7.4 – EXERCÍCIOS – pg. 309

Nos exercícios de 1 a 35, calcular a integral indefinida.

1. $\int \frac{sen\,\sqrt{x}}{\sqrt{x}}dx$

Fazendo

$u = \sqrt{x} \quad \Rightarrow \quad du = \frac{1}{2}x^{-1/2}dx = \frac{dx}{2\sqrt{x}}$

Temos:

$I = -2\cos\sqrt{x} + C$.

2. $\int \cos x\; cox\,(sen\; x)\, dx$

Fazendo:

$u = sen\; x$

$du = \cos x\; dx$

Temos que:

$\int \cos x\; cox\,(sen\; x)\, dx = sen\,(sen\; x) + C$.

3. $\int \frac{sen\; 2x}{\cos x}\, dx$

Temos:

$I = \int \frac{2\; sen\; x \cos x}{\cos x}\, dx$

$= 2\int sen\; x\; dx = -2\cos x + C$

4. $\int x\; tg\,(x^2+1)dx$

Fazendo:

$u = x^2 + 1$

$du = 2xdx$

Temos:

$I = \frac{1}{2}\int tg\; u\; du$

$= \frac{1}{2}\int \frac{sen\; u}{\cos u}du = -\frac{1}{2}\ln|\cos u| + C$

$= -\frac{1}{2}\ln\left|\cos\left(x^2+1\right)\right| + C = \frac{1}{2}\ln\left|\sec\left(x^2+1\right)\right| + C.$

5. $\int \frac{\cot g\left(1/x\right)}{x^2}dx$

Fazendo:

$u = \frac{1}{x} \quad \rightarrow \quad du = -\frac{1}{x^2}\,dx$

Temos:

$I = -\int \cot\ g\ u\ du$

$= -\int \frac{\cos u}{sen\ u}\,du$

$= -\ln\left|sen\ u\right| + C$

$= -\ln\left|sen\ 1/x\right| + C.$

6. $\int \sec\left(x+1\right)dx$

Fazendo:

$u = x+1 \quad \rightarrow \quad du = dx$

Temos:

$I = \int \frac{\sec u\left(\sec u + tg\ u\right)}{\sec u + tg\ u}du$

$= \int \frac{\sec^2 u + \sec u\ .\ tg\ u}{\sec u + tg\ u}du.$

Considerando:

$u^* = \sec u + tg\ u$

$du^* = \left(\sec^2 u + \sec u\ .\ tg\ u\right)du$

Finalizamos:

$I = \ln\left|\sec u + tg\ u\right| + C$

$= \ln\left|\sec\left(x+1\right) + tg\ \left(x+1\right)\right| + C.$

7. $\int sen\left(wt+\theta\right)dt$

Fazendo:

$u = wt+\theta \quad \rightarrow \quad du = wdt$

Temos:

$I = -\frac{1}{w}\cos\left(wt+\theta\right) + C.$

8. $\int x\cos ec\ x^2\ dx$

Fazendo:

$u = x^2 \quad \rightarrow \quad du = 2xdx$

Temos:

$$I = \frac{1}{2}\int \operatorname{cos}ec\ u\ du$$

$$= \frac{1}{2}\ln\left|\operatorname{cos}ec\ u - \cot g\ u\right| + C$$

$$= \frac{1}{2}\ln\left|\operatorname{cos}ec\ x^2 - \cot\ g\ x^2\right| + C.$$

9. $\int \cos x \,.\, tg\ (sen\ x)\, dx$

Fazendo:

$u = sen\ x \quad \rightarrow \quad du = \cos x\, dx$

Temos:

$$I = \int tg\ u\ du$$

$$= -\ln\left|\cos u\right| + C$$

$$= -\ln\left|\cos\ (sen\ x)\right| + C$$

$$= \ln\left|\sec\ (sen\ x)\right| + C.$$

10. $\int sen^3 (2x+1)\, dx$

Fazendo:

$u = 2x+1$

$du = 2dx$

Temos:

$$I = \frac{1}{2}\int sen^3 u\ du$$

$$= \frac{1}{2}\int sen\ u\ \left(1 - \cos^2 u\right) du$$

$$= \frac{1}{2}\int \left(sen\ u - \cos^2 u\ sen\ u\right) du$$

$$= \frac{1}{2}\left[-\cos u + \frac{\cos^3 u}{3}\right] + C$$

$$= \frac{1}{2}\left(-\cos\ (2x+1) + \frac{1}{3}\cos^3 (2x+1)\right) + C.$$

11. $\int \cos^5 (3-3x)\, dx$

Fazendo:

$u = 3-3x \quad \rightarrow \quad du = -3dx$

Temos:

$$I = -\frac{1}{3}\int \cos^5 u\ du$$

$$= -\frac{1}{3}\int \cos^4 u \cos u\ du$$

$$= -\frac{1}{3}\int \left(\cos u - 2sen^2 u \cos u + sen^4 u \cos u\right) du$$

$$= -\frac{1}{3}\left(senu - \frac{2}{3}sen^3 u + \frac{1}{5}sen^5 u\right) + C$$

$$= -\frac{1}{3}sen\ (3-3x) + \frac{2}{9}\ sen^3 (3-3x) - \frac{1}{15}sen^5\ (3-3x) + C.$$

12. $\int 2x\ sen^4 \left(x^2 - 1\right) dx$

Fazendo:

$u = x^2 - 1$

$du = 2xdx$

Temos:

$$\int sen^4 u\ du = \int \left(sen^2 u\right)^2 du$$

$$= \int \left(\frac{1-\cos 2u}{2}\right)^2\ du$$

$$= \int \frac{1}{4}\left(1 - 2\cos\ 2u + \cos^2\ 2u\right) du$$

$$= \frac{1}{4}u - \frac{1}{2}\cdot\frac{1}{2}\ sen\ 2u + \frac{1}{4}\int \cos^2\ 2u\ du$$

$$= \frac{1}{4}u - \frac{1}{4}sen\ 2u + \frac{1}{4}\int \frac{1+\cos 4\ u}{2}\ du$$

$$= \frac{1}{4}u - \frac{1}{4}sen\ 2u + \frac{1}{8}u + \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{4}sen\ 4u + C$$

$$= \frac{1}{4}u - \frac{1}{4}sen\ 2u + \frac{1}{8}u + \frac{1}{32}sen\ 4u + C$$

$$= \frac{3}{8}u - \frac{1}{4}sen\ 2u + \frac{1}{32}sen\ 4u + C$$

$$= \frac{3}{8}\left(x^2 - 1\right) - \frac{1}{4}sen\ 2\left(x^2 - 1\right) + \frac{1}{32}sen\ 4\ \left(x^2 - 1\right) + C$$

13. $\int e^{2x} \cos^2 \left(e^{2x} - 1\right) dx$

Fazendo:

$u = e^{2x} - 1$

$du = 2\, e^{2x} dx$

Temos:

$$I = \frac{1}{2}\int \cos^2 u\, du$$

$$= \frac{1}{2}\left[\frac{1}{2}u + \frac{1}{4}\, sen\, 2u\right] + C$$

$$= \frac{1}{4}u + \frac{1}{8}\, sen\, 2u + C$$

$$= \frac{1}{4}\left(e^{2x} - 1\right) + \frac{1}{8} sen\,\left(2\, e^{2x} - 2\right) + C.$$

14. $\int sen^3\ 2\theta \cos^4\ 2\theta\, d\theta$

Fazendo:

$u = 2\theta \quad \rightarrow \quad du = 2d\theta$

Temos:

$$I = \frac{1}{2}\int sen^3 u \cos^4 u\, du$$

$$= \frac{1}{2}\int \left(1 - \cos^2 u\right) sen\, u \cos^4 u\, du$$

$$= \frac{1}{2}\int \left(\cos^4 u\ sen\, u - \cos^6 u\ sen\, u\right) du$$

$$= \frac{1}{2}\left[-\frac{\cos^5 u}{5} + \frac{\cos^7 u}{7}\right] + C$$

$$= \frac{-1}{10}\cos^5 2\theta + \frac{1}{14}\cos^7 2\theta + C.$$

15. $\int sen^3 (1 - 2\theta) \cos^3 (1 - 2\theta)\, d\theta$

Fazendo:

$u = 1 - 2\theta$

$du = -2d\theta$

Temos:

$$I = -\frac{1}{2}\int sen^3 u \cos^3 u\, du$$

$$= -\frac{1}{2}\int sen^3 u\left(1 - sen^2 u\right)\cos u\, du$$

$$= -\frac{1}{2}\left(sen^3 u \cos u - sen^5 u \cos u\right) du$$

$$= -\frac{1}{2}\left[\frac{sen^4 u}{4} - \frac{sen^6 u}{6}\right] + C$$

$$= -\frac{1}{8} sen^4 u + \frac{1}{12} sen^6 u + C$$

$$= -\frac{1}{8} sen^4 (1 - 2\theta) + \frac{1}{12} sen^6 (1 - 2\theta) + C.$$

Outra maneira

$$I = -\frac{1}{2}\int sen^3 u \cos^3 u\, du$$

$$= -\frac{1}{2}\int \left(1 - \cos^2 u\right) sen\, u \cos^3 u\, du$$

$$= \frac{1}{8}\cos^4 u - \frac{1}{12}\cos^6 u + C$$

$$= \frac{1}{8}\cos^4 (1 - 2\theta) - \frac{1}{12}\cos^6 (1 - 2\theta) + C$$

16. $\int sen^{19}(t-1)\cos\,(t-1)\,dt$

Fazendo:

$u = t - 1$

$du = dt$

Temos:

$$I = \int sen^{19} u \cos u\, du$$

$$= \frac{sen^{20} u}{20} + C = \frac{sen^{20}(t-1)}{20} + C.$$

17. $\int \frac{1}{\theta} tg^3 (\ln \theta) d\theta$

Fazendo:

$u = \ln \theta$

$du = \dfrac{d\theta}{\theta}$

Temos:

$I = \int tg^3 u \, du$

$= \int tgu\left(\sec^2 u - 1\right)du = \dfrac{1}{2} tg^2 u - \int tg \, u \, du$

$= \dfrac{1}{2} tg^2 u + \ln\left|\cos u\right| + C$

$= \dfrac{1}{2} tg^2 (\ln \theta) + \ln \left|\cos (\ln \theta)\right| + C.$

18. $\int tg^3 x \cos^4 x \, dx$

Temos:

$I = \int \dfrac{sen^3 x}{\cos^3 x} \, . \cos^4 x \, dx$

$= \int sen^3 x \cos x \, dx$

$= \dfrac{sen^4 x}{4} + C.$

19. $\int \cos^4 x \, dx$

Temos:

$I = \dfrac{1}{4} \cos^3 x \, sen \, x + \dfrac{3}{4} \int \cos^2 x \, dx$

$= \dfrac{1}{4} \cos^3 x \, sen \, x + \dfrac{3}{4}\left[\dfrac{1}{2} \cos x \, sen \, x + \dfrac{1}{2} \int dx\right]$

$= \dfrac{1}{4} \cos^3 x \, sen \, x + \dfrac{3}{8} \cos x \, sen \, x + \dfrac{3}{8} x + C.$

20. $\int tg^4 x \, dx$

Temos:

$$
\begin{aligned}
I &= \int \frac{sen^4 x}{\cos^4 x}\,dx \\
&= \int \frac{sen^2 x\left(1-\cos^2 x\right)}{\cos^4 x}dx \\
&= \int \frac{sen^2 x}{\cos^4 x}dx - \int \frac{sen^2 x \cos^2 x}{\cos^4 x}dx \\
&= \int \frac{sen^2 x}{\cos^4 x}dx - \int \frac{sen^2 x}{\cos^2 x}dx \\
&= \int tg^2 x \sec^2 x\,dx - \int \frac{1-\cos^2 x}{\cos^2 x}dx \\
&= \frac{tg^3 x}{3} - \int \sec^2 x\,dx + \int dx \\
&= \frac{tg^3 x}{3} - tg\ x + x + C.
\end{aligned}
$$

21. $\int \frac{sen^2 x}{\cos^4 x}dx$

Temos:

$$
\begin{aligned}
I &= \int tg^2 x \sec^2 x\,dx \\
&= \frac{tg^2 x}{3} + C.
\end{aligned}
$$

22. $\int 15 sen^5 x\,dx$

Temos:

$$
\begin{aligned}
I &= 15\int \left(sen^2 x\right)^2 sen\ x\,dx \\
&= 15\int \left(1-\cos^2 x\right)^2 sen\ x\,dx \\
&= 15\int \left(1-2\cos^2 x+\cos^4 x\right) sen\ x\,dx \\
&= 15\left[-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x\right] + C \\
&= -15\cos x + 10\cos^3 x - 3\cos^5 x + C.
\end{aligned}
$$

23. $\int 15\, sen^2 x \cos^3 x\,dx$

Temos:

$$I = 15\int sen^2 x\left(1 - sen^2 x\right)\cos x\, dx$$
$$= 15\int sen^2 x \cos x\, dx - 15\int sen^4 x \cos x\, dx$$
$$= 15\frac{sen^3 x}{3} - 15\frac{sen^5 x}{5} + C$$
$$= 5\, sen^3 x - 3\, sen^5 x + C.$$

24. $\int 48\, sen^2 x \cos^4 x\, dx$

Temos:

$$I = 48\int \left(1 - \cos^2 x\right)\cos^4 x\, dx$$
$$= 48\int \cos^4 x\, dx - 48\int \cos^6 x\, dx$$
$$= 48\left(I_4 - I_6\right)$$
$$= 48\left(I_4 - \left(\frac{1}{6}\cos^5 x\, sen\, x + \frac{5}{6}I_4\right)\right)$$
$$= 48\left(\frac{1}{6}\, I_4 - \frac{1}{6}\cos^5 x sen\, x\right)$$
$$= 8\left(\frac{1}{4}\cos^3 x\, sen\, x + \frac{3}{4}I_2 - \cos^5 x\, sen\, x\right)$$
$$= 2\cos^3 x\, sen\, x - 8\cos^5 x\, sen\, x + 6\left(\frac{1}{2}sen\, x\cos x + \frac{1}{2}x\right) + C$$
$$= 2\cos^3 x\, sen\, x - 8\cos^5 x\, sen\, x + 3sen\, x\cos x + 3x + C.$$

25. $\int \cos^6 3x\, dx$

Fazendo:

$u = 3x \rightarrow du = 3dx$

Temos:

$$\int \cos^6 u\, .\frac{du}{3} = \frac{1}{3}\, I_6$$
$$= \frac{1}{3}\left[\frac{1}{6}\cos^5 u\, sen\, u + \frac{5}{24}\cos^3 u\, sen\, u + \frac{15}{48}sen\, u\cos u + \frac{15}{48}\, u\right] + C$$
$$= \frac{1}{18}\cos^5 3x\, sen\, 3x + \frac{5}{72}\cos^3 3x sen 3x + \frac{5}{48}\cos 3x\, sen\, 3x + \frac{5}{16}x + C.$$

26. $\int \frac{-3\cos^2 x}{sen^4 x}\, dx$

Temos:

$$I = -3\int \frac{\cos^2 x}{sen^2 x\; sen^2 x}\, dx$$

$$= -3\int \cot g^2 x \,.\, \cos ec^2 x\, dx$$

$$= 3\frac{\cot g^3 x}{3} + C$$

$$= \cot g^3 x + C.$$

27. $\int sen\, 3x \cos 5x\, dx$

Temos:

$$I = \frac{1}{2}\int sen\, 8x\, dx - \frac{1}{2}\int sen\, 2x\, dx$$

$$= \frac{-1}{16}\cos 8x + \frac{1}{4}\cos 2x + C.$$

28. $\int tg^2 5x\, dx$

Temos:

$$I = \int \frac{sen^2 5x}{\cos^2 5x} dx$$

$$= \int \frac{1-\cos^2 5x}{\cos^2 5x} dx$$

$$= \int \frac{1}{\cos^2 5x} dx - \int dx$$

$$= \int \sec^2 5x\, dx - x + C$$

$$= \frac{1}{5} tg\, 5x - x + C.$$

29. $\int sen\; w\, t\; sen(w\, t + \theta)\, dt$

Temos:

$$I = \int \frac{1}{2}\left[\cos\left(w\,t - w\,t - \theta\right) - \cos\left(w\,t - w\,t - \theta\right)\right]dt$$

$$= \int \frac{1}{2}\left(\cos(-\theta) - \cos\left(2w\,t + \theta\right)\right)dt$$

$$= \frac{1}{2}\cos\theta\, t - \frac{1}{2}\cdot\frac{1}{2w}\, sen\left(2w\,t + \theta\right) + C$$

$$= \frac{1}{2}t\cos\theta - \frac{1}{4w}\, sen\left(2w\,t + \theta\right) + C.$$

30. $\int \frac{\cos^3 x}{sen^4 x}\,dx$

Temos:

$$I = \int \frac{\left(1 - sen^2 x\right)}{sen^4 x}\cos x\,dx$$

$$= \int sen^{-4}x\cos x\,dx - \int sen^{-2}x\cos x\,dx$$

$$= \frac{sen^{-3}x}{-3} - \frac{sen^{-1}}{-1} + C$$

$$= \frac{-1}{3\,sen^3 x} + \frac{1}{sen\,x} + C.$$

31. $\int \sec^4 t\,\cot g^6 t\; sen^8 t\,dt$

Temos:

$$I = \int \frac{1}{\cos^4 t}\cdot\frac{\cos^6 t}{sen^6 t}\cdot sen^8 t\,dt$$

$$= \int \cos^2 t\; sen^2 t\,dt$$

$$= \int \left(\cos t\; sen\,t\right)^2 dt$$

$$= \int \left(\frac{sen\,2t}{2}\right)^2 dt$$

$$= \frac{1}{4}\int sen^2 2t\,dt$$

$$I = \frac{1}{4}\int\left(\frac{1}{2} - \frac{1}{2}\cos 4t\right)dt$$

$$= \frac{1}{8}t - \frac{1}{32}\,sen\,4t + C.$$

32. $\int \frac{x}{\sqrt{x^2-1}}\, tg^3 \sqrt{x^2-1}\, dx$

Fazendo:

$u=\sqrt{x^2-1}$

$du=\frac{xdx}{\sqrt{x^2-1}}$

Temos:

$$\begin{aligned}
I &= \int tg^3 u\, du \\
&= \frac{1}{2} tg^2 u - \int tg\, u\, du \\
&= \frac{1}{2} tg^2 u + \ln\left|\cos u\right| + C \\
&= \frac{1}{2} tg^2 \sqrt{x^2-1} + \ln\left|\cos \sqrt{x^2-1}\right| + C.
\end{aligned}$$

33. $\int \sec^3(1-4x)dx$

Fazendo:

$u=1-4x \rightarrow du=-4dx$

Temos:

$$\begin{aligned}
I &= -\frac{1}{4}\int \sec^3 u\, du \\
&= -\frac{1}{4}\left[\frac{1}{2}\sec u\, tg\, u + \frac{1}{2}\int \sec u\, du\right] \\
&= -\frac{1}{8}\sec u\, tg\, u - \frac{1}{8}\ln\left|\sec u + tg\, u\right| + C \\
&= -\frac{1}{8}\sec(1-4x) tg(1-4x) - \frac{1}{8}\ln\left|\sec(1-4x) + tg(1-4x)\right| + C.
\end{aligned}$$

34. $\int \cos ec^4(3-2x)dx$

Fazendo:

$u=3-2x \rightarrow du=-2dx$

Temos:

$$I = -\frac{1}{2}\int \operatorname{cosec}^4 u\ du$$

$$= -\frac{1}{2}\int \left(1 + \operatorname{cotg}^2 u\right)\operatorname{cosec}^2 u\ du$$

$$= -\frac{1}{2}\left[-\operatorname{cotg} u - \frac{\operatorname{cotg}^3 u}{3}\right] + C$$

$$= \frac{1}{2}\operatorname{cotg} u + \frac{1}{6}\operatorname{cotg}^3 u + C$$

$$= \frac{1}{2}\operatorname{cotg}\left(3 - 2x\right) + \frac{1}{6}\operatorname{cotg}^3\left(3 - 2x\right) + C.$$

35. $\int x \operatorname{cotg}^2\left(x^2 - 1\right)\operatorname{cosec}^2\left(x^2 - 1\right) dx$

Fazendo:

$u = x^2 - 1$

$du = 2xdx$

Temos:

$$I = \frac{1}{2}\int \operatorname{cotg}^2 u\ \operatorname{cosec}^2 u\ du$$

$$I = -\frac{1}{2}\ \frac{\operatorname{cotg}^3 u}{3} + C$$

$$= -\frac{1}{6}\operatorname{cotg}^3\left(x^2 - 1\right) + C.$$

36. Verificar as fórmulas de recorrência (8), (9) e (10) da secção 7.2.11.

Verificando a fórmula (8):

$$\int \cos^n u\ du = \frac{1}{n}\cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\ du$$

Fazendo:

$u^* = \cos^{n-1} u \quad \Rightarrow \quad du^* = -\left(n-1\right)\cos^{n-2} u\ .\ \operatorname{sen} u\ du$

$dv = \cos u\ du \quad \Rightarrow \quad v = \operatorname{sen} u$

Temos:

$$\int \cos^n u\, du = \cos^{n-1} u\ sen\, u + \int sen\, u\, .\, (n-1)\cos^{n-2} u\, .\, sen\, u\, du$$

$$\int \cos^n u\, du = \cos^{n-1} u\ sen\, u + (n-1)\int \cos^{n-2} u\,\left(1-\cos^2 u\right) du$$

$$\int \cos^n u\, du = \cos^{n-1} u\ sen\, u + (n-1)\int \cos^{n-2} u du - (n-1)\int \cos^n u\, du$$

$$\int \cos^n u\, du + (n-1)\int \cos^n u\, du = \cos^{n-1} u\ sen\, u + (n-1)\int \cos^{n-2} u\, du$$

$$\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u\, .\, sen\, u + \frac{n-1}{n}\int \cos^{n-2} u\, du$$

Verificando a fórmula (9):

$$\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u\ tg\ u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$$

Fazendo:

$$u^* = \sec^{n-2} u \quad \Rightarrow \quad du^* = (n-2)\sec^{n-3} u\, .\, \sec u\, .\, tg\ u\, du$$

$$dv = \sec^2 u\, du \quad \Rightarrow \quad v = tg\ u$$

Temos:

$$\int \sec^n u\, du = \sec^{n-2} u\, .\, tg\ u - \int tg^2 u\,(n-2)\sec^{n-2} u\, .\, du$$

$$\int \sec^n u\, du = \sec^{n-2} u\, .\, tg\ u - (n-2)\int \left(\sec^2 u - 1\right)\sec^{n-2} u\, du$$

$$\int \sec^n u\, du = \sec^{n-2} u\, .\, tg\ u - (n-2)\int \sec^n u\, .\, du + (n-2)\int \sec^{n-2} u\, du$$

$$\int \sec^n u\, du + (n-2)\int \sec^n u\, du = \sec^{n-2} u\ tgu + (n-2)\int \sec^{n-2} u\, du$$

$$\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u\ tg\ u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$$

Verificando a fórmula (10):

$$\int \cos c^n u\, du = \frac{-1}{n-1}\cos ec^{n-2} u\, .\, \cot g\ u + \frac{n-2}{n-1}\int \cos ec^{n-2} u\, du$$

Fazendo:

$$u^* = \cos ec^{n-2} u\, du \quad \Rightarrow \quad du^* = -(n-2)\cos ec^{n-3} u\, .\, \cos ec\ u\, .\, \cot g\ u\, du$$

$$dv = \cos ec^2 u\, du \quad \Rightarrow \quad v = -\cot g\ u$$

Temos:

$$\int \cos ec^n u\, du = -\cos ec^{n-2} u\ \cot g\ u - \int \cot^2 g\ u\, .\, (n-2)\cos ec^{n-2}\ u\, du$$

$$\int \cos ec^n u\, du = -\cos ec^{n-2} u\ \cot g\ u - (n-2)\int \left(\cos ec^2 u - 1\right)\cos ec^{n-2} u\, du$$

$$\int \cos ec^n u\, du + (n-2)\int \cos ec^2 u\, du = -\cos ec^n u.\ \cot g\ u + (n-2)\int \cos ec^{n-2} u\, du$$

$$\int \cos ec^n u\, du = \frac{-1}{n-1}\cos ec^{n-2} u\, .\, \cot g\ u + \frac{n-2}{n-1}\int \cos ec^{n-2} u\, du$$

37. Verificar as fórmulas.

a) $\int tg^n u\, du = \dfrac{1}{n-1}\, tg^{n-1} u - \int tg^{n-2} u\, du$

b) $\int \cot g^n u\, du = -\dfrac{1}{n-1} \cot g^{n-1} u - \int \cot g^{n-2} u\, du$

Solução (a)

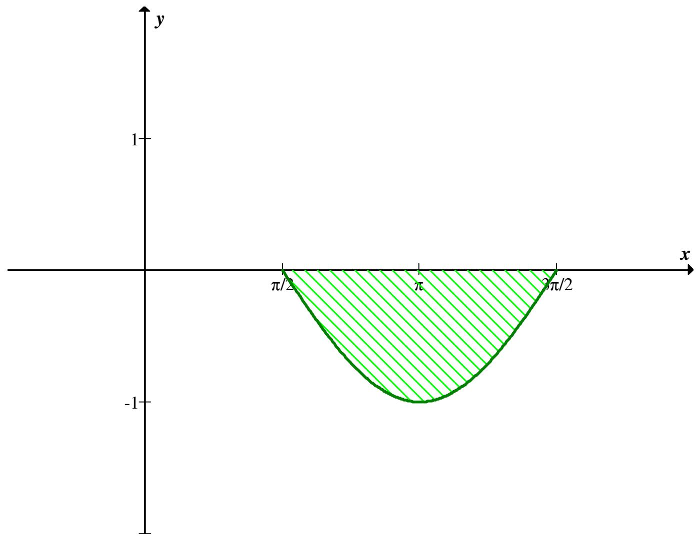
$$\int tg^n u\, du = \int tg^2 u \,.\, tg^{n-2} u\, du$$
$$= \int \left(\sec^2 u - 1\right) .\, tg^{n-2} u\, du$$
$$= \int tg^{n-2} u\ \sec^2 u du - \int tg^{n-2} u\, du$$
$$= \frac{tg^{n-1} u}{n-1} - \int tg^{n-2} u\, du$$

Solução (b)

$$\int \cot g^n u\, du = \int \cot g^2 u \,.\, \cot g^{n-2} u\, du$$
$$= \int \left(\cos ec^2 u - 1\right) .\, \cos tg^{n-2} u\, du$$
$$= \int \cot g^{n-2} u\ \ \cos ec^2 u\, du - \int \cot g^{n-2} u\, du$$
$$= -\frac{\cot g^{n-1}}{n-1} - \int \cot g^{n-2} u\, du$$

38. Calcular a área limitada pela curva $y = \cos x$, pelas retas $x = \dfrac{\pi}{2}$ e $x = \dfrac{3\pi}{2}$ e o eixo dos x.

A Figura que segue mostra a área.



$$A = -2\int_{\frac{\pi}{2}}^{\pi} \cos\, x\, dx$$

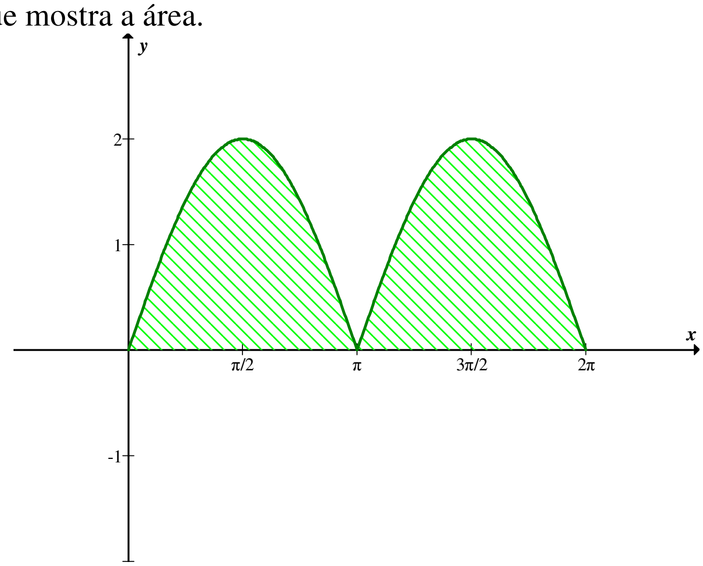
$$= -2\, sen\, x\Big|_{\frac{\pi}{2}}^{\pi}$$

$$= -2\left(sen\, \pi - sen\frac{\pi}{2}\right)$$

$$= 2\; u\; a$$

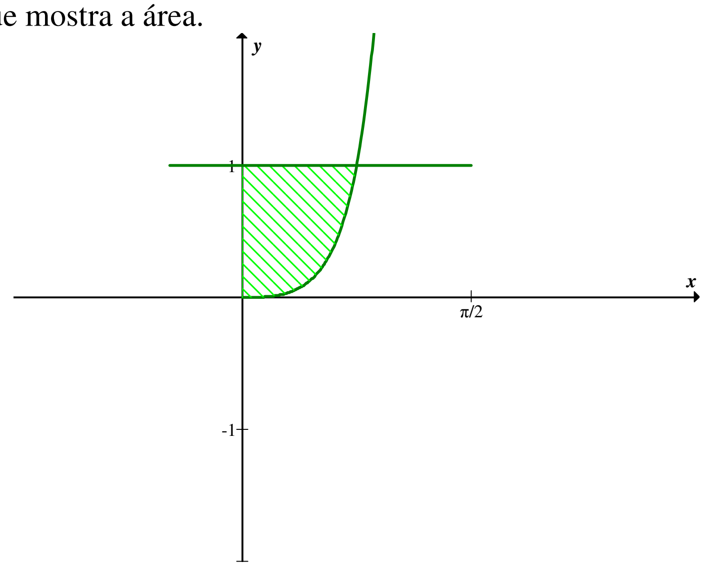
39. Calcular a área limitada por $y = 2\left|sen\, x\right|$, $x = 0$, $x = 2\pi$ e o eixo dos x

A Figura que segue mostra a área.



$$A = 2\int_0^{\pi} 2\ sen\ x\ dx$$
$$= -4\cos x\Big|_0^{\pi}$$
$$= -4(\cos\pi - \cos 0)$$
$$= -4(-1-1)$$
$$= 8\ u.\ a$$

40. Calcular a área da região limitada por $y = tg^3 x$, $y = 1$ e $x = 0$
A Figura que segue mostra a área.

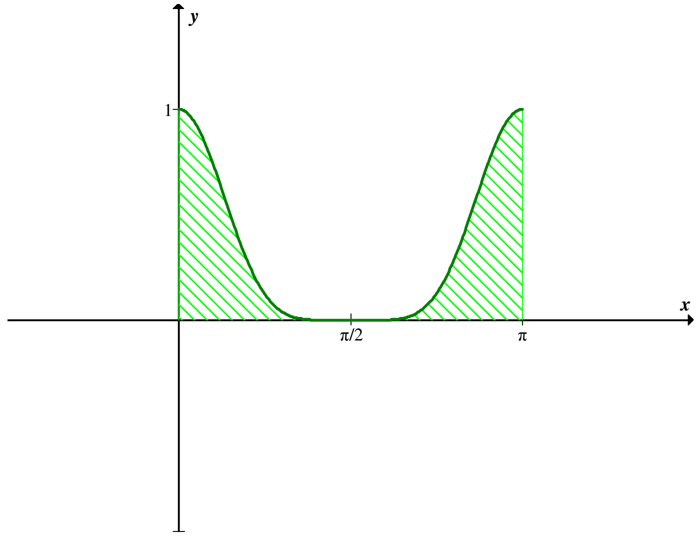


$$A_1 = \int_0^{\frac{\pi}{4}} tg^3 x\ dx$$
$$= \frac{1}{2} tg^2 x + \ln|\cos x|\Big|_0^{\frac{\pi}{4}}$$
$$= \frac{1}{2}\left(tg^2\frac{\pi}{4} - tg^2 0\right) + \ln\left|\cos\frac{\pi}{4}\right| - \ln|\cos 0|$$
$$= \frac{1}{2}(1-0) + \ln\frac{\sqrt{2}}{2}$$
$$= \frac{1}{2} + \ln\frac{\sqrt{2}}{2}$$

Assim,
$$A = \left(\frac{\pi}{4} - \frac{1}{2} - \ln\frac{\sqrt{2}}{2}\right)$$
$$= \left(\frac{\pi}{4} - \frac{1}{2} + \frac{1}{2}\ln 2\right) u.\ a$$

41. Calcular a área sob o gráfico de $y = \cos^6 x$ de 0 ate π.

A Figura que segue mostra a área.



$$\int \cos^6 x\, dx = \frac{1}{6}\cos^5 x\ sen\ x + \frac{5}{6}\int \cos^4 x\, dx$$

$$= \frac{1}{6}\cos^5 x\ sen\ x + \frac{5}{6}\left(\frac{1}{4}\cos^3 x\ sen\ x + \frac{3}{4}\int \cos^2 x\, dx\right)$$

$$= \frac{1}{6}\cos^5 x\ sen\ x + \frac{5}{24}\cos^3 x\ sen\ x + \frac{15}{24}\int \cos^2 x\, dx$$

$$= \frac{1}{6}\cos^5 x\ sen\ x + \frac{5}{24}\cos^3 x\ sen\ x + \frac{15}{24}\left(\frac{1}{2}x + \frac{1}{4}sen\ 2x\right) + C$$

Assim,

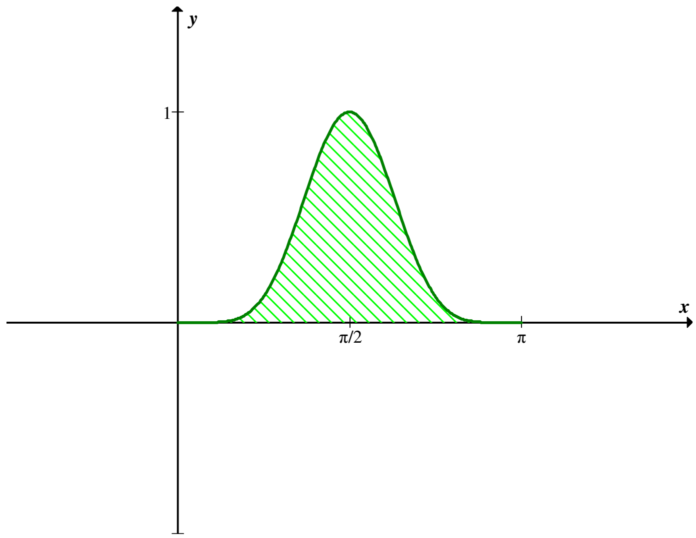
$$A = \int_0^{\pi} \cos^6 x\, dx$$

$$= \frac{1}{6}\cos^5 x\ sen\ x + \frac{5}{24}\cos^3 x\ sen\ x + \frac{15}{24}\left(\frac{1}{2}x + \frac{1}{4}sen\ 2x\right)\Bigg|_0^{\pi}$$

$$= \frac{5}{16}\pi\ u.a$$

42. Calcular a área sob o gráfico de $y = sen^6 x$ de 0 ate π.

A Figura que segue mostra a área.



$$\int sen^6 x\, dx = -\frac{1}{6} sen^5 x \cos x + \frac{5}{6}\int sen^4 x\, dx$$

$$= -\frac{1}{6} sen^5 x \cos x + \frac{5}{6}\left(-\frac{1}{4} sen^3 x \cos x + \frac{3}{4}\int sen^2 x\, dx\right)$$

$$= -\frac{1}{6} sen^5 x \cos x - \frac{5}{24}\, sen^3 x \cos x + \frac{15}{24}\left(\frac{1}{2}x - \frac{1}{4} sen\, 2x\right) + C$$

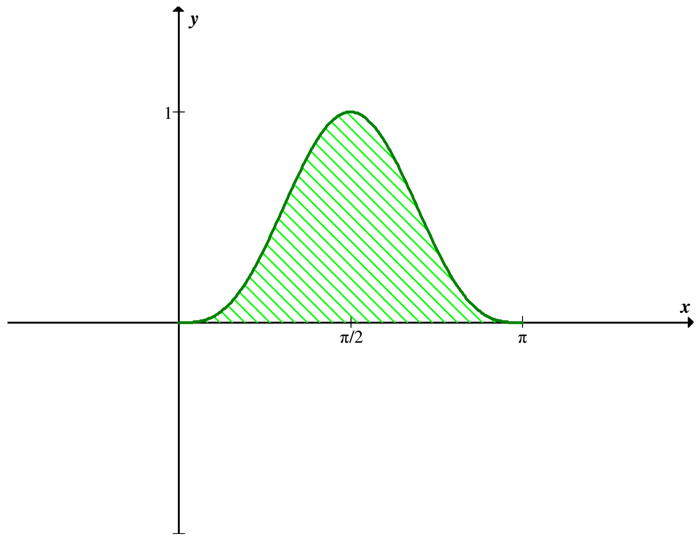
Assim,

$$A = \int_0^{\pi} sen^6 x\, dx$$

$$= -\frac{1}{6} sen^5 x \cos x - \frac{5}{24} sen^3 x \cos x + \frac{15}{24}\left(\frac{1}{2}x - \frac{1}{4} sen\, 2x\right)\Bigg|_0^{\pi}$$

$$= \frac{5}{16}\pi\ u.a$$

43. Calcular a área sob o gráfico de $y = sen^3 x$ de 0 ate π.
A Figura que segue mostra a área.



$$\int sen^3 x\, dx = -\frac{1}{3} sen^2 \cos x + \frac{2}{3} \int sen\ x\, dx$$

$$= -\frac{1}{3} sen^2 \cos x - \frac{2}{3} \cos x + C$$

Assim,

$$A = \int_0^{\pi} sen^3 x\, dx$$

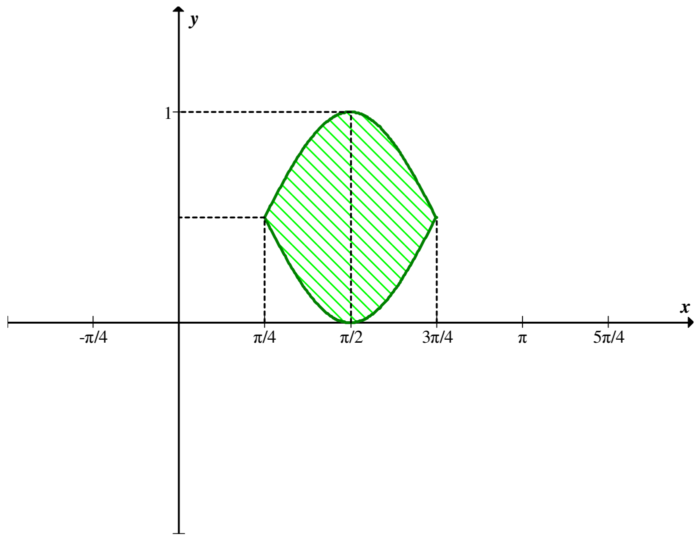
$$= -\frac{1}{3} sen^2 x \cos x - \frac{2}{3} \cos x \Big|_0^{\pi}$$

$$= -\frac{2}{3} (\cos \pi - \cos 0)$$

$$= \frac{4}{3} u.a.$$

44. Calcular a área entre as curvas $y = sen^2 x$ e $y = \cos^2 x$, de $\frac{\pi}{4}$ ate $\frac{3\pi}{4}$

A Figura que segue mostra a área.



$$A = 2.\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(sen^2 x - \cos^2 x\right) dx$$

$$= 2.\left(\frac{1}{2}x - \frac{1}{4}\ sen\ 2x - \frac{1}{2}x - \frac{1}{4} sen\ 2x\right)\Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= 2.\frac{-1}{2} sen\ 2x\Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= -\left(sen\pi - sen\frac{\pi}{2}\right)$$

$$= sen\frac{\pi}{2}$$

$$= 1\ u.a$$

Nos exercícios de 45 a 67, calcular a integral indefinida.

45. $\int \frac{dx}{x^2\sqrt{x^2-5}}$

Fazendo:

$$\sqrt{x^2-5} = \sqrt{5}\ tg\ \theta$$

$$x = \sqrt{5}\sec\theta$$

$$dx = \sqrt{5}\sec\theta\ tg\ \theta\ d\theta$$

Temos:

$$I = \int \frac{\sqrt{5}\sec\theta\ tg\ \theta\ d\theta}{5\sec^2\theta\sqrt{5}tg\ \theta}$$

$$= \int \frac{d\theta}{5\sec\theta} = \frac{1}{5}\int \cos\theta d\theta$$

$$= \frac{1}{5} sen\ \theta + C$$

$$= \frac{1}{5}\frac{\sqrt{x^2-5}}{x} + C$$

46. $\int \frac{dt}{\sqrt{9-16t^2}}$

Fazendo:

$u^2 = 16t^2$

$u = 4t \quad \rightarrow \quad du = 4dt$

Temos:

$$I = \int \frac{\frac{1}{4}\, du}{\sqrt{9-u^2}}$$

Fazendo:

$\sqrt{9-u^2} = 3\cos\theta$

$u = 3\ sen\ \theta \quad \therefore \quad \theta = arc\ sen\frac{u}{3}$

$du = 3\cos\theta\ d\theta$

Obtemos:

$$I = \frac{1}{4}\int \frac{3\cos\theta\ d\theta}{3\cos\theta}$$

$$= \frac{1}{4}\int d\theta = \frac{1}{4}\theta + C$$

$$= \frac{1}{4} arc\ sen\ \frac{u}{3} + C$$

$$= \frac{1}{4} arc\ sen\ \frac{4t}{3} + C$$

47. $\int \frac{x^3 dx}{\sqrt{x^2-9}}$

Fazendo:

$$\sqrt{x^2-9} = 3\, tg\ \theta$$
$$x = 3 \sec \theta$$
$$dx = 3 \sec \theta\ tg\theta\ d\theta$$

Temos:

$$\int \frac{27 \sec^3 \theta \,.\, 3 \sec \theta \,.\, tg\ \theta\ d\theta}{3\, tg\ \theta}$$
$$= 27 \int \sec^4 \theta\ d\theta$$
$$= 27 \left(\frac{1}{3} \sec^2 \theta\ tg\ \theta + \frac{2}{3} \int \sec^2 \theta\ d\theta \right)$$
$$= 9 \sec^2 \theta\ tg\ \theta + 18\, tg\ \theta + C$$
$$= 9 \left(\frac{x}{3} \right)^2 \frac{\sqrt{x^2-9}}{3} + 18 \frac{\sqrt{x^2-9}}{3} + C$$
$$= \left(\frac{1}{3} x^2 + 6 \right) \sqrt{x^2-9} + C$$

48. $\int \left(1-4t^2\right)^{3/2} dt$

$$= \int \left(1-4t^2\right)\sqrt{1-4t^2}\ dt$$
$$= \frac{1}{2} \int \left(1-u^2\right)\sqrt{1-u^2}\, du$$

onde: $u^2 = 4t^2 \quad \rightarrow \quad u = 2t$

$du = 2dt$

Fazendo:

$$\sqrt{1-u^2} = \cos \theta$$
$$u = sen\ \theta$$
$$du = \cos\theta\ d\theta$$

Temos:

$$I = \frac{1}{2}\int \cos\theta\left(1 - sen^2\theta\right).\cos\theta\, d\theta$$

$$= \frac{1}{2}\int \cos\theta\,.\cos^2\theta\,.\cos\theta\, d\theta$$

$$= \frac{1}{2}\int \cos^4\theta\, d\theta$$

$$= \frac{1}{2}\left[\frac{1}{4}\cos^3\theta\, sen\theta + \frac{3}{4}\int \cos^2\theta\, d\theta\right]$$

$$= \frac{1}{8}\cos^3\theta\, sen\,\theta + \frac{3}{8}\left(\frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta\right) + C$$

$$= \frac{1}{8}\cos^3\theta\, sen\,\theta + \frac{3}{16}\theta + \frac{3}{32}\, sen\, 2\theta + C.$$

Considerando:

$$sen\, 2\theta = 2\, sen\,\theta\cos\theta$$

$$= 2\, u.\sqrt{1 - u^2}$$

Finalizamos:

$$I = \frac{1}{8}\left(\sqrt{1 - u^2}\right)^3 u + \frac{3}{16}\, arc\ sen\, u + \frac{3}{16} u\sqrt{1 - u^2} + C$$

$$= \frac{1}{8} u\left(1 - u^2\right)\sqrt{1 - u^2} + \frac{3}{16}\, arc\ sen\, u + \frac{3}{16} u\sqrt{1 - u^2} + C$$

$$= \frac{1}{8}\,.\,2t\left(1 - 4t^2\right)\sqrt{1 - 4t^2} + \frac{3}{16}\, arc\ sen\, 2t + \frac{3}{16}\,.\,2t\sqrt{1 - 4t^2} + C$$

$$= \frac{1}{4} t\left(1 - 4t^2\right)\sqrt{1 - 4t^2} + \frac{3}{16}\, arc\ sen\, 2t + \frac{3}{8}\,.\,t\sqrt{1 - 4t^2} + C$$

49. $\int x^2\sqrt{4 - x^2}\, dx$

Fazendo:

$$\sqrt{4 - x^2} = 2\cos\theta$$

$$x = 2\, sen\,\theta$$

$$dx = 2\cos\theta\, d\theta$$

Temos:

$$I = \int 4\, sen^2\theta \,.\, 2\cos\theta \,.\, 2\cos\theta\, d\theta$$
$$= 16\int sen^2\theta \cos^2\theta\, d\theta$$
$$= 16\int (sen\theta \cos\theta)^2\, d\theta$$
$$= 16\int (1-\cos^2\theta)\cos^2\theta\, d\theta$$
$$= 16\int (\cos^2\theta - \cos^4\theta)\, d\theta$$
$$= 16\left[\frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta - \left(\frac{1}{4}\cos^3\theta\, sen\theta + \frac{3}{4}\int \cos^2\theta\, d\theta\right)\right]$$
$$= 16\left[\frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta - \frac{1}{4}\cos^3\theta\, sen\theta - \frac{3}{4}\left(\frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta\right)\right] + C$$
$$= 16\left[\frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta - \frac{1}{4}\cos^3\theta\, sen\theta - \frac{3}{8}\theta - \frac{3}{16} sen\, 2\theta\right] + C$$

$$= 16\left[\frac{4-3}{8}\theta + \frac{4+(-3)}{16} sen\, 2\theta - \frac{1}{4}\cos^3\theta\, sen\theta\right] + C$$
$$= 2\theta + sen\, 2\theta - 4\cos^3\theta\, sen\theta + C$$
$$= 2\, arc\, sen\, \frac{x}{2} + 2\,\frac{x}{2}\,.\,4\frac{\sqrt{4-x^2}}{2} - 4\,.\left(\frac{\sqrt{4-x^2}}{2}\right)^3 .\,\frac{x}{2} + C$$
$$= 2\, arc\, sen\, \frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} - \frac{x(4-x^2)\sqrt{4-x^2}}{4} + C$$

50. $\int x^3\sqrt{x^2+3}\, dx$

Fazendo:

$$\sqrt{x^2+3} = \sqrt{3}\sec\theta$$
$$x = \sqrt{3}\, tg\theta$$
$$dx = \sqrt{3}\sec^2\theta\, d\theta$$

Temos:

$$I = \int 3\sqrt{3}\, tg^3\theta \,.\, \sqrt{3}\sec^3\theta \, d\theta$$

$$= 9\sqrt{3}\int tg^3\theta \,.\, \sec^3\theta \, d\theta$$

$$= 9\sqrt{3}\int \left(\sec^2\theta - 1\right) tg\ \theta \,.\, \sec^2\theta \,.\, \sec\theta \, d\theta$$

$$= 9\sqrt{3}\int \left(\sec^4\theta - \sec^2\theta\right).\ tg\ \theta \,.\, \sec\theta \, d\theta$$

$$= 9\sqrt{3}\left(\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3}\right) + C$$

$$= \frac{9\sqrt{3}}{5}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)^5 - \frac{9\sqrt{3}}{3}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)^3 + C$$

$$= \frac{1}{5}\left(\sqrt{x^2+3}\right)^5 - \left(\sqrt{x^2+3}\right)^3 + C$$

51. $\int \frac{5x+4\ dx}{x^3\sqrt{x^2+1}}$

Fazendo

$$\sqrt{x^2+1} = \sec\theta$$

$$x = tg\ \theta$$

$$dx = \sec^2\ \theta\ d\theta$$

Temos:

$$I = \int \frac{5tg\theta + 4}{tg^3\theta.\sec\theta}.\sec^2\theta.d\theta$$

$$= \int \frac{5tg\theta + 4}{tg^3\theta}.\sec\theta.d\theta$$

$$= \int \frac{5\sec\theta}{tg^2\theta}d\theta + \int \frac{4\sec\theta}{tg^3\theta}d\theta$$

$$= \int 5sen^{-2}\theta\cos\theta\ d\theta + \int \frac{4\cos^2\theta}{sen^3\theta}d\theta$$

$$= \int 5sen^{-2}\theta\cos\theta\ d\theta + 4\int \cos\sec^3\theta d\theta - 4\int \cos\sec\theta d\theta$$

$$= \frac{-5}{sen\theta} - 2\cos\sec\theta\cot g\theta - 2\ln|\cos\sec\theta - \cot g\theta| + C$$

$$= \frac{-5\sqrt{x^2+1}}{x} - \frac{2\sqrt{x^2+1}}{x^2} - 2\ln\left|\frac{\sqrt{x^2+1}-1}{x}\right| + C.$$

52. $\int (x+1)^2\ \sqrt{x^2+1}dx$

Fazendo

$\sqrt{x^2+1} = \sec\theta$

$x = tg\ \theta$

$dx = \sec^2\ \theta\ d\theta$

Temos:

$$I = \int (tg\ \theta + 1)^2 . \sec\theta . \sec^2\theta\ d\theta$$

$$= \int (tg^2\theta + 2\ tg\ \theta + 1)\sec^3\theta\ d\theta$$

$$= \int (tg^2\theta + 1)\sec^3\theta\ d\theta + \int 2tg\ \theta \sec^3\theta\ d\theta$$

$$= \int \sec^5\theta\ d\theta + 2\int \sec^2\theta . \sec\theta . tg\ \theta\ d\theta$$

$$= \frac{1}{4}\sec^3\theta\ tg\ \theta + \frac{3}{4}\int \sec^3\theta\ d\theta + 2 . \frac{\sec^3\theta}{3}$$

$$= \frac{1}{4}\sec^3\theta\ tg\ \theta + \frac{3}{4}\left(\frac{1}{2}\sec\theta\ tg\theta + \frac{1}{2}\int \sec\theta\ d\theta\right) + \frac{2}{3}\sec^3\theta + C$$

$$= \frac{1}{4}\sec^3\theta\ tg\ \theta + \frac{3}{8}\ \sec\theta\ tg + \frac{3}{8}\ln\left|\sec\theta + tg\ \theta\right| + \frac{2}{3}\sec^3\theta + C$$

$$= \frac{1}{4}x\left(\sqrt{x^2+1}\right)^3 + \frac{3}{8}\sqrt{x^2+1} . x + \frac{3}{8}\ln\left|\sqrt{x^2+1} + x\right| + \frac{2}{3}\left(\sqrt{x^2+1}\right)^3 + C$$

$$= \frac{1}{4}x\left(x^2+1\right)\sqrt{x^2+1} + \frac{3}{8}x\sqrt{x^2+1} + \frac{2}{3}\left(x^2+1\right)\sqrt{x^2+1} + \frac{3}{8}\ln\left|\sqrt{x^2+1} + x\right| + C$$

53. $\displaystyle\int \frac{t^5}{\sqrt{t^2+16}}\,dt$

Fazendo

$\sqrt{t^2+16} = 4\sec\theta$

$t = 4\ tg\ \theta$

$dt = 4\sec^2\ \theta\ d\theta$

Temos:

$$I = \int \frac{4^5\ tg^5\theta\ .\ 4\sec^2\theta\ d\theta}{4\sec\theta}$$

$$= \int 4^5\ tg^5\theta \sec\theta\ d\theta$$

$$= 4^5 \int tg^2\theta\ .\ tg^2\theta\ .\ tg\ \theta\ .\sec\theta\ d\theta$$

$$= 4^5 \int \left(\sec^2\theta - 1\right)^2 .\ tg\ \theta\ .\sec\theta\ d\theta$$

$$= 4^5 \int \left(\sec^4\theta - 2\sec^2\theta + 1\right) tg\ \theta \sec\theta\ d\theta$$

$$= 4^5 \left[\int \sec^4\theta\ tg\ \theta \sec\theta\ d\theta - 2\int \sec^2\theta\ \ .\ tg\ \theta\ .\sec\theta\ d\theta + \int tg\ \theta\ .\sec\theta\ d\theta\right]$$

$$= 4^5 \left[\frac{\sec^5\theta}{5} - 2\ .\frac{\sec^3\theta}{3} + \sec\theta\right] + C$$

$$= \frac{4^5}{5}\left(\frac{\sqrt{t^2+16}}{4}\right)^5 - \frac{4^4 . 8}{3}\left(\frac{\sqrt{t^2+16}}{4}\right)^3 + 4^5\left(\frac{\sqrt{t^2+16}}{4}\right) + C$$

$$= \frac{1}{5}\left(t^2+16\right)^2\sqrt{t^2+16} - \frac{32}{3}\left(t^2+16\right)^2\sqrt{t^2+16} + 256\sqrt{t^2+16} + C$$

54. $\int \frac{e^x}{\sqrt{e^{2x}+1}}\, dx$

Fazendo

$u = e^x \quad \rightarrow \quad u^2 = e^{2x}$

$du = e^x dx$

Temos:

$$I = \int \frac{du}{\sqrt{u^2+1}}$$

Considerando:

$\sqrt{u^2+1} = \sec\theta$

$u = tg\ \theta$

$du = \sec^2\theta\ d\theta$

Finalizamos:

$$I = \int \frac{\sec^2 \theta \, d\theta}{\sec \theta}$$
$$= \sec \theta \, d\theta$$
$$= \ln \left| \sec \theta + tg \ \theta \right| + C$$
$$= \ln \left| \sqrt{u^2 + 1} + u \right| + C$$
$$= \ln \left| \sqrt{e^{2x} + 1} + e^x \right| + C$$

55. $\int \frac{x^2}{\sqrt{2 - x^2}} dx$

Fazendo

$$\sqrt{2 - x^2} = \sqrt{2} \cos \theta$$
$$x = \sqrt{2} \ sen \ \theta$$
$$dx = \sqrt{2} \cos \theta \ d\theta$$

Temos:

$$I = \int \frac{2 \ sen^2\theta}{\sqrt{2} \cos \theta} . \sqrt{2} \cos \theta \ d\theta$$
$$= 2 \int sen^2\theta \ d\theta$$
$$= 2 \int \frac{1 - \cos 2 \theta}{2} d\theta$$
$$= \int (1 - \cos 2\theta) \, d\theta$$
$$= \theta - \frac{1}{2} sen \ 2\theta + C$$
$$= \theta - sen \ \theta \cos \theta + C$$
$$= arc \ sen \ \frac{x}{\sqrt{2}} - \frac{1}{2} x \sqrt{2 - x^2} + C$$

56. $\int \frac{e^x}{\sqrt{4 - e^{2x}}} dx$

Fazendo

$$u^2 = e^{2x}$$
$$u = e^x$$
$$du = e^x dx$$

Temos:

$$I = \int \frac{du}{\sqrt{4-u^2}}$$

Considerando:

$$\sqrt{4-u^2} = 2\cos\theta$$

$$u = 2\, sen\, \theta$$

$$du = 2\cos\theta\, d\theta$$

Obtemos:

$$I = \int \frac{2\cos\theta\, d\theta}{2\cos\theta} = \theta + C$$

$$= arc\ sen\ \frac{u}{2} + C$$

$$= arc\ sen\left(\frac{e^x}{2}\right) + C$$

57. $\int \frac{x+1}{\sqrt{x^2-1}}dx$

Fazendo

$$\sqrt{x^2-1} = tg\ \theta$$

$$x = \sec\theta$$

$$dx = \sec\theta\, .\, tg\ \theta\ d\theta$$

Temos:

$$I = \int \frac{\sec\theta + 1}{tg\theta}\, .\sec\theta\, tg\ \theta\, .\, d\theta$$

$$= \int \left(\sec^2\theta + \sec\theta\right)d\theta$$

$$= tg\theta + \ln\left|\sec\theta + tg\theta\right| + c$$

$$= \sqrt{x^2-1} + \ln\left|x + \sqrt{x^2-1}\right| + C$$

58. $\int \frac{\sqrt{x^2-1}}{x^2}dx$

Fazendo:

$$\sqrt{x^2-1} = tg\theta$$

$$x = \sec\theta$$

$$dx = \sec\theta\, tg\theta\, d\theta$$

Temos:

$$I=\int\frac{tg\theta}{\sec^2\theta}\sec\theta . tg\theta . d\theta$$
$$=\int tg^2\theta .\frac{1}{\sec\theta}d\theta$$
$$=\int\frac{\sec^2\theta-1}{\sec\theta}d\theta$$
$$=\int\sec\theta d\theta-\int\cos\theta d\theta$$
$$=\ln\left|\sec\theta+tg\theta\right|-sen\theta+C$$
$$=\ln\left|x+\sqrt{x^2-1}\right|-\frac{\sqrt{x^2-1}}{x}+C$$

59. $\int\frac{\sqrt{1+x^2}}{x^3}dx$

Fazendo:

$$\sqrt{x^2-1}=\sec\theta$$
$$x=tg\theta$$
$$dx=\sec^2\theta d\theta$$

$$I=\int\frac{\sec^3\theta}{tg^3\theta}\,d\theta$$
$$=\int\frac{1}{\cos^3\theta}\cdot\frac{\cos^3\theta}{sen^3\theta}\,d\theta$$
$$=\int\cos\sec^3\theta\,d\theta$$
$$=-\frac{1}{2}\cos\sec\theta\cot g\ \theta+\frac{1}{2}\ln\left|\cos\sec\theta-\cot g\ \theta\right|+C$$
$$=-\frac{1}{2}\ \frac{\sqrt{1+x^2}}{x}\cdot\frac{1}{x}+\frac{1}{2}\ln\left|\frac{\sqrt{1+x^2}}{x}-\frac{1}{x}\right|+C$$

60. $\int\frac{x+1}{\sqrt{4-x^2}}dx$

Fazendo:

$$\sqrt{4-x^2}=2\cos\theta$$
$$x=2\ sen\ \theta$$
$$dx=2\cos\theta\ d\theta$$

Temos:

$$I = \int \frac{2 sen\,\theta + 1}{2\cos\theta} . 2\cos\theta\, d\theta$$

$$= \int (2\, sen\,\theta + 1)\, d\theta$$

$$= -2\cos\theta + C$$

$$= -\sqrt{4 - x^2} + arc\ sen\frac{x}{2} + C$$

61. $\int \frac{6x+5}{\sqrt{9x^2+1}}dx$

Fazendo:

$u^2 = 9x^2$

$u = 3x$

$du = 3dx$

Temos:

$$I = \int \frac{(2u+5)}{\sqrt{u^2+1}} \frac{1}{3} du$$

Considerando:

$\sqrt{u^2+1} = \sec\theta$

$u = tg\ \theta$

$du = \sec^2\theta\, d\theta$

Obtemos:

$$I = \frac{1}{3}\int \frac{(2tg\ \theta + 5)\sec^2\theta\, d\theta}{\sec\theta}$$

$$= \frac{1}{3}\int 2tg\ \theta \sec\theta\, d\theta + \frac{1}{3}\int 5\sec\theta\, d\theta$$

$$= \frac{2}{3}\sec\theta + \frac{5}{3}\ln|\sec\theta + tg\theta| + C$$

$$= \frac{2}{3}\sqrt{u^2+1} + \frac{5}{3}\ln\left|\sqrt{u^2+1} + u\right| + C$$

$$= \frac{2}{3}\sqrt{9x^2+1} + \frac{5}{3}\ln\left|\sqrt{9x^2+1} + 3x\right| + C$$

62. $\int \frac{(x+3)dx}{\sqrt{x^2+2x}}$

Fazendo:

$$x^2 + 2x = (x+1)^2 - 1$$
$$u^2 = (x+1)^2$$
$$u = x+1$$
$$du = dx$$
Temos:
$$I = \int \frac{(u-1)+3}{\sqrt{u^2-1}}\,du$$
Considerando:
$$\sqrt{u^2-1} = tg\ \theta$$
$$u = \sec\theta$$
$$du = \sec\theta \,.\, tg\ \theta\ d\theta$$
Obtemos:
$$= \int \frac{\sec\theta + 2}{tg\ \theta} \,.\, \sec\theta \,.\, tg\ \theta\ d\theta$$
$$= \int \left(\sec^2\theta + 2\sec\theta\right) d\theta$$
$$= tg\theta + 2\ln\left|\sec\theta + tg\ \theta\right| + C$$
$$= \sqrt{u^2-1} + 2\ln\left|u + \sqrt{u^2-1}\right| + C$$
$$= \sqrt{x^2+2x} + 2\ln\left|x+1+\sqrt{x^2+2x}\right| + C$$

63. $\int \sqrt{4-x^2}\,dx$

Fazendo:
$$\sqrt{4-x^2} = 2\cos\theta$$
$$x = 2\ sen\ \theta$$
$$dx = 2\cos\theta\ d\theta$$
Temos:
$$I = \int 2\cos\theta \,.\, 2\cos\theta\ d\theta$$
$$= 4\int \cos^2\theta\ d\theta$$
$$= 4\int \frac{1+\cos 2\theta}{2} d\theta$$
$$= 2\left(\theta + \frac{1}{2} sen\ 2\theta\right) + C$$
$$= 2\,\theta + sen\ 2\theta + C$$
$$= 2\ arc\ sen\ \frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + C$$

64. $\int \sqrt{x^2 - 4}\, dx$

Fazendo:

$\sqrt{x^2 - 4} = 2\, tg\ \theta$

$x = 2 \sec \theta$

$dx = 2 \sec \theta \,.\, tg\ \theta\, d\theta$

Temos:

$$I = \int 2\, tg\ \theta \,.\, 2 \sec \theta \,.\, tg\ \theta\, d\theta$$

$$= 4 \int tg^2 \theta \,.\, \sec \theta\, d\theta$$

$$= 4 \int \left(\sec^2 \theta - 1\right) . \sec \theta\, d\theta$$

$$= 4 \int \left(\sec^3 \theta - \sec \theta\right) d\theta$$

$$= 4 \left[\frac{1}{2} \sec \theta \,.\, tg\ \theta + \frac{1}{2} \int \sec \theta\, d\theta - \int \sec \theta\, d\theta\right]$$

$$= 2 \sec \theta \,.\, tg\ \theta - 2 \ln \left|\sec \theta + tg\ \theta\right| + C$$

$$= 2 \,.\, \frac{x}{2}\, \frac{\sqrt{x^2 - 4}}{2} - 2 \ln \left|\frac{x + \sqrt{x^2 - 4}}{2}\right| + C$$

65. $\int \sqrt{4 + x^2}\, dx$

Fazendo:

$\sqrt{4 + x^2} = 2 \sec \theta$

$x = 2\, tg\ \theta$

$dx = 2 \sec^2 \theta\, d\theta$

Temos:

$$I = \int 2 \sec \theta \,.\, 2 \sec^2 \theta\, d\theta$$

$$= 4 \int \sec^3 \theta\, d\theta$$

$$= 4 \left(\frac{1}{2} \sec \theta\, tg\ \theta + \frac{1}{2} \int \sec \theta\, d\theta\right)$$

$$= 2 \sec \theta\, tg\ \theta + 2 \ln \left|\sec \theta + tg\ \theta\right| + \overline{C}$$

$$= 2\, \frac{\sqrt{4 + x^2}}{2} \,.\, \frac{x}{2} + 2 \ln \left|\frac{\sqrt{4 + x^2}}{2} + \frac{x}{2}\right| + \overline{C}$$

$$= \frac{x\sqrt{4 + x^2}}{2} + 2 \ln \left|\frac{\sqrt{4 + x^2} + x}{2}\right| + \overline{C}$$

$$= \frac{x\sqrt{4 + x^2}}{2} + 2 \ln \left|\sqrt{4 + x^2} + x\right| + C$$

66. $\int \left(\sqrt{1+x^2}+2x\right)dx$

Fazendo:

$\sqrt{1+x^2}=\sec\theta$

$x = tg\ \theta$

$dx=\sec^2\theta\ d\theta$

Temos:

$I=\int(\sec\theta+2\ tg\ \theta)\sec^2\theta\ d\theta$

$=\int\left(\sec^3+2\ tg\ \theta\sec\theta\sec\theta\right)d\theta$

$=\frac{1}{2}\sec\theta\ tg\ \theta+\frac{1}{2}\int\sec\theta\ d\theta+2\frac{\sec^2\theta}{2}+\overline{C}$

$=\frac{1}{2}\sec\theta\ tg\ \theta+\frac{1}{2}\ln\left|\sec\theta+tg\ \theta\right|+\sec^2\theta+\overline{C}$

$=\frac{1}{2}\sqrt{1+x^2}\ .\ x+\frac{1}{2}\ln\left|\sqrt{1+x^2}+x\right|+1+x^2+\overline{C}$

$=\frac{1}{2}x\sqrt{1+x^2}+x^2+\frac{1}{2}\ln\left|x+\sqrt{1+x^2}\right|+C$

67. $\int\left(sen\ x+\frac{x^2}{\sqrt{1+x^2}}\right)dx$

$=\int sen\ x\ dx+\int\frac{x^2dx}{\sqrt{1+x^2}}$

$=-\cos\ x+\int\frac{x^2dx}{\sqrt{1+x^2}}$

Fazendo:

$\sqrt{1+x^2}=\sec\theta$

$x=tg\ \theta$

$dx=\sec^2\theta\ d\theta$

Temos:

$$I = -\cos x + \int \frac{tg^2\theta \,.\, \sec^2\theta \, d\theta}{\sec\theta}$$

$$= -\cos x + \int tg^2\theta \,.\, \sec\theta \, d\theta$$

$$= -\cos x + \int \left(\sec^2\theta - 1\right) \sec\theta \, d\theta$$

$$= -\cos x + \int \left(\sec^3\theta - \sec\theta\right) d\theta$$

$$= -\cos x + \frac{1}{2}\sec\theta \, tg\,\theta + \frac{1}{2}\int \sec\theta \, d\theta - \int \sec\theta \, d\theta$$

$$= -\cos x + \frac{1}{2}\sec\theta \, tg\,\theta - \frac{1}{2}\ln\left|\sec\theta + tg\,\theta\right| + C$$

$$= -\cos x + \frac{1}{2}\sqrt{1+x^2} \,.\, x - \frac{1}{2}\ln\left|\sqrt{1+x^2} + x\right| + C$$

Nos exercícios de 68 a 72, calcular a integral definida.

68. $\displaystyle\int_0^1 \frac{dx}{\sqrt{3x^2+2}}$

Fazendo:

$u^2 = 3x^2$

$u = \sqrt{3}\, x$

$du = \sqrt{3}\, dx$

Temos:

$$I = \int \frac{dx}{\sqrt{3x^2+2}} = \int \frac{\frac{1}{\sqrt{3}}du}{\sqrt{u^2+2}} = \frac{1}{\sqrt{3}}\int \frac{du}{\sqrt{u^2+2}}$$

Considerando:

$\sqrt{u^2+2} = \sqrt{2}\sec\theta$

$u = \sqrt{2}\, tg\,\theta$

$du = \sqrt{2}\sec^2\theta \, d\theta$

Obtemos:

$$I = \frac{1}{\sqrt{3}} \int \frac{\sqrt{2}\,\sec^2\theta\, d\theta}{\sqrt{2}\,\sec\theta}$$

$$= \frac{1}{\sqrt{3}} \int \sec\theta\, d\theta = \frac{1}{\sqrt{3}} \ln\left|\sec\theta + tg\ \theta\right| + C$$

$$= \frac{1}{\sqrt{3}} \ln\left|\frac{\sqrt{u^2+2}}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{3}} \ln\left|\frac{\sqrt{3x^2+2}}{\sqrt{2}} + \frac{\sqrt{3}x}{\sqrt{2}}\right| + C$$

Assim,

$$\int_0^1 \frac{dx}{\sqrt{3x^2+2}} = \frac{1}{\sqrt{3}} \ln\left|\frac{\sqrt{3x^2+2}+\sqrt{3}x}{\sqrt{2}}\right|\Bigg|_0^1$$

$$= \frac{1}{\sqrt{3}}\left[\ln\left|\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}\right| - \ln\left|\frac{\sqrt{2}+0}{\sqrt{2}}\right|\right]$$

$$= \frac{1}{\sqrt{3}} \ln\left(\frac{\sqrt{3}+\sqrt{5}}{\sqrt{2}}\right)$$

69. $\int_0^{\frac{a}{2b}} \sqrt{a^2 - b^2x^2}\, dx$, $0 < a < b$

Fazendo:

$u^2 = b^2x^2$

$u = b\ x$

$du = b\ dx$

Temos:

$$I = \int \sqrt{a^2-u^2}\ .\frac{1}{b}\, du$$

Considerando:

$\sqrt{a^2-u^2} = a\cos\theta$

$u = asen\theta$

$du = a\cos\theta d\theta$

$$I = \frac{1}{b}\int a\cos\theta \,.\, a\cos\theta\, d\theta$$

$$= \frac{a^2}{b}\int \cos^2\theta\, d\theta$$

$$= \frac{a^2}{b}\left(\frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta\right) + C$$

$$= \frac{a^2}{b}\left(\frac{1}{2} arc\ sen\, \frac{u}{a} + \frac{1}{4} \,.\, \frac{2u\sqrt{a^2-u^2}}{a^2}\right) + C$$

$$= \frac{a^2}{b}\left(\frac{1}{2} arc\ sen\, \frac{bx}{a} + \frac{bx\sqrt{a^2-b^2x^2}}{2a^2}\right) + C$$

Portanto,

$$\int_0^{\frac{a}{2b}} \sqrt{a^2-b^2x^2}\, dx = \frac{a^2}{b}\left(\frac{1}{2} arc\ sen\, \frac{bx}{a} + \frac{bx\sqrt{a^2-b^2x^2}}{2a^2}\right)\Bigg|_0^{\frac{a}{2b}}$$

$$= \frac{a^2}{b}\left(\frac{1}{2} arc\ sen \frac{1}{2} + \frac{b \,.\, \frac{a}{2b}\sqrt{a^2 - b^2 \,.\, \frac{a^2}{4b^2}}}{2a^2}\right)$$

$$= \frac{a^2}{b}\left(\frac{1}{2} \,.\, \frac{\pi}{6} + \frac{\frac{a}{2}\sqrt{\frac{4a^2-a^2}{4}}}{2a^2}\right)$$

$$= \frac{a^2}{b}\left(\frac{\pi}{12} + \frac{\sqrt{3}}{8}\right)$$

70. $\displaystyle\int_1^2 \frac{dt}{t^4\sqrt{4+t^2}}$

Fazendo:

$\sqrt{4+t^2} = 2\sec\theta$

$t = 2\, tg\ \theta$

$dt = 2\sec^2\theta\, d\theta$

Temos:

$$I=\int\frac{dt}{t^4\sqrt{4+t^2}}=\int\frac{2\sec^2\theta\,d\theta}{16\,tg^4\theta\,.\,2\sec\theta}$$

$$=\frac{1}{16}\int\frac{\sec\theta\,d\theta}{tg^4\theta}$$

$$=\frac{1}{16}\int\frac{1}{\cos\theta}\cdot\frac{\cos^4\theta}{sen^4\theta}d\theta$$

$$=\frac{1}{16}\int\frac{\cos^3\theta}{sen^4\theta}\,d\theta$$

$$=\frac{1}{16}\int sen^{-4}\theta\left(1-sen^2\theta\right)\cos\theta\,d\theta$$

$$=\frac{1}{16}\int\left(sen^{-4}\theta\cos\theta-sen^{-2}\theta\cos\theta\right)d\theta$$

$$=\frac{1}{16}\,\frac{sen^{-3}\theta}{-3}-\frac{1}{16}\,\frac{sen^{-1}\theta}{-1}+C$$

$$=-\frac{1}{48\,sen^3\theta}+\frac{1}{16sen\theta}+C$$

$$=-\frac{1}{48}\cos\sec^3\theta+\frac{1}{16}\cos\sec\theta+C$$

$$=-\frac{1}{48}\,\frac{\sqrt{4+t^2}^{\,3}}{t^3}+\frac{1}{16}\frac{\sqrt{4+t^2}}{t}+C$$

Assim,

$$\int_1^2\frac{dt}{t^4\sqrt{4+t^2}}=\left[\frac{-1}{48}\,\frac{\sqrt{4+t^2}^{\,3}}{t^3}+\frac{\sqrt{4+t^2}}{16t}\right]_1^2$$

$$=\frac{-1}{48}\left(\frac{\sqrt{8^3}}{8}-\frac{\sqrt{5^3}}{1}\right)+\frac{1}{16}\left(\frac{\sqrt{8}}{2}-\frac{\sqrt{5}}{1}\right)$$

$$=\frac{-\sqrt{2}}{24}+\frac{5\sqrt{5}}{48}+\frac{\sqrt{2}}{16}-\frac{\sqrt{5}}{16}$$

$$=\frac{1}{48}\left(\sqrt{2}+2\sqrt{5}\right)$$

71. $\displaystyle\int_{\sqrt{2}}^{\sqrt{3}}\frac{dt}{t^2\sqrt{9t^2+16}}$

Fazendo:

$u^2 = 9t^2$

$u = 3t$

$du = 3dt$

Temos:

$$I = \int \frac{\frac{1}{3}du}{\frac{u^2}{9}\sqrt{u^2+16}}$$

Considerando:

$\sqrt{u^2+16} = 4\sec\theta$

$u = 4tg\ \theta$

$du = 4\sec^2\theta\, d\theta$

Obtemos:

$$I = \frac{\frac{1}{3}}{\frac{1}{9}}\int \frac{4\sec^2\theta\, d\theta}{16\, tg^2\theta\, .\, 4\sec\theta} = 3\, .\, \frac{1}{4\, .\, 4}\int \frac{\sec\theta\, d\theta}{tg^2\theta}$$

$$= \frac{3}{16}\int \frac{1}{\cos\theta}\, .\, \frac{\cos^2\theta}{sen^2\theta} d\theta$$

$$= \frac{3}{16}\int \frac{\cos\theta\, d\theta}{sen^2\theta}$$

$$= \frac{3}{16}\int sen^{-2}\theta\, .\cos\theta\, d\theta \qquad \text{I}$$

$$= -\frac{3}{16}\cos\sec\theta + C$$

$$= \frac{-3}{16}\, .\, \frac{\sqrt{u^2+16}}{u} + C$$

$$= \frac{-3}{16}\, .\, \frac{\sqrt{9t^2+16}}{3t} + C$$

Assim,

$$\int_{\sqrt{2}}^{\sqrt{3}} \frac{dt}{t^2\sqrt{9t^2+16}} = \frac{-3}{16}\, .\, \frac{\sqrt{9t^2+16}}{3t}\Bigg|_{\sqrt{2}}^{\sqrt{3}}$$

$$= \frac{-1}{16}\left(\frac{\sqrt{43}}{3} - \sqrt{17}\right)$$

72. $\displaystyle\int_{6}^{7} \frac{dt}{(t-1)^2\sqrt{(t-1)^2-9}}$

Fazendo:

$u^2 = (t-1)^2$

$u = t-1$

$du = dt$

Temos:

$$I = \int \frac{du}{u^2\sqrt{u^2-9}}$$

Considerando:

$\sqrt{u^2-9} = 3\,tg\,\theta$

$u = 3\sec\theta$

$u = 3\sec\theta\, tg\,\theta\, d\theta$

Obtemos:

$$I = \int \frac{3\sec\theta\,.\,tg\,\theta\,d\theta}{9\sec^2\theta\,.\,3\,.\,tg\,\theta}$$

$$= \int \frac{d\theta}{9\sec\theta} = \frac{1}{9}\int \cos\theta\,d\theta$$

$$= \frac{1}{9}\,sen\theta + C$$

$$= \frac{1}{9}\frac{\sqrt{u^2-9}}{u} + C$$

$$= \frac{1}{9}\frac{\sqrt{(t-1)^2-9}}{t-1} + C$$

Assim,

$$\int_6^7 \frac{dt}{(t-1)^2\sqrt{(t-1)^2-9}} = \frac{1}{9}\left.\frac{\sqrt{(t-1)^2-9}}{t-1}\right|_6^7$$

$$= \frac{1}{9}\left(\frac{\sqrt{27}}{6} - \frac{4}{5}\right)$$

Nos exercícios 73 a 76, verificar se a integral imprópria converge. Em caso positivo, determinar seu valor.

73. $\displaystyle\int_3^{10} \frac{dx}{x^2\sqrt{x^2-9}}$

$$I = \int_3^{10} \frac{dx}{x^2\sqrt{x^2-9}} = \lim_{r\to 3^+}\int_r^{10} \frac{dx}{x^2\sqrt{x^2-9}}$$

$$I = \int \frac{dx}{x^2 \sqrt{x^2 - 9}}$$

Fazendo:

$x = 3\sec\theta \qquad\qquad dx = 3\sec\theta \, tg\, \theta \, d\theta$

Temos:

$$I_1 = \int \frac{3\sec\theta \, tg\theta \, d\theta}{9\sec^2\theta\sqrt{9\sec^2\theta - 9}} = \frac{1}{3}\int \frac{tg\theta \, d\theta}{\sec\theta \, 3\, tg\,\theta}$$

$$= \frac{1}{9}\int \frac{d\theta}{\sec\theta} = \frac{1}{9}\int \cos\theta \, d\theta = \frac{1}{9} sen\theta + c$$

$$= \frac{1}{9}\frac{\sqrt{x^2 - 9}}{x} + c$$

$$I = \lim_{r\to 3^+} \frac{1}{9}\frac{\sqrt{x^2 - 9}}{x}\Bigg|_r^{10}$$

$$= \frac{1}{9}\frac{\sqrt{10^2 - 9}}{10} - \lim_{r\to 3^+}\frac{1}{9}\frac{\sqrt{r^2 - 9}}{r}$$

$$= \frac{\sqrt{91}}{90} - \frac{1}{9}\frac{\sqrt{9 - 9}}{3} = \frac{\sqrt{91}}{90}$$

Portanto, a integral converge e tem como resultado $\frac{\sqrt{91}}{90}$.

74. $\int_3^{+\infty} \frac{dx}{\sqrt{x^2 - 4}}$

$$I = \int_3^{+\infty} \frac{dx}{\sqrt{x^2 - 4}} = \lim_{b\to+\infty}\int_3^b \frac{dx}{\sqrt{x^2 - 4}}$$

Fazendo:

$x = 2\sec\theta$

$dx = 2\sec\theta \, tg\, \theta \, d\theta$

$$I_1 = \int \frac{dx}{\sqrt{x^2-4}} = \int \frac{2\sec\theta\, tg\theta\, d\theta}{\sqrt{4\sec^2\theta - 4}}$$

$$= \int \frac{2\sec\theta\, tg\theta\, d\theta}{2\, tg\,\theta} = \int \sec\theta\, d\theta = \ln\left|\sec\theta + tg\,\theta\right| + c$$

$$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + c$$

$$I = \lim_{b\to+\infty} \ln\left|\frac{x}{2} + \sqrt{\frac{x^2-4}{2}}\right|\Bigg|_3^b$$

$$= \lim_{b\to+\infty} \ln\left|\frac{b}{2} + \sqrt{\frac{b^2-4}{2}}\right| - \ln\left|\frac{3}{2} + \sqrt{\frac{5}{2}}\right|$$

$$= +\infty$$

Portanto, a integral diverge.

75. $\displaystyle\int_0^1 \frac{dx}{(1-x^2)^{3/2}}$

$$I = \int_0^1 \frac{dx}{\left(1-x^2\right)^{3/2}} = \lim_{s\to1^-}\int_0^s \frac{dx}{\left(1-x^2\right)^{3/2}}$$

$$I_1 = \int \frac{dx}{\left(1-x^2\right)^{3/2}} = \int \frac{dx}{\left(\sqrt{1-x^2}\right)^3}$$

$x = sen\theta$
$dx = \cos\theta\, d\theta$

$$I_1 = \int \frac{\cos\theta\, d\theta}{\left(1-sen^2\theta\right)^{3/2}} = \int \frac{\cos\theta\, d\theta}{\left(\cos\theta\right)^3} = \int \frac{d\theta}{\cos^2\theta}$$

$$= \int \sec^2\theta\, d\,\theta = tg\,\theta + c = \frac{x}{\sqrt{1-x^2}} + c$$

$$I = \lim_{s\to1^-} \frac{x}{\sqrt{1-x^2}}\Bigg|_0^s = \lim_{s\to1^-}\frac{s}{\sqrt{1-s^2}} - 0 = +\infty$$

Portanto, a integral diverge.

76. $\int_{1}^{+\infty} \frac{dx}{x\sqrt{x^2+4}}$

$$I = \int_{1}^{+\infty} \frac{dx}{x\sqrt{x^2+4}} = \lim_{b \to +\infty} \int_{1}^{b} \frac{dx}{x\sqrt{x^2+4}}$$

$$I_1 = \int \frac{dx}{x\sqrt{x^2+4}}$$

$$x = 2\, tg\theta$$
$$dx = 2 \sec^2 \theta \, d\theta$$

$$I_1 = \int \frac{2\sec^2\theta \, d\theta}{2\, tg\theta \sqrt{4\, tg^2\theta + 4}} = \frac{1}{2}\int \frac{\sec^2\theta \, d\theta}{tg\theta \, . \sec\theta}$$
$$= \frac{1}{2}\int \frac{\sec\theta}{tg\theta}\, d\theta = \frac{1}{2}\int \frac{1}{\cos\theta}\,\frac{\cos\theta}{sen\,\theta}\, d\theta$$
$$= \frac{1}{2}\int \cos ec\,\theta \, d\theta = \frac{1}{2}\ln\left|\cos ec\,\theta - \cot g\,\theta\right| + c$$
$$= \frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} - \frac{2}{x}\right| + c$$

$$I = \lim_{b \to +\infty} \frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} - \frac{2}{x}\right|\Bigg|_1^b$$
$$= \lim_{b \to +\infty} \frac{1}{2}\ln\left(\frac{\sqrt{b^2+4}}{b} - \frac{2}{b}\right) - \frac{1}{2}\ln\left|\sqrt{5}-2\right|$$
$$= \frac{1}{2}\lim_{b \to +\infty} \ln\left(\sqrt{\frac{b^2+4}{b^2}} - \frac{2}{b}\right) - \frac{1}{2}\ln\left|\sqrt{5}-2\right|$$
$$= \frac{1}{2}\, .\, 0 - \frac{1}{2}\ln\left|\sqrt{5}-2\right|$$
$$= -\frac{1}{2}\ln\left|\sqrt{5}-2\right|$$

Portanto, a integral converge e tem como resultado $-\frac{1}{2}\ln\left|\sqrt{5}-2\right| \cong 0{,}7218$.