$T: \mathbb{R}^4 \to \mathbb{R}^4$

$\ker(T) = \text{span}\{(1,1,0,0),(0,0,1,1)\}$

Sol: se $\dim(\ker(T)) = 2 \to \dim(\text{Im}(T)) = 4-2 = 2$

Temos que $T(1,1,0,0) = (0,0,0,0)$ e $T(0,0,1,1) = (0,0,0,0)$ } pois $(1,1,0,0)$ e $(0,0,1,1) \in \ker(T)$.

Precisamos encontrar dois vetores em $\mathbb{R}^4$ que sejam l.i (temos que achar uma base p/ $\text{Im}(T)$)

Considere a matriz:

$$\begin{array}{c} v_1 \\ v_2 \\ \\ \\ \end{array}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ & ? & & \\ & ? & & \end{bmatrix} \to \begin{array}{c} \\ \\ v_3 \\ v_4 \end{array}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

os vetores v_3 e v_4 formam juntamente com v_1 e v_2 um conjunto l.i em $\mathbb{R}^4$

seja $u = (x,y,z,w) \in \mathbb{R}^4$

$(x,y,z,w) = a(1,1,0,0) + b(0,0,1,1) + c(0,1,0,0) + d(0,0,1,0)$

$$\begin{cases} \boxed{a = x} \\ a + c = y \quad \to \boxed{c = y - x} \\ b + d = z \quad \to \boxed{d = z - w} \\ \boxed{b = w} \end{cases}$$

Suponha que $T(0,1,0,0) = (1,0,0,0)$ e $T(0,0,1,0) = (0,1,0,0)$ (v_3 e v_4 geram a $\text{Im}(T)$, $T(v_3)$ e $T(v_4)$ podem ser escolhidos aleatoriamente, a únicas restrições são $T(v_3) \neq 0$ e $T(v_4) \neq 0$, $T(v_3) \neq T(v_4)$)

Assim,

$T(x,y,z,w) = T(x(1,1,0,0) + w(0,0,1,1) + (y-x)(0,1,0,0) + (z-w)(0,0,1,0))$

$= x \cdot \cancel{T(1,1,0,0)}^{0} + w\cancel{T(0,0,1,1)}^{0} + (y-x)T(0,1,0,0) + (z-w)T(0,0,1,0)$

$= (y-x)(1,0,0,0) + (z-w)(0,0,1,0)$

$= (y-x, z-w, 0, 0)$

9) W gerado por $B = \{1, x, e^x, xe^x\}$.

$[D]_B = \,?$ $\qquad D(p)\cancel{(x)} = p'(x)$

$$D(1) = 0 \qquad = 0 \cdot 1 + 0 \cdot x + 0 \cdot e^x + 0 \cdot xe^x$$

$$D(x) = 1 \qquad = 1 \cdot 1 + 0 \cdot x + 0 \cdot e^x + 0\, xe^x$$

$$D(e^x) = e^x \qquad = 0 \cdot 1 + 0 \cdot x + 1 \cdot e^x + 0 \cdot xe^x$$

$$D(xe^x) = e^x + xe^x \qquad = 0 \cdot 1 + 0 \cdot x + 1 \cdot e^x + 1 \cdot xe^x.$$

$$\rightsquigarrow [D]_B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$