1 Introduction

The seminal model of Hodgkin and Huxley (Hodgkin and Huxley 1952) of neuronal voltage conductance describes how action potential occurs and propagates. It is a landmark model and presents an outstanding combination of modeling based on physical arguments and experimental data, needed to determine the behavior of ion channels. As a part of their work, they modeled the macroscopic behavior of the conductances by designing several mathematical functions (the α's and β's) that make the computed voltage to behave as the data. In this paper, we propose a numerical procedure to approximate the conductances of the ion channels, using an iterative method to obtain the unknown parameters.

Finding the conductances is crucial if one wants to emulate the neuronal voltage propagation using computational models, since the conductances are part of the data required by the Hodgkin and Huxley model. Using simpler models might be an alternative, but it is always necessary to find out what are the physiological parameters. What we would like to offer is a computational way to determine the conductances based on experimental data, and we consider our method a step towards that final goal. The method can

Action Editor: Alain Destexhe

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Electronic supplementary material The online version of this article (https://doi.org/10.1007/s10827-020-00752-7) contains supplementary material, which is available to authorized users.

✉ Jemy A. Mandujano Valle
jhimyunac@gmail.com

Alexandre L. Madureira
alm@lncc.br

Antonio Leitão
acgleitao@gmail.com

1 Laboratório Nacional de Computação Científica, Petrópolis - RJ, Brazil

2 EPGE - Fundação Getúlio Vargas, Rio de Janeiro - RJ, Brazil

3 Department of Mathematics, Federal University of St. Catarina, P.O. Box 476,88040-900 Florianópolis, Brazil

INVERSE PROBLEMS IN SCIENCE AND ENGINEERING
https://doi.org/10.1080/17415977.2019.1662001

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On the choice of Lagrange multipliers in the iterated Tikhonov method for linear ill-posed equations in Banach spaces

M. P. Machado[a], F. Margotti[b] and A. Leitão[b]

[a]Department of Mathematics, Federal University of Bahia, Salvador, Brazil; [b]Department of Mathematics, Federal University of St. Catarina, Florianópolis, Brazil

ABSTRACT

This article is devoted to the study of *nonstationary Iterated Tikhonov* (nIT) type methods (Hanke M, Groetsch CW. Nonstationary iterated Tikhonov regularization. J Optim Theory Appl. 1998;98(1):37–53; Engl HW, Hanke M, Neubauer A. Regularization of inverse problems. Vol. 375, Mathematics and its Applications. Dordrecht: Kluwer Academic Publishers Group; 1996. MR 1408680) for obtaining stable approximations to linear ill-posed problems modelled by operators mapping between Banach spaces. Here we propose and analyse an *a posteriori* strategy for choosing the sequence of regularization parameters for the nIT method, aiming to obtain a predefined decay rate of the residual. Convergence analysis of the proposed nIT type method is provided (convergence, stability and semi-convergence results). Moreover, in order to test the method's efficiency, numerical experiments for three distinct applications are conducted: (i) a 1D convolution problem (smooth Tikhonov functional and Banach parameter-space); (ii) a 2D deblurring problem (nonsmooth Tikhonov functional and Hilbert parameter-space); (iii) a 2D elliptic inverse potential problem.

ARTICLE HISTORY
Received 21 February 2019
Accepted 15 August 2019

KEYWORDS
Ill-posed problems; Banach spaces; linear operators; iterated Tikhonov method

2010 MATHEMATICS SUBJECT CLASSIFICATIONS
65J20; 47J06

1. Introduction

In this article we investigate *nonstationary Iterated Tikhonov* (nIT) type methods [1,2] for obtaining stable approximations of linear ill-posed problems modelled by operators mapping between Banach spaces. The novelty of our approach consists in the introduction of an *a posteriori* strategy for choosing the sequence of regularization parameters (or, equivalently, the Lagrange multipliers) for the nIT iteration, which play a key role in the convergence speed of the nIT iteration.

This new *a posteriori* strategy aims to enforce a pre-defined decay of the residual in each iteration; it differs from the classical choice for the Lagrange multipliers (see, e.g. [2,3]), which is based on an *a priori* strategy (typically geometrical) and leads to an unknown decay rate of the residual.

The *inverse problem* we are interested in consists of determining an unknown quantity $x \in X$ from given data $y \in Y$, where X, Y are Banach spaces. We assume that data are

CONTACT A. Leitão fabiomarg@gmail.com Department of Mathematics, Federal University of St. Catarina, P.O. Box 476, 88040-900 Florianópolis, Brazil

On Nonstationary Iterated Tikhonov Methods for Ill-Posed Equations in Banach Spaces

M. P. Machado, F. Margotti, and Antonio Leitão

Abstract In this article we propose a novel *nonstationary iterated Tikhonov* (nIT) type method for obtaining stable approximate solutions to ill-posed operator equations modeled by linear operators acting between Banach spaces. We propose a novel a posteriori strategy for choosing the sequence of regularization parameters (or, equivalently, the Lagrange multipliers) for the nIT iteration, aiming to obtain a fast decay of the residual.

Numerical experiments are presented for a 1D convolution problem (smooth Tikhonov functional and Banach parameter-space), and for a 2D deblurring problem (nonsmooth Tikhonov functional and Hilbert parameter-space).

1 Introduction

In this article we propose and (numerically) investigate a new *nonstationary Iterated Tikhonov* (nIT) type method [6, 9] for obtaining stable approximations of linear ill-posed problems modeled by operators mapping between Banach spaces.

The novelty of our approach consists in adopting an a posteriori strategy for the choice of the Lagrange multipliers, which aims to achieve a predefined decay of the residual in each iteration. This strategy differs from the classical choice for the Lagrange multipliers in [9, 10], which propose an a priori strategy, and leads to an unknown decay rate of the residual.

The *inverse problem* we are interested in consists of determining an unknown quantity $x \in X$ from the set of data $y \in Y$, where X, Y are Banach spaces. In practical situations, one does not know the data exactly; instead, only approximate

M. P. Machado
IMPA, Estrada Dona Castorina 110, 22460-320 Rio de Janeiro, Brazil
e-mail: majentcha@gmail.com

F. Margotti · A. Leitão (✉)
Department of Mathematics, Federal University of St. Catarina, P.O. Box 476, 88040-900 Florianópolis, Brazil
e-mail: fabiomarg@gmail.com; acgleitao@gmail.com

B. Hofmann et al. (eds.), *New Trends in Parameter Identification for Mathematical Models*, Trends in Mathematics, https://doi.org/10.1007/978-3-319-70824-9_10

IMA Journal of Numerical Analysis (2018) **00**, 1–22
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Range-relaxed criteria for choosing the Lagrange multipliers in nonstationary iterated Tikhonov method

R. Boiger
Materials Center Leoben Forschung Gmbh, Roseggerstrae 12, 8700 Leoben, Austria; former: Insitut für Mathematik, Alpen-Adria Universität Klagenfurt, Universitätsstrasse 65-67, 9020 Klagenfurt, Austria
romana.boiger@mcl.at

A. Leitão*
Department of Mathematics, Federal University of St. Catarina, P.O. Box 476, 88040-900 Florianópolis, Brazil
*Corresponding author: acgleitao@gmail.com

and

B. F. Svaiter
IMPA, Estr. Dona Castorina 110, 22460-320 Rio de Janeiro, Brazil
benar@impa.br

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In this article we propose a novel *nonstationary iterated Tikhonov* (NIT)-type method for obtaining stable approximate solutions to ill-posed operator equations modeled by linear operators acting between Hilbert spaces. Geometrical properties of the problem are used to derive a new strategy for choosing the sequence of regularization parameters (Lagrange multipliers) for the NIT iteration. Convergence analysis for this new method is provided. Numerical experiments are presented for two distinct applications: (I) a two-dimensional elliptic parameter identification problem (inverse potential problem); and (II) an image-deblurring problem. The results obtained validate the efficiency of our method compared with standard implementations of the NIT method (where a geometrical choice is typically used for the sequence of Lagrange multipliers).

Keywords: ill-posed problems; linear operators; iterated Tikhonov method; nonstationary methods.

1. Introduction

In this article we propose a new *nonstationary iterated Tikhonov* (NIT)-type method (Brill & Schock, 1987, Section 1.2) for obtaining stable approximations of linear ill-posed problems. The Lagrange multiplier is chosen so as to guarantee the residual of the next iterate to be in a *range*. Previous strategies for choosing the Lagrange multiplier in each iteration of NIT-type methods either prescribe (*a priori*) a geometrical increase of this multiplier (Hanke & Groetsch, 1998) or require (*a posteriori*) the residual at the next iterate to assume a prescribed value which depends on the current residual.

In those NIT methods that prescribe a geometrical increase of the Lagrange multipliers, the use of a too large geometric factor may lead to numerical instabilities and failure of convergence, whereas the

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On a Family of Gradient-Type Projection Methods for Nonlinear Ill-Posed Problems

Antonio Leitão[a] and Benar F. Svaiter[b]

[a]Department of Mathematics, Federal University of Santa Catarina, Florianópolis, Brazil; [b]National Institute for Pure and Applied Mathematics, Estrada Dona Castorina, Rio de Janeiro, Brazil

ABSTRACT

We propose and analyze a family of successive projection methods whose step direction is the same as the Landweber method for solving nonlinear ill-posed problems that satisfy the *Tangential Cone Condition* (TCC). This family encompasses the Landweber method, the minimal error method, and the steepest descent method; thus, providing an unified framework for the analysis of these methods. Moreover, we define new methods in this family, which are convergent for the constant of the TCC in a range *twice as large* as the one required for the Landweber and other gradient type methods. The TCC is widely used in the analysis of iterative methods for solving nonlinear ill-posed problems. The key idea in this work is to use the TCC in order to construct special convex sets possessing a separation property, and to successively project onto these sets. Numerical experiments are presented for a nonlinear two-dimensional elliptic parameter identification problem, validating the efficiency of our method.

ARTICLE HISTORY
Received 7 April 2016
Revised 28 September 2016
Accepted 22 February 2018

KEYWORDS
Ill-posed problems; nonlinear equations; projection methods; tangential cone condition

AMS CLASSIFICATION
65J20; 47J06

1. Introduction

In this article, we propose a family of successive orthogonal projection methods for obtaining stable approximate solutions to nonlinear ill-posed operator equations.

The *inverse problems*, we are interested in, consist of determining an unknown quantity $x \in X$ from the data set $y \in Y$, where X, Y are Hilbert spaces. The problem data y are obtained by indirect measurements of the parameter x, this process being described by the model $F(x) = y$, where $F : D \subset X \to Y$ is a non-linear ill-posed operator with domain $D = D(F)$.

In practical situations, the exact data y is not known. Instead, what is available is only approximately measured data $y^\delta \in Y$ satisfying

CONTACT Antonio Leitão acgleitao@gmail.com Department of Mathematics, Federal University of Santa Catarina, P.O. Box 476, 88040-900, Florianópolis, Brazil; B. F. Svaiter benar@impa.br National Institute for Pure and Applied Mathematics, Estrada Dona Castorina 110, 22460-320, Rio de Janeiro, Brazil.
Color versions of one or more of the figures in this article can be found online at www.tandfonline.com/lnfa.

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A regularization method based on level sets and augmented Lagrangian for parameter identification problems with piecewise constant solutions

J P Agnelli[1], **A De Cezaro**[2] **and A Leitão**[3]

[1] FaMAF-CIEM (CONICET), Universidad Nacional de Córdoba, Medina Allende S/N 5000, Córdoba, Argentina
[2] Institute of Mathematics Statistics and Physics, Federal University of Rio Grande, Av. Italia km 8, 96201-900 Rio Grande, Brazil
[3] Department of Mathematics, Federal University of St. Catarina, PO Box 476, 88040-900 Florianópolis, Brazil

E-mail: agnelli@famaf.unc.edu.ar, decezaromtm@gmail.com and acgleitao@gmail.com

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Published 1 October 2018

CrossMark

Abstract

We propose and analyse a regularization method for parameter identification problems modeled by ill-posed nonlinear operator equations, where the parameter to be identified is a piecewise constant function taking known values.

Following (De Cezaro *et al* 2013 *Inverse Problems* **29** 015003), a piecewise constant level set approach is used to represent the unknown parameter, and a corresponding Tikhonov functional is defined on an appropriated space of level set functions. Additionally, a suitable constraint is enforced, resulting that minimizers of our Tikhonov functional belong to the set of piecewise constant level set functions. In other words, the original parameter identification problem is rewritten in the form of a constrained optimization problem, which is solved using an augmented Lagrangian method.

We prove the existence of zero duality gaps and the existence of generalized Lagrangian multipliers. Moreover, we extend the analysis in De Cezaro *et al*'s work (2013 *Inverse Problems* **29** 015003), proving convergence and stability of the proposed parameter identification method.

A primal-dual algorithm is proposed to compute approximate solutions of the original inverse problem, and its convergence is proved. Numerical examples are presented: this algorithm is applied to a 2D diffuse optical tomography problem. The numerical results are compared with the ones in

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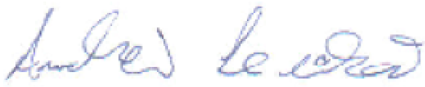

Assinatura Proponente

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Assinatura Coordenador de Pesquisa
Departamento de Matemática – UFSC

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