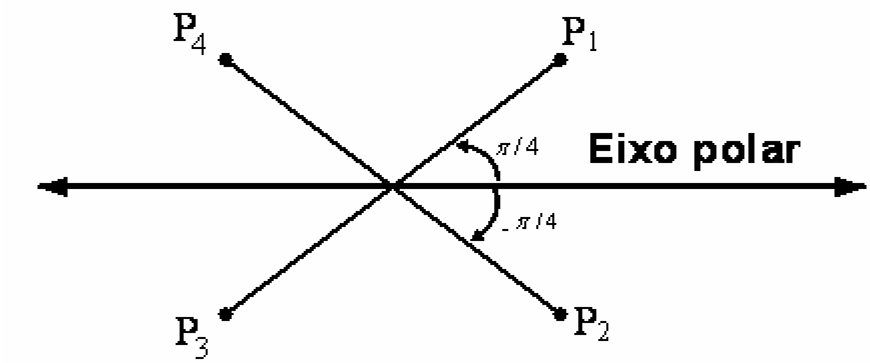
8.11 – EXERCÍCIOS – pg. 379

1. Demarcar os seguintes pontos no sistema de coordenadas polares.

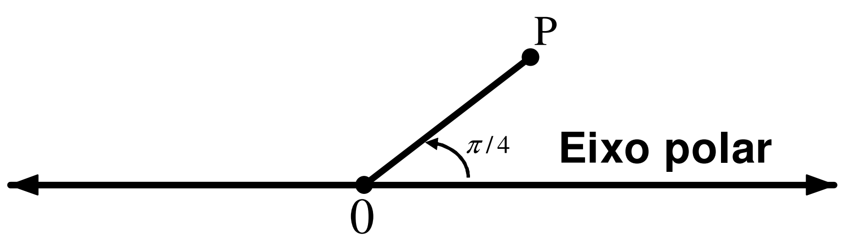
(a) $P_1\left(4, \frac{\pi}{4}\right)$
(b) $P_2\left(4, -\frac{\pi}{4}\right)$
(c) $P_3\left(-4, \frac{\pi}{4}\right)$
(d) $P_4\left(-4, -\frac{\pi}{4}\right)$



2. Em cada um dos itens, assinalar o ponto dado em coordenadas polares e depois escrever as coordenadas polares para o mesmo ponto tais que:

(i) r tenha sinal contrario
(ii) θ tenha sinal contrario

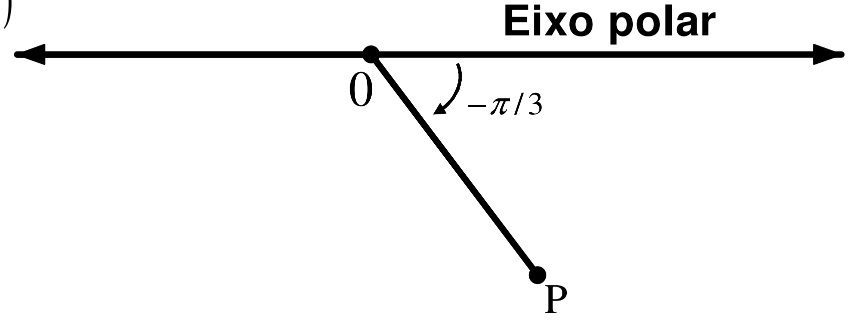
(a) $\left(2, \frac{\pi}{4}\right)$
(i) $\left(-2, \frac{5\pi}{4}\right)$
(ii) $\left(2, -\frac{7\pi}{4}\right)$



b) $\left(\sqrt{2}, -\pi/3\right)$

(i) $\left(-\sqrt{2}, -4\pi/3\right)$

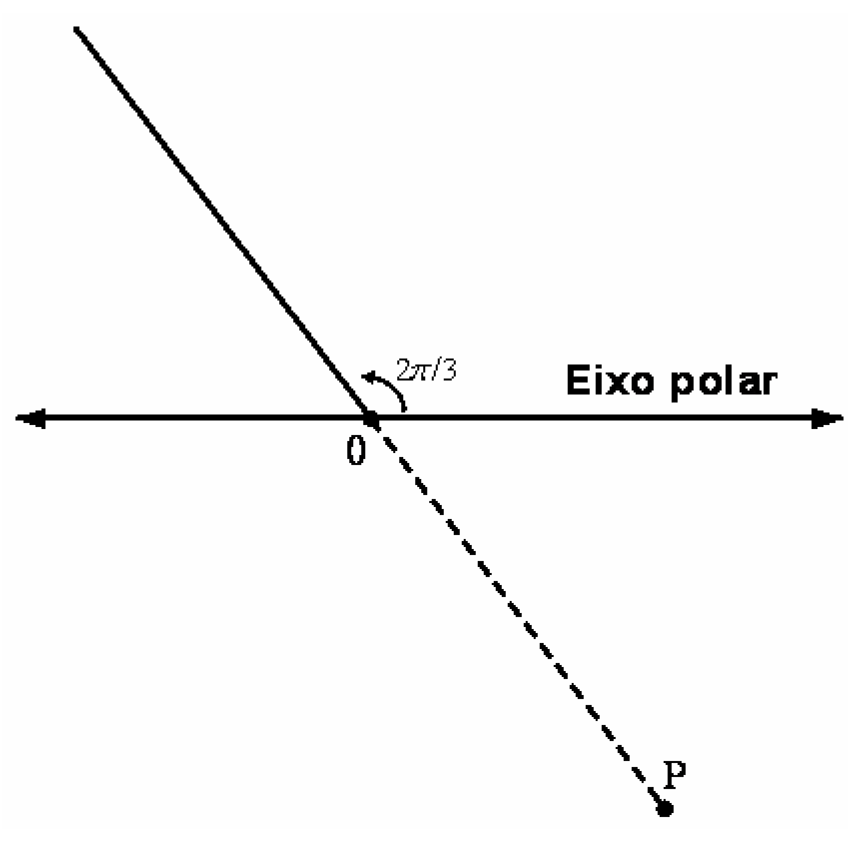
(ii) $\left(\sqrt{2}, 5\pi/3\right)$



(c) $\left(-5, 2\pi/3\right)$

(i) $\left(5, 5\pi/3\right)$

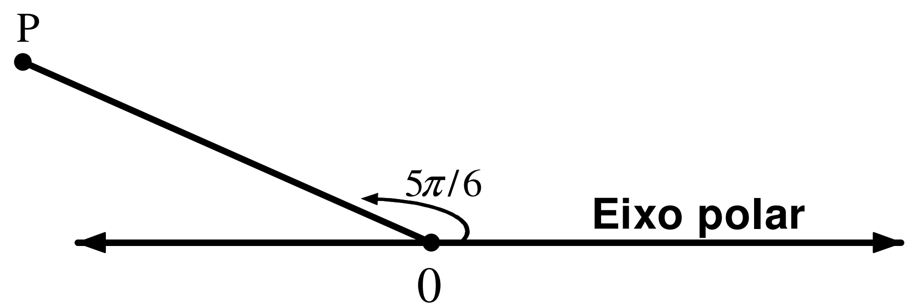
(ii) $\left(-5, -4\pi/3\right)$



(d) $\left(4, 5\pi/6\right)$

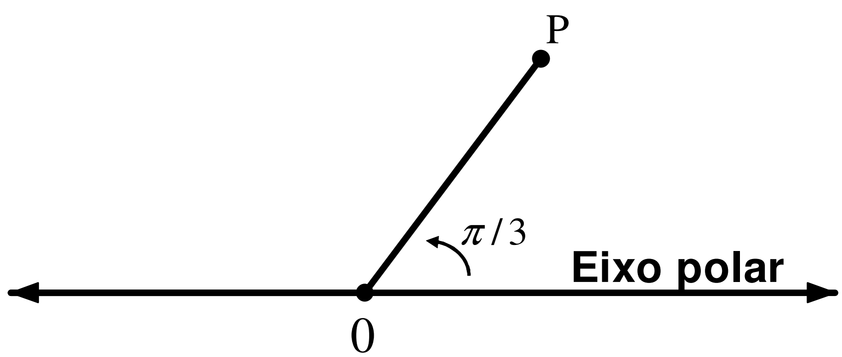
(i) $\left(-4, 11\pi/6\right)$

(ii) $\left(4, -7\pi/6\right)$



3. Demarcar os seguintes pontos no sistema de coordenadas polares e encontrar suas coordenadas cartesianas.

a) $\left(3, \pi/3\right)$



$x = r \cos \theta$

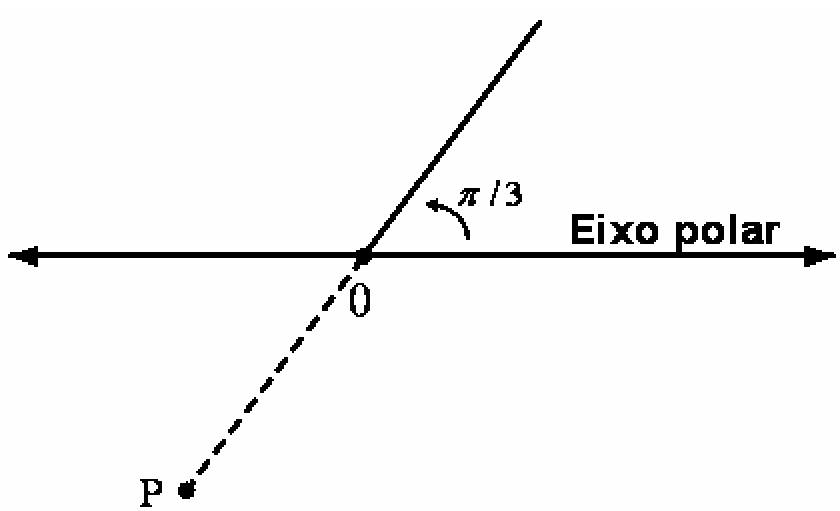
$x = 3 \cos \pi/3$ $\qquad$ $y = r \; sen \; \theta$

$x = 3 \, . \, \dfrac{1}{2}$ $\qquad$ $y = 3 \, . \, \dfrac{\sqrt{3}}{2} \cong 2{,}59$

$x = 1{,}5$

$$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right) \cong (1{,}5;\ 2{,}59)$$

b) $\left(-3, +\pi/3\right)$



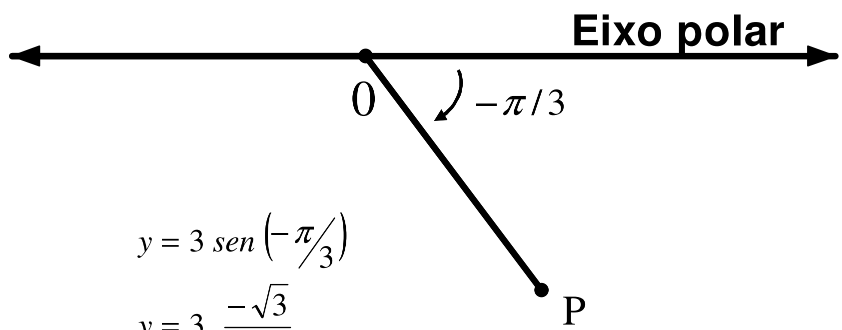
$x = -3 \cos \pi/3$ $\quad$ $y = -3\ sen\ \pi/3$

$x = -3 \,.\, \frac{1}{2}$ $\quad$ $y = -3 \,.\, \frac{\sqrt{3}}{2}$

$x = -1{,}5$ $\quad$ $y \cong -2{,}59$

$$\left(-\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right) \cong (-1{,}5; -2{,}59)$$

c) $\left(3, -\pi/3\right)$



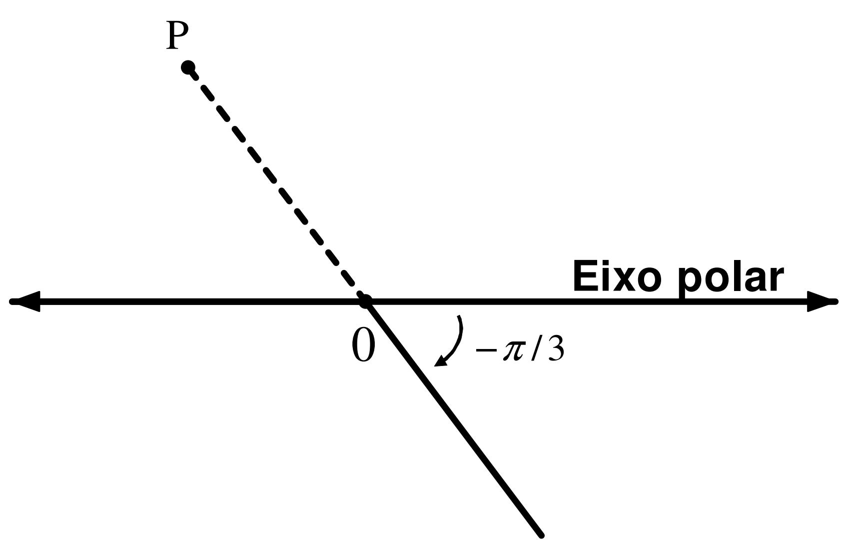
$x = 3 \cos\left(-\pi/3\right)$ $\quad$ $y = 3\ sen\left(-\pi/3\right)$

$x = 3 \,.\, \frac{1}{2}$ $\quad$ $y = 3 \,.\, \frac{-\sqrt{3}}{2}$

$x = 1{,}5$ $\quad$ $y \cong -2{,}59$

$$\left(\frac{3}{2}; \frac{-3\sqrt{3}}{2}\right) \cong (1{,}5; -2{,}59)$$

d) $\left(-3, -\pi/3\right)$



$x = -3\cos\left(-\pi/3\right)$ $\quad$ $y = -3\, sen\left(-\pi/3\right)$

$x = -3 \,.\, \frac{1}{2}$ $\quad$ $y = -3 \,.\, \frac{-\sqrt{3}}{2}$

$x = -1{,}5$ $\quad$ $y \cong 2{,}59$

$$\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right) \cong (-1{,}5;\ 2{,}59)$$

4. Encontrar as coordenadas cartesianas dos seguintes pontos dados em coordenadas polares.

a) $\left(-2, 2\pi/3\right)$

$x = -2\cos 2\pi/3 = -2 \,.\, -1/2 = 1$

$y = -2\, sen\, 2\pi/3 = -2 \,.\, \sqrt{3}/2 = -\sqrt{3}$

$\left(1, -\sqrt{3}\right)$

b) $\left(4, 5\pi/8\right)$

$x = 4\cos 5\pi/8 \cong -1{,}5307$

$y = 4\, sen\, 5\pi/8 \cong 3{,}6955$

$(-1{,}5307,\ 3{,}6955)$

c) $\left(3, 13\pi/4\right)$

$x = 3\cos 13\pi/4 = -3\frac{\sqrt{2}}{2}$

$y = 3\, sen\, 13\pi/4 = \frac{-3\sqrt{2}}{2}$

$$\left(\frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$$

d) $\left(-10, \pi/2\right)$

$x = -10 \cos \pi/2 = -10 \,.\, 0 = 0$

$(0, -10)$

$y = -10\, sen\, \pi/2 = -10 \,.\, 1 = -10$

e) $\left(-10, 3\pi/2\right)$

$x = -10 \cos 3\pi/2 = -10 \,.\, 0 = 0$

$(0, 10)$

$y = -10\, sen\, 3\pi/2 = -10 \,.\, -1 = 10$

f) $(1, 0)$

$x = 1 \cos 0 = 1$

$(1, 0)$

$y = 1\, sen\, 0 = 0$

5. Encontrar um par de coordenadas polares dos seguintes pontos:

a) $(1, 1)$

$r = \sqrt{2}$

$$\left.\begin{array}{l} \cos\theta = \dfrac{1}{\sqrt{2}} \\ sen\theta = \dfrac{1}{\sqrt{2}} \end{array}\right\} \Rightarrow \theta = \frac{\pi}{4}$$

$\left(\sqrt{2}, \pi/4\right)$

b) $(-1, 1)$

$r = \sqrt{2}$

$$\left.\begin{array}{l} \cos\theta = -\dfrac{1}{\sqrt{2}} \\ sen\theta = \dfrac{1}{\sqrt{2}} \end{array}\right\} \Rightarrow \theta = \frac{3\pi}{4}$$

$\left(\sqrt{2}, {}^{3\pi}\!/_{4}\right)$

c) $(-1, -1)$

$r = \sqrt{2}$

$$\left.\begin{array}{l} \cos\theta = -\dfrac{1}{\sqrt{2}} \\ sen\theta = -\dfrac{1}{\sqrt{2}} \end{array}\right\} \Rightarrow \theta = \frac{5\pi}{4}$$

$\left(\sqrt{2}, {}^{5\pi}\!/_{4}\right)$

d) $(1, -1)$

$r = \sqrt{2}$

$$\left.\begin{array}{l} \cos\theta = \dfrac{1}{\sqrt{2}} \\ sen\theta = -\dfrac{1}{\sqrt{2}} \end{array}\right\} \Rightarrow \theta = \frac{7\pi}{4}$$

$\left(\sqrt{2}, -{}^{\pi}\!/_{4}\right)$ ou $\left(\sqrt{2}, {}^{7\pi}\!/_{4}\right)$

6. Usar.

a) $r > 0$ e $0 \leq \theta < 2\pi$;

b) $r < 0$ e $0 \leq \theta < 2\pi$;

c) $r > 0$ e $-2\pi < \theta \leq 0$;

d) $r < 0$ e $-2\pi < \theta \leq 0$;

para escrever os pontos $P_1\left(\sqrt{3}, -1\right)$ e $P_2\left(-\sqrt{2}, -\sqrt{2}\right)$ em coordenadas polares.

$P_1\left(\sqrt{3}, -1\right)$

$r = 2$

$\cos\theta = \dfrac{\sqrt{3}}{2}$ $\qquad$ $sen\ \theta = \dfrac{-1}{2}$ $\qquad$ $\theta = \dfrac{-\pi}{6}$ ou $\theta = \dfrac{11\pi}{6}$

a) $\left(2, \dfrac{11\pi}{6}\right)$ $\qquad$ b) $\left(-2, \dfrac{5\pi}{6}\right)$ $\qquad$ c) $\left(2, \dfrac{-\pi}{6}\right)$ $\qquad$ d) $\left(-2, \dfrac{-7\pi}{6}\right)$

$P_2\left(-\sqrt{2}, -\sqrt{2}\right)$

$r = 2$

$\cos\theta = \dfrac{-\sqrt{2}}{2}$ $\qquad$ $sen\ \theta = \dfrac{-\sqrt{2}}{2}$ $\qquad$ $\theta = \dfrac{5\pi}{4}$

a) $\left(2, \dfrac{5\pi}{4}\right)$ $\qquad$ b) $\left(-2, \dfrac{\pi}{4}\right)$ $\qquad$ c) $\left(2, \dfrac{-3\pi}{4}\right)$ $\qquad$ d) $\left(-2, \dfrac{-7\pi}{4}\right)$

7. Transformar as seguintes equações para coordenadas polares.

a) $x^2 + y^2 = 4$

$$\begin{aligned} r^2\cos^2\theta + r^2 sen^2\theta &= 4 \\ r^2 &= 4 \\ r &= \pm 2 \end{aligned}$$

b) $x = 4$

$r\cos\theta = 4$

c) $y = 2$

$r\ sen\ \theta = 2$

d) $y + x = 0$

$$\begin{aligned} r\ sen\ \theta + r\cos\theta &= 0 \\ r\left(sen\ \theta + \cos\theta\right) &= 0 \end{aligned} \qquad \begin{cases} r = \text{qualquer} \\ sen\ \theta = -\cos\theta \end{cases}$$

$$\theta = \frac{3\pi}{4} + k\pi, k \in Z$$

e) $x^2 + y^2 - 2x = 0$

$$r^2 - 2r\cos\theta = 0$$
$$r - 2\cos\theta = 0$$
$$r = 2\cos\theta$$

f) $x^2 + y^2 - 6y = 0$

$$r^2 - 6r\, sen\theta = 0$$
$$r = 6\, sen\,\theta$$

8. Transformar as seguintes equações para coordenadas cartesianas

a) $r = \cos\theta$

$$\sqrt{x^2 + y^2} = \frac{x}{\sqrt{x^2 + y^2}}$$
$$x^2 + y^2 - x = 0$$

b) $r = 2\, sen\,\theta$

$$\sqrt{x^2 + y^2} = 2 \,.\, \frac{y}{\sqrt{x^2 + y^2}}$$
$$x^2 + y^2 - 2y = 0$$

c) $r = \dfrac{1}{\cos\theta + sen\,\theta}$

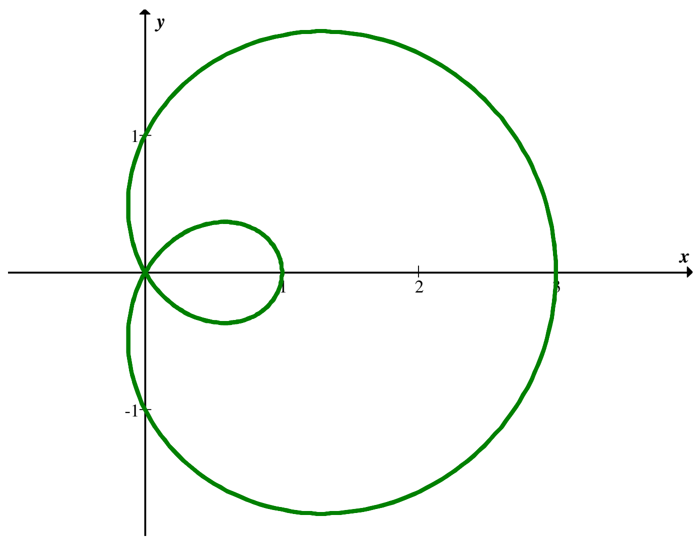
$$\sqrt{x^2 + y^2} = \frac{1}{\dfrac{x}{\sqrt{x^2 + y^2}} + \dfrac{y}{\sqrt{x^2 + y^2}}}$$
$$x + y = 1$$

d) $r = a,\ a > 0$

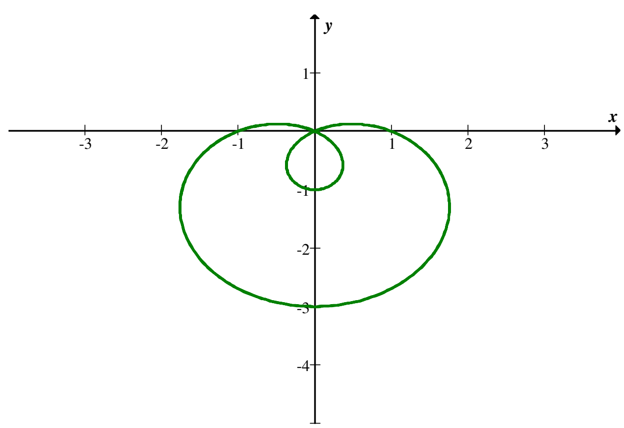
$$\sqrt{x^2 + y^2} = a$$
$$x^2 + y^2 = a^2$$

Nos exercícios de 9 a 32 esboçar o gráfico das curvas dadas em coordenadas polares.

9. $r = 1 + 2\cos\theta$



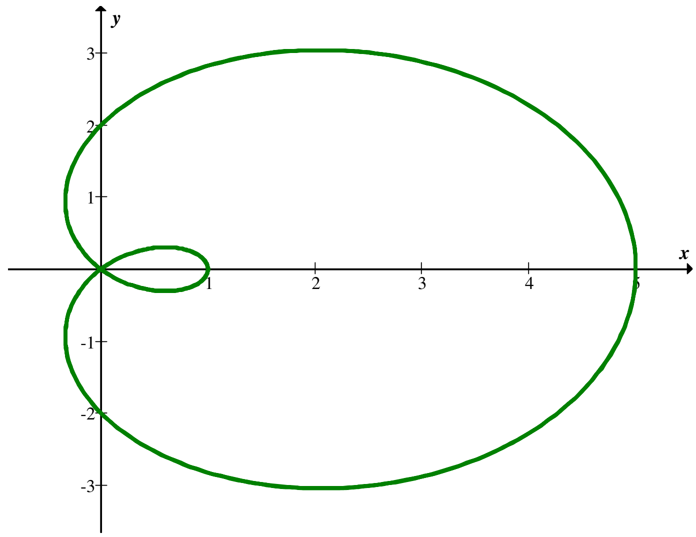
10. $r = 1 - 2\, sen\, \theta$



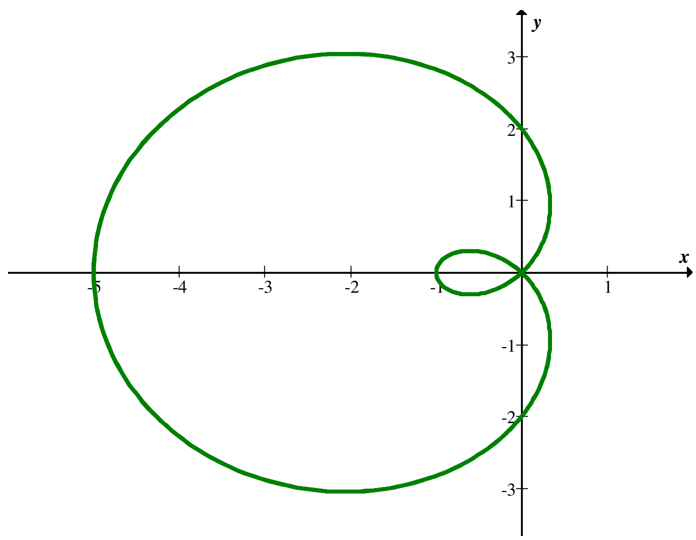
11. $r = a \pm b\cos\theta$

$a = 2$ e $b = 3$
$a = 3$ e $b = 2$
$a = b = 3$

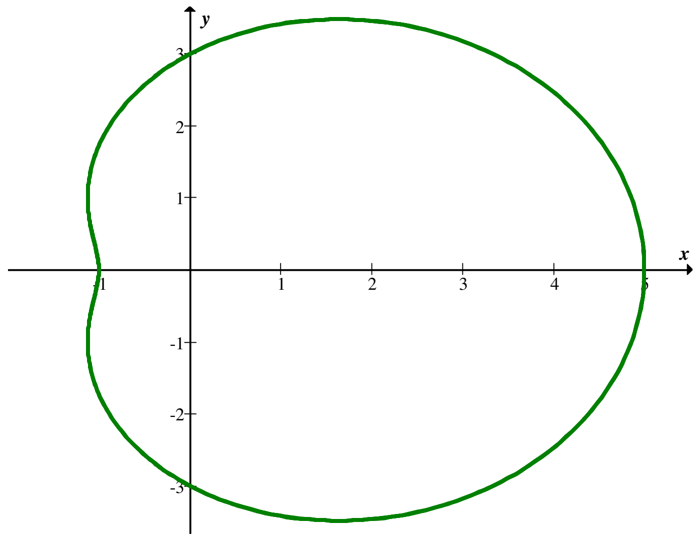
$r = 2 + 3\cos\theta$



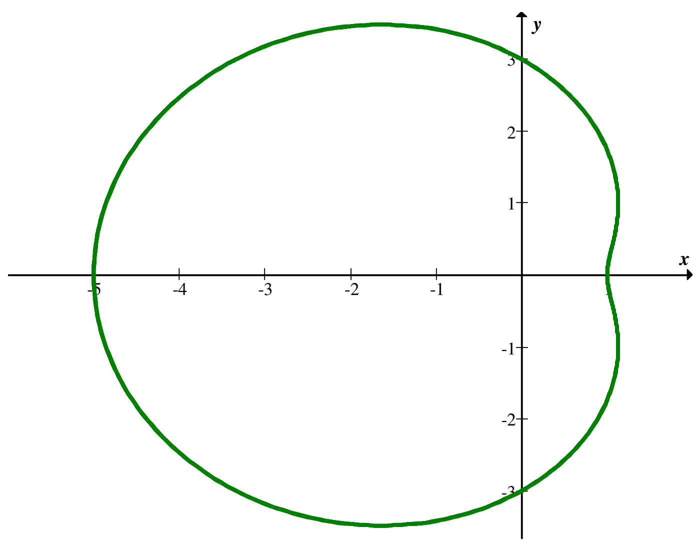
$r = 2 - 3\cos\theta$



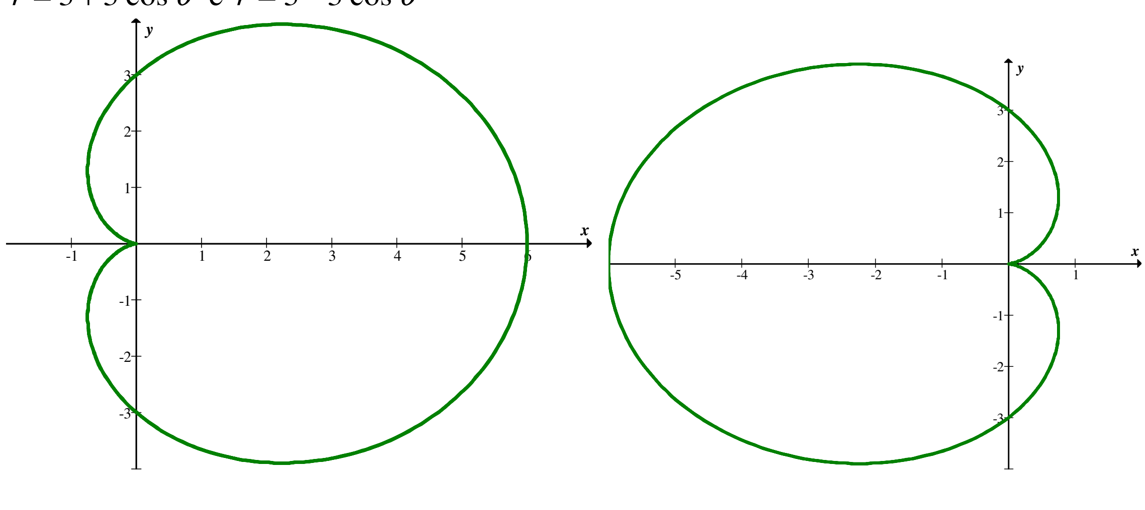
$r = 3 + 2\cos\theta$



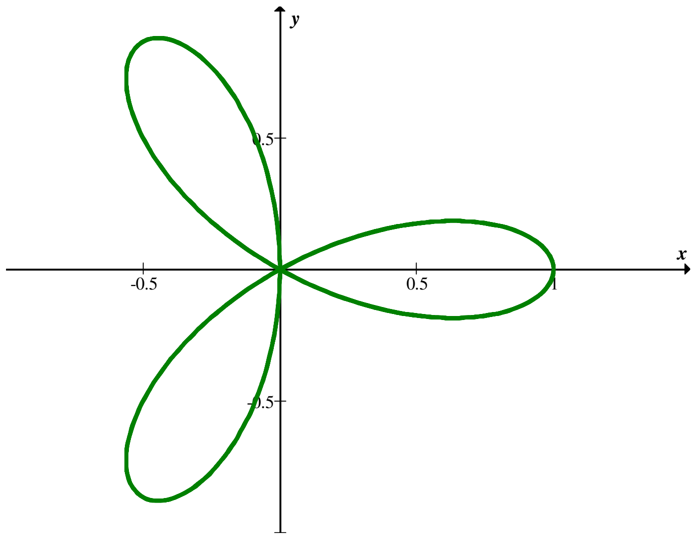
$r = 3 - 2\cos\theta$



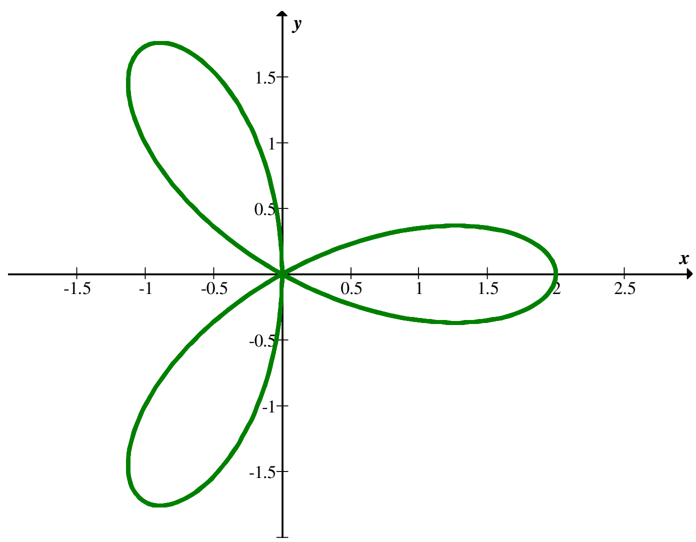
$r = 3 + 3\cos\theta$ e $r = 3 - 3\cos\theta$



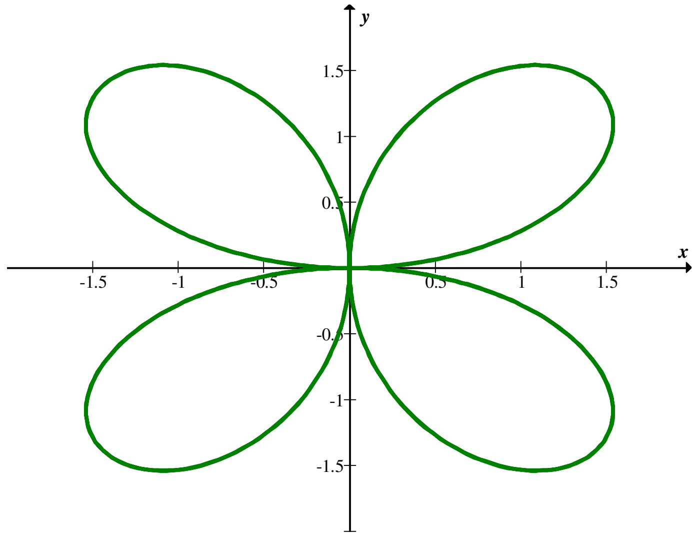
12. $r = \cos 3\theta$



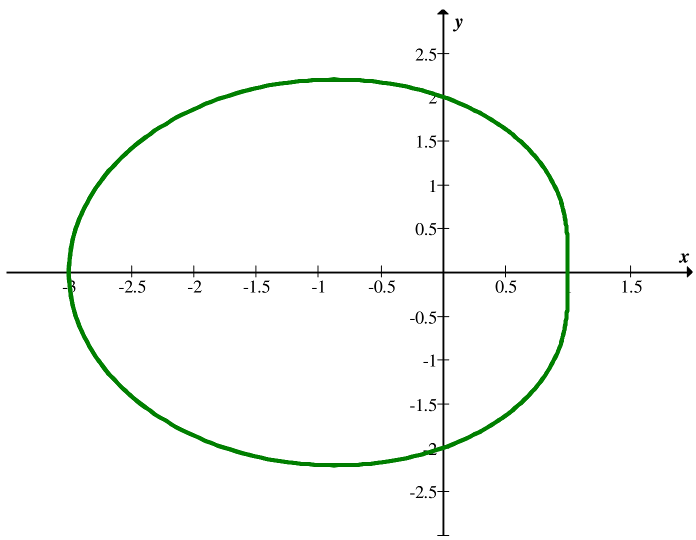
13. $r = 2\cos 3\theta$



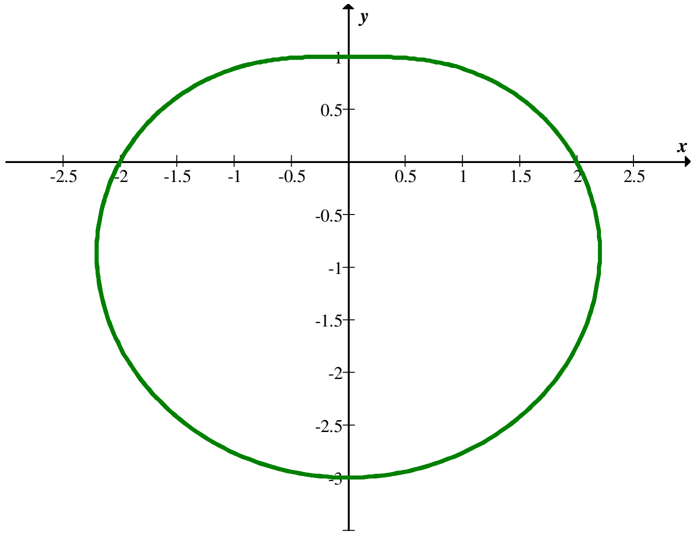
14. $r = 2\ sen\ 2\theta$



15. $r = 2 - \cos\theta$



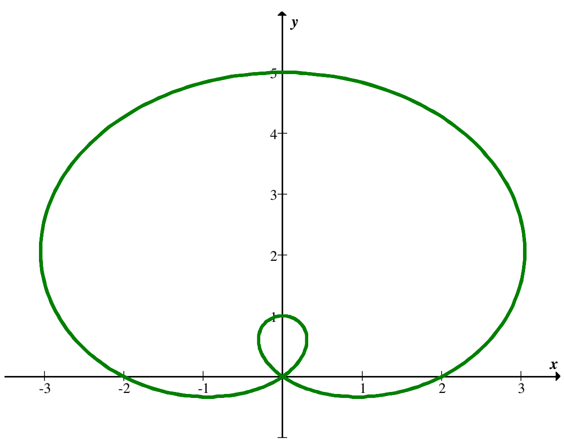
16. $r = 2 - sen\,\theta$

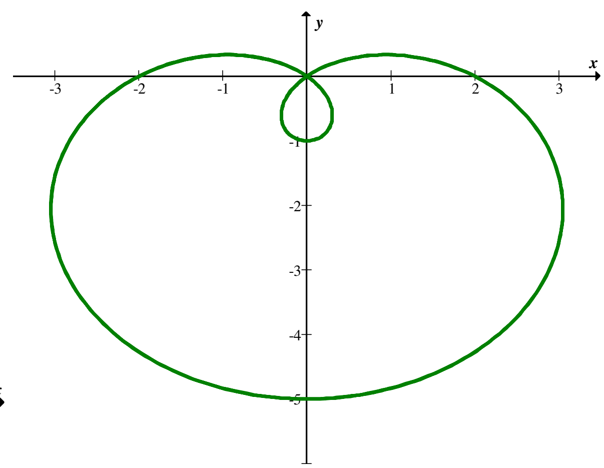


17. $r = a \pm b\ sen\ \theta$

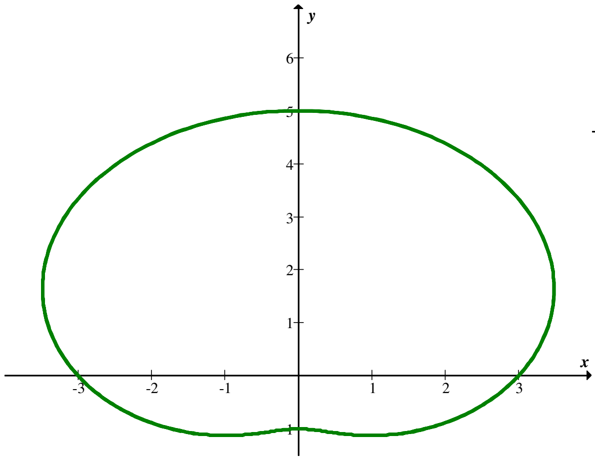
$a = 2$ e $b = 3$
$a = 3$ e $b = 2$
$a = b = 2$

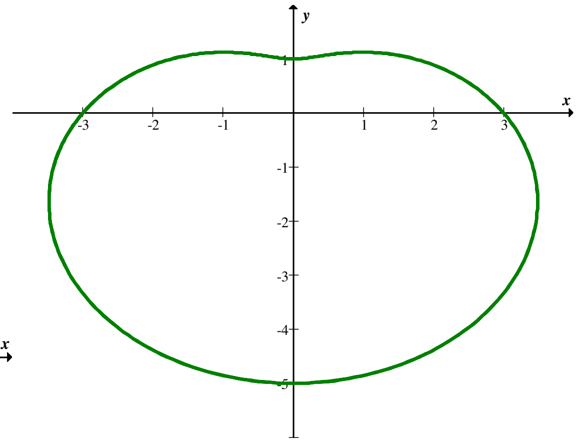
$r = 2 + 3\ sen\ \theta$ $\qquad$ $r = 2 - 3\ sen\ \theta$



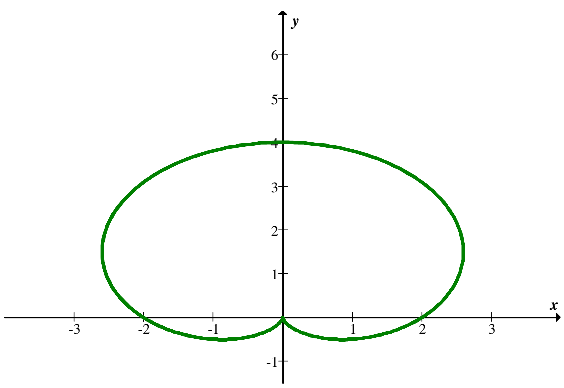


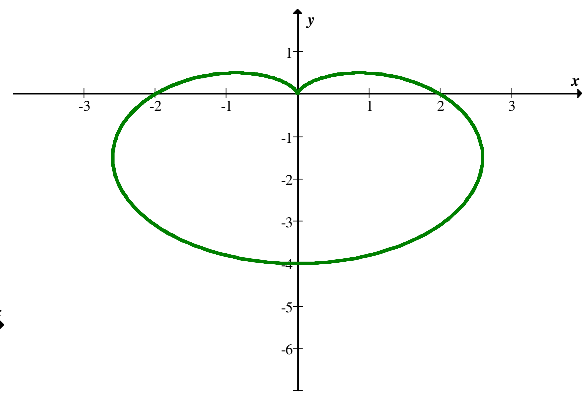
$r = 3 + 2\ sen\ \theta$ $\qquad$ $r = 3 - 2\ sen\ \theta$



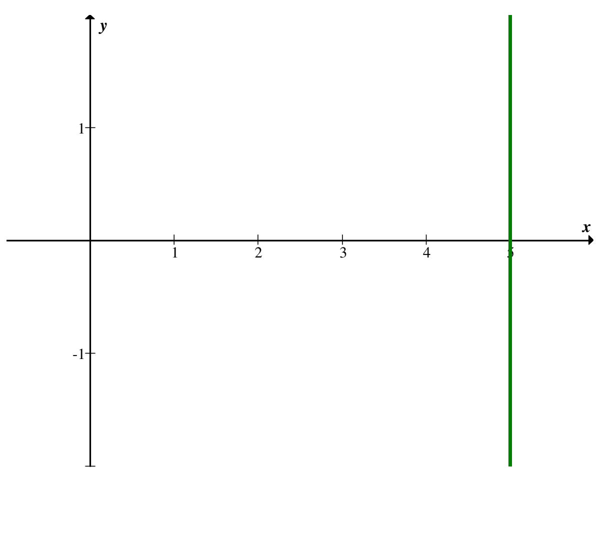


$r = 2 + 2\ sen\ \theta$ $\qquad\qquad$ $r = 2 - 2\ sen\ \theta$

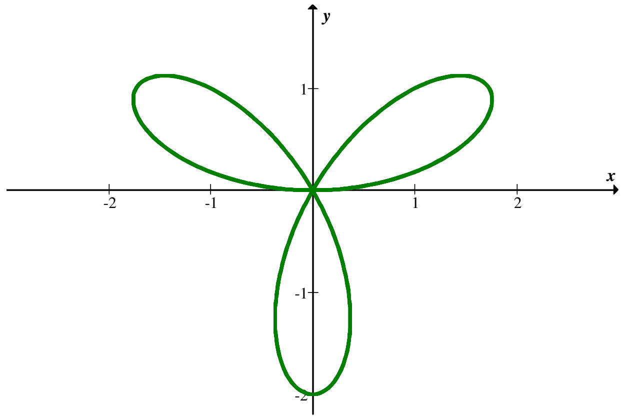




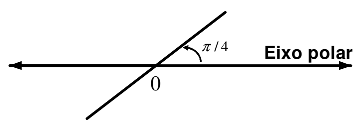
18. $r \cos \theta = 5$



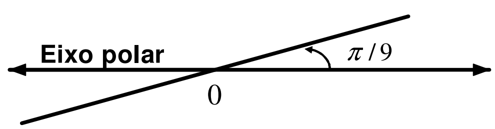
19. $r = 2\ sen\ 3\theta$



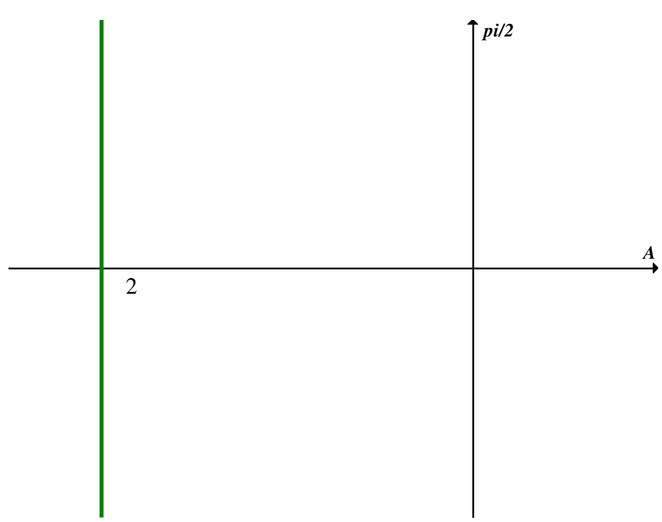
20. $\theta = \dfrac{\pi}{4}$



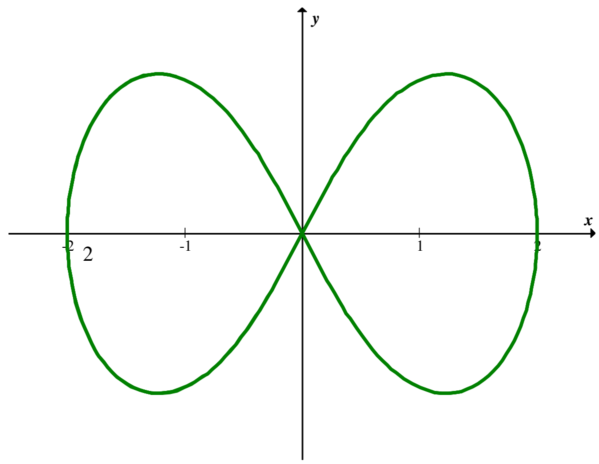
21. $\theta = \dfrac{\pi}{9}$



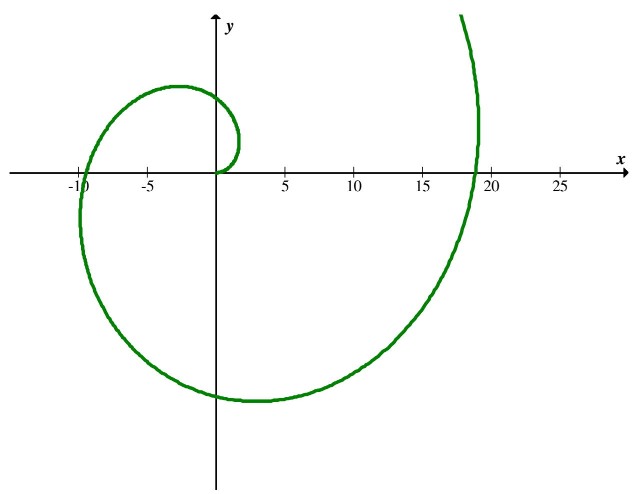
22. $5r \cos \theta = -10$



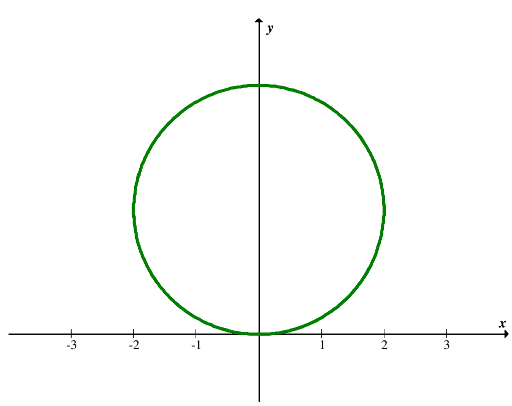
23. $r^2 = 4 \cos 2\theta$



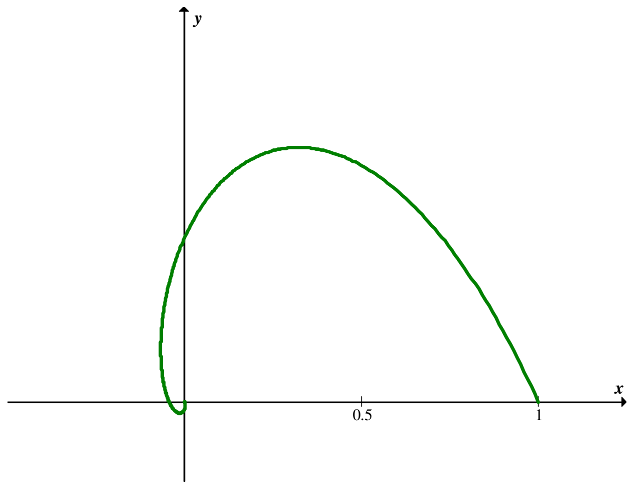
24. $r = 3\theta,\ \theta \geq 0$



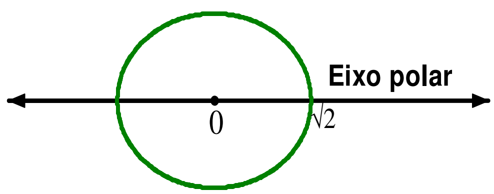
25. $r = 4\ sen\ \theta$



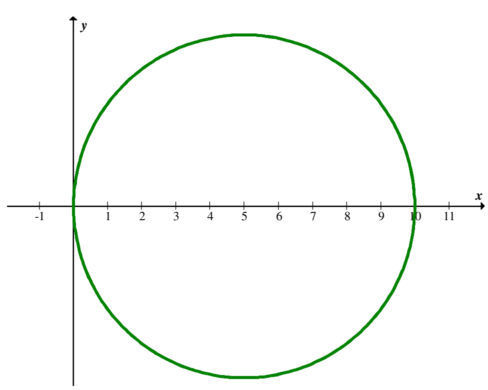
26. $r = e^{-\theta}, \theta \geq 0$



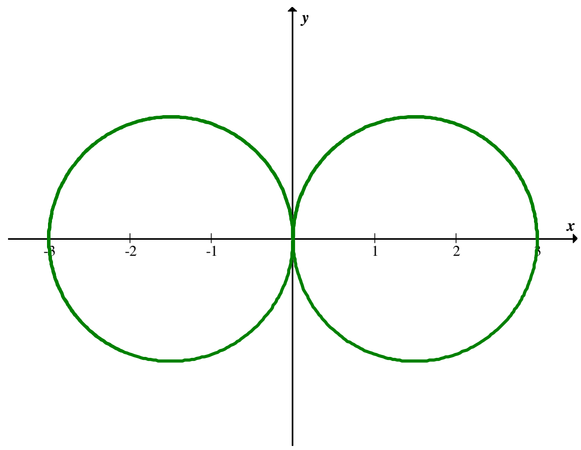
27. $r = \sqrt{2}$



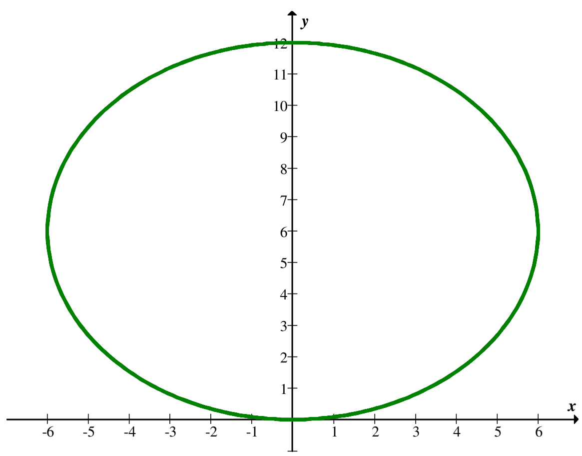
28. $r = 10 \cos \theta$



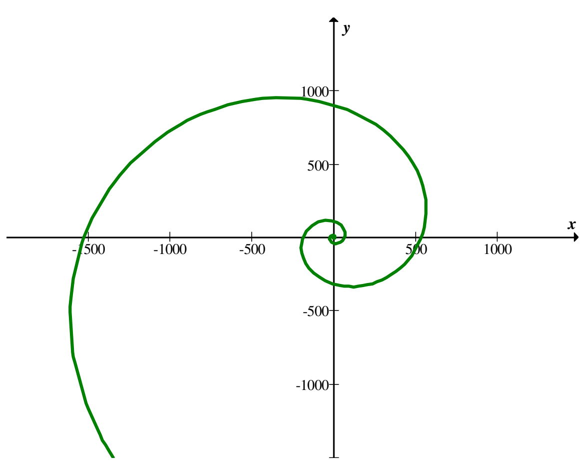
29. $r = 2\left|\cos\theta\right|$



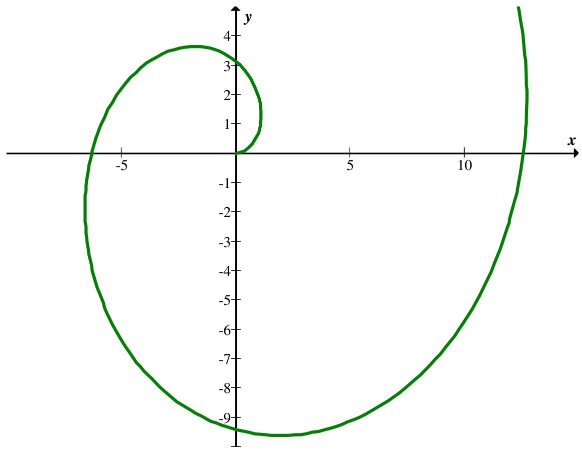
30. $r = 12\ sen\ \theta$



31. $r = e^{\theta/3}$



32. $r = 2\theta$



Nos exercícios 33 a 37, encontrar o comprimento de arco da curva dada.

33. $r = e^{\theta}$, entre $\theta = 0$ e $\theta = \pi/3$

$$s = \int_0^{\pi/3} \sqrt{e^{2\theta} + e^{2\theta}}\, d\theta$$

$$= \int_0^{\pi/3} \sqrt{2e^{2\theta}}\, d\theta$$

$$= \sqrt{2} \int_0^{\pi/3} e^{\theta}\ d\theta$$

$$= \sqrt{2}\ e^{\theta}\Big|_0^{\pi/3} = \sqrt{2}\left(e^{\pi/3} - 1\right) u.\ c.$$

34. $r = 1 + \cos\theta$

$$s = 2\int_0^{\pi} \sqrt{(-sen\theta)^2 + (1+\cos\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi} \sqrt{sen^2\theta + 1 + 2\cos\theta + \cos^2\theta}\, d\theta$$

$$= 2\int_0^{\pi} \sqrt{2 + 2\cos\theta}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{1 + \cos\theta}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{2\cos^2\frac{\theta}{2}}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{2}\cos\frac{\theta}{2} d\theta$$

$$= 4 \,.\, 2\, sen\,\frac{\theta}{2}\Big|_0^{\pi}$$

$$= 8\left(sen\frac{\pi}{2} - sen\,0\right)$$

$$= 8 \;\; u.c.$$

35. $r = 2\, a\, sen\,\theta$

$$s = 2\int_0^{\pi/2} \sqrt{(2a\cos\theta)^2 + (2a\, sen\,\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi/2} \sqrt{4a^2}\, d\theta = 2\int_0^{\pi/2} 2a\; d\theta$$

$$= 4a\,\frac{\pi}{2} = 2a\pi\, u.c$$

36. $r = 3\,\theta^2$, de $\theta = 0$ até $\theta = 2\pi/3$

$$s = \int_0^{2\pi/3} \sqrt{36\,\theta^2 + 9\theta^4}\,d\theta$$

$$= \int_0^{2\pi/3} \sqrt{9\,\theta^2\left(4+\theta^2\right)}\,d\theta$$

$$= \int_0^{2\pi/3} 3\theta\,\left(4+\theta^2\right)^{1/2} d\theta$$

$$= 3\,.\,\frac{1}{2}\frac{\left(4+\theta^2\right)^{3/2}}{3/2}\Bigg|_0^{2\pi/3}$$

$$= \frac{8}{27}\left(9+\pi^2\right)^{3/2} - 8\,u.c$$

37. $r = e^{2\theta}$, de $\theta = 0$ até $\theta = 3\pi/2$

$$s = \int_0^{3\pi/2} \sqrt{\left(2e^{2\theta}\right)^2 + e^{4\theta}}\,d\theta$$

$$= \int_0^{3\pi/2} \sqrt{4e^{4\theta} + e^{4\theta}}\,d\theta$$

$$= \int_0^{3\pi/2} \sqrt{5}\;e^{2\theta}d\theta = \frac{\sqrt{5}}{2}e^{2\theta}\Bigg|_0^{3\pi/2}$$

$$= \frac{\sqrt{5}}{2}\left(e^{3\pi} - 1\right)u.\,c.$$

38. Achar o comprimento da cardióide $r = 10(1 - \cos\theta)$.

$$s = 2\int_0^{\pi} \sqrt{(10\ sen\ \theta)^2 + 100(1-\cos\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi} \sqrt{200 - 200\cos\theta}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{200}\ \sqrt{1-\cos\theta}\ d\theta$$

$$= 2\int_0^{\pi} 10\sqrt{2}\ \sqrt{2\ sen^2\ \frac{\theta}{2}}\ d\theta$$

$$= 2\,.\,10\sqrt{2}\,.\,\sqrt{2}\int_0^{\pi} sen\ \frac{\theta}{2}\ d\theta$$

$$= -40\,.\,2\cos\frac{\theta}{2}\Big|_0^{\pi}$$

$$= -80\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= 80\ u.\,c.$$

Nos exercícios 39 a 46, encontrar a integral que dá o comprimento total da curva dada.

39. $r^2 = 9\cos 2\,\theta$

$$r = (9\cos 2\,\theta)^{1/2} = 3(\cos 2\theta)^{1/2}$$

$$r' = \frac{3}{2}(\cos 2\theta)^{-1/2}(-\,sen\ 2\theta)2 = \frac{-3sen\ 2\theta}{\sqrt{\cos 2\theta}}$$

$$s = 4\int_0^{\pi/4} \sqrt{\frac{9\ sen^2\, 2\theta}{\cos 2\theta} + 9\cos 2\theta}\ d\theta$$

$$= 4\int_0^{\pi/4} \sqrt{\frac{9\ sen^2\, 2\theta + 9\cos^2 2\theta}{\cos 2\theta}}\ d\theta$$

$$= 12\int_0^{\pi/4} \frac{d\theta}{\sqrt{\cos 2\theta}}$$

40. $r = 3\ sen\ 3\theta$

$$s = 6\int_0^{\pi/6} \sqrt{(9\cos 3\theta)^2 + 9\,sen^2 3\theta}\; d\theta$$

$$= 18\int_0^{\pi/6} \sqrt{9\cos^2 3\theta + sen^2 3\theta}\; d\theta$$

41. $r = 4\cos 4\theta$

$$s = 16\int_0^{\pi/8} \sqrt{(-16 sen\, 4\theta)^2 + 16\cos^2 4\theta}\; d\theta$$

$$= 64\int_0^{\pi/8} \sqrt{16\, sen^2 4\theta + \cos^2 4\theta}\; d\theta$$

42. $r^2 = 9\, sen\, 2\theta$

$$r = 3(sen\, 2\,\theta)^{1/2}$$

$$r' = 3\,\frac{1}{2}\,(sen\, 2\,\theta)^{-1/2}.\cos 2\,\theta\,.\,2$$

$$s = 4\int_0^{\pi/4} \sqrt{\frac{9\cos^2 2\theta}{sen\, 2\theta} + 9\, sen 2\theta}\; d\theta$$

$$= 4\int_0^{\pi/4} \sqrt{\frac{9\cos^2 2\theta + 9\, sen^2 2\theta}{sen\, 2\theta}}\; d\theta$$

$$= 4\int_0^{\pi/4} \sqrt{\frac{9}{sen\, 2\theta}}\; d\theta$$

$$= 12\int_0^{\pi/4} \frac{d\theta}{\sqrt{sen\, 2\theta}}$$

43. $r = 2 - 3\cos\theta$

$$s = 2\int_0^{\pi} \sqrt{(3\ sen\ \theta)^2 + (2 - 3\cos\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi} \sqrt{9\ sen^2\ \theta + 4 - 12\cos\theta + 9\cos^2\theta}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{13 - 12\cos\theta}\ d\theta$$

44. $r = 4 - 2\ sen\ \theta$

$$s = 2\int_{-\pi/2}^{\pi/2} \sqrt{(-2\cos\theta)^2 + (4 - 2\ sen\ \theta)^2}\, d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} \sqrt{4\cos^2\theta + 16 - 16\ sen\ \theta + 4\ sen^2\ \theta}\ d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2} \sqrt{20 - 16\ sen\ \theta}\ d\theta$$

$$= 4\int_{-\pi/2}^{\pi/2} \sqrt{5 - 4\ sen\ \theta}\ d\theta$$

45. $r = 3 + 2\cos\theta$

$$s = 2\int_0^{\pi} \sqrt{(-2\ sen\ \theta)^2 + (3 + 2\cos\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi} \sqrt{4\ sen^2\ \theta + 9 + 12\cos\theta + 4\cos^2\theta}\ d\theta$$

$$= 2\int_0^{\pi} \sqrt{13 + 12\cos\theta}\ d\theta$$

46. $r = 4 + 2\ sen\ \theta$

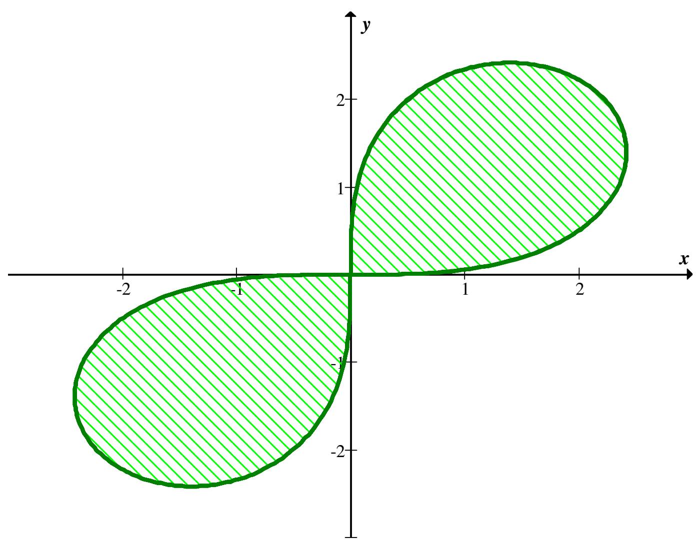
$$s = 2\int_{-\pi/2}^{\pi/2}\sqrt{(2\cos\theta)^2 + (4 + 2\, sen\,\theta)^2}\,d\theta$$

$$= 2\int_{-\pi/2}^{\pi/2}\sqrt{20 + 16\, sen\,\theta}\; d\theta$$

$$= 4\int_{-\pi/2}^{\pi/2}\sqrt{5 + 4\, sen\,\theta}\; d\theta$$

Nos exercícios 47 a 56, calcular a área limitada pela curva dada.

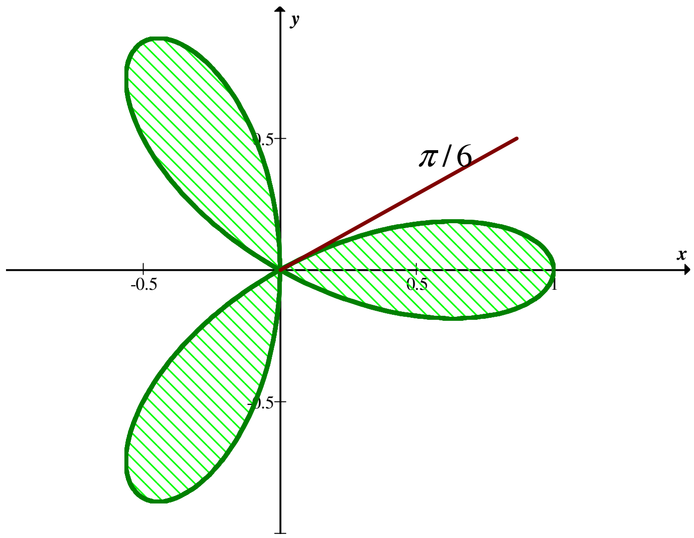
47. $r^2 = 9\; sen\; 2\theta$



$$A = 2\,.\,\frac{1}{2}\int_0^{\pi/2} 9\; sen\;\; 2\theta\; d\theta$$

$$= -9\,.\,\frac{1}{2}\cos\; 2\theta\Big|_0^{\pi/2} = 9\; u.\,a.$$

48. $r = \cos 3\theta$



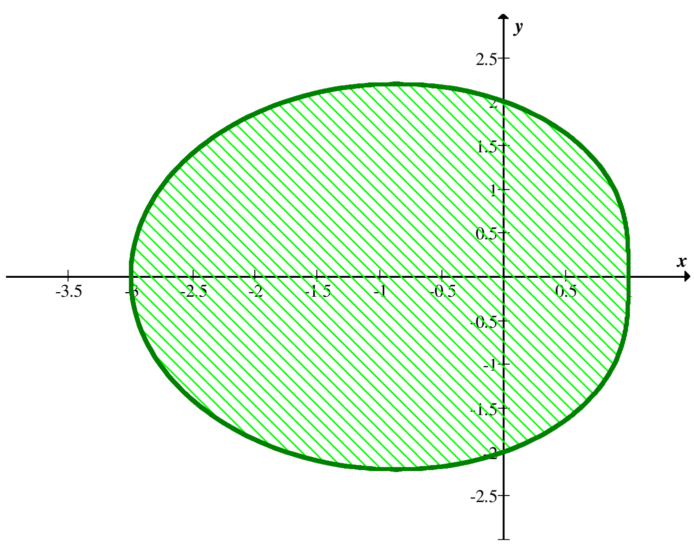
$$A = \frac{1}{2}\,.\,6\int_0^{\pi/6} \cos^2 3\theta \, d\theta$$

$$= 3\int_0^{\pi/6}\left(\frac{1}{2}+\frac{1}{2}\cos 6\theta\right)d\theta$$

$$= 3\,.\left[\frac{1}{2}\theta+\frac{1}{12}\,sen\,6\theta\right]_0^{\pi/6}$$

$$= \frac{\pi}{4}\,u.\,a.$$

49. $r = 2 - \cos\theta$

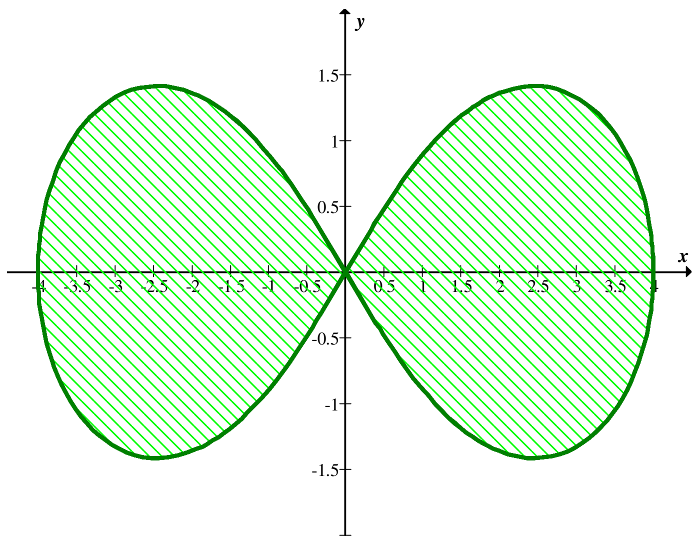


$$A = 2 \cdot \frac{1}{2}\int_0^{\pi} (2 - \cos\theta)^2 \, d\theta$$

$$= \int_0^{\pi} \left(4 - 4\cos\theta + \cos^2\theta\right) d\theta$$

$$= 4\theta - 4\, sen\,\theta + \frac{1}{2}\theta + \frac{1}{4}\, sen\, 2\theta \Big|_0^{\pi} = \frac{9\pi}{2}\, u.\, a.$$

50. $r^2 = 16\cos 2\theta$

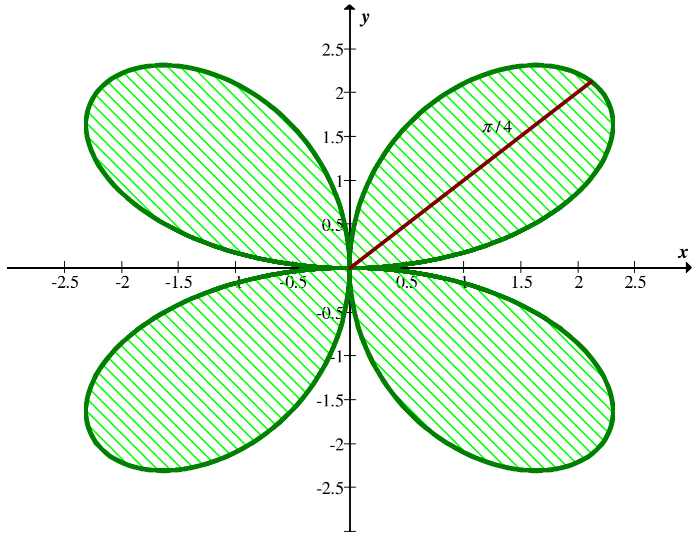


$$A_1 = \frac{1}{2} \cdot 4\int_0^{\pi/4} 16\cos 2\theta\, d\theta$$

$$= 32\frac{1}{2}\, sen\, 2\theta \Big|_0^{\pi/4}$$

$$= 16\, u.\, a$$

51. $r = 3\, sen\, 2\theta$

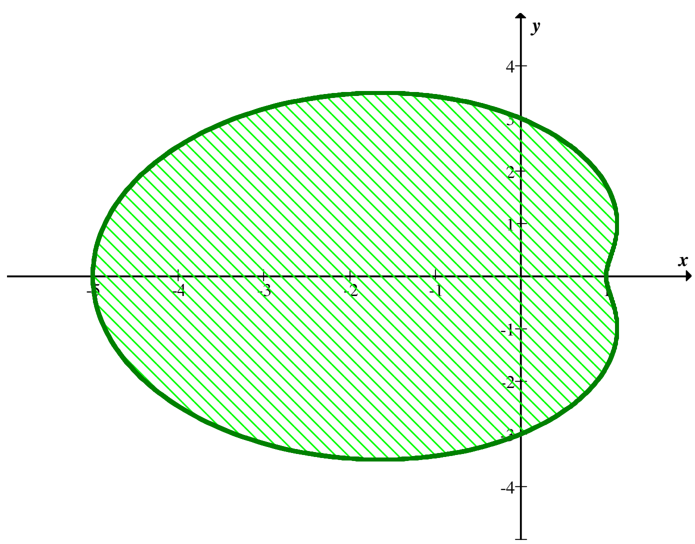


$$A = 8 \,.\, \frac{1}{2} \int_0^{\pi/4} 9\ sen^2\ 2\theta\ d\theta$$

$$= 36\left(\frac{1}{2}\,.\,\theta - \frac{1}{8}\ sen\ 4\theta\right)\Bigg|_0^{\pi/4}$$

$$= \frac{9\pi}{2}\ u.\ a.$$

52. $r = 3 - 2\cos\theta$



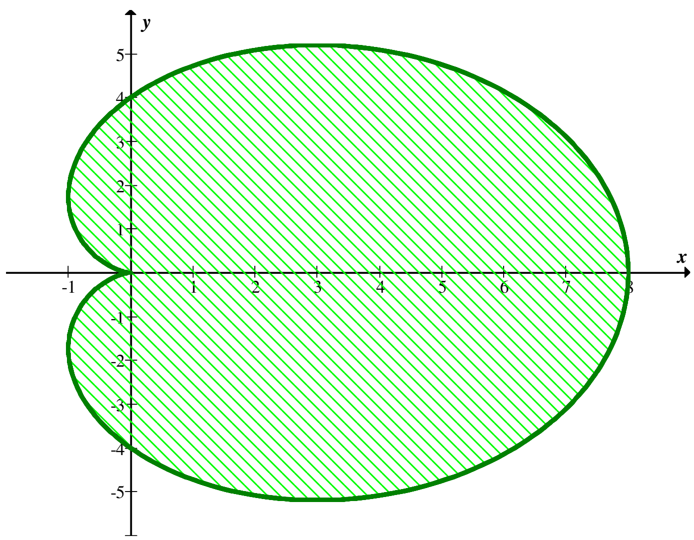
$$s = 2 \cdot \frac{1}{2}\int_0^{\pi} (3 - 2\cos\theta)^2 d\theta$$

$$= \int_0^{\pi} \left(9 - 12\cos\theta + 4\cos^2\theta\right) d\theta = \int_0^{\pi} (9 - 12\cos\theta + 2 + 2\cos 2\theta) d\theta$$

$$= \left(11\theta - 12\, sen\,\theta + sen\, 2\theta\right)\Big|_0^{\pi}$$

$$= 11\pi\, u.\, a.$$

53. $r = 4(1 + \cos\theta)$



$$A = 2 \cdot \frac{1}{2}\int_0^{\pi} 16(1 + \cos\theta)^2 d\theta$$

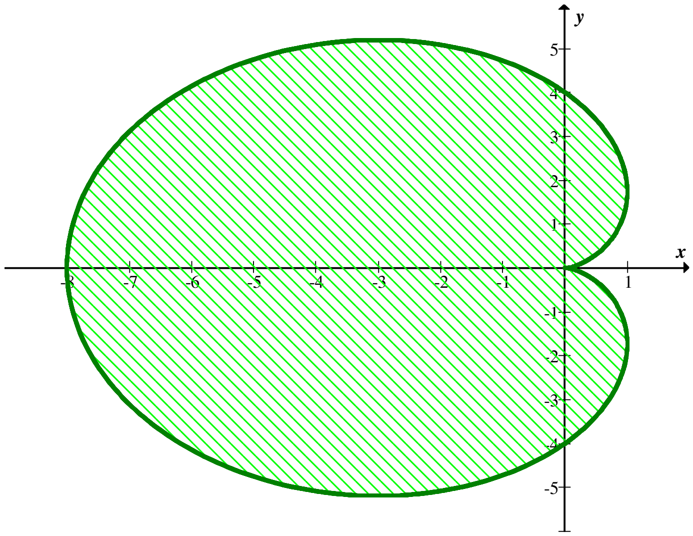
$$= 16\int_0^{\pi} \left(1 + 2\cos\theta + \cos^2\theta\right) d\theta$$

$$= 16\left(\theta + 2\, sen\,\theta + \frac{1}{2}\theta + \frac{1}{4} sen\, 2\theta\right)\Big|_0^{\pi}$$

$$= 24\pi\, u.\, a.$$

.

54. $r = 4(1 - \cos\theta)$



$$A = 2\,.\,\frac{1}{2}\int_0^{\pi} 16(1-\cos\theta)^2 d\theta$$

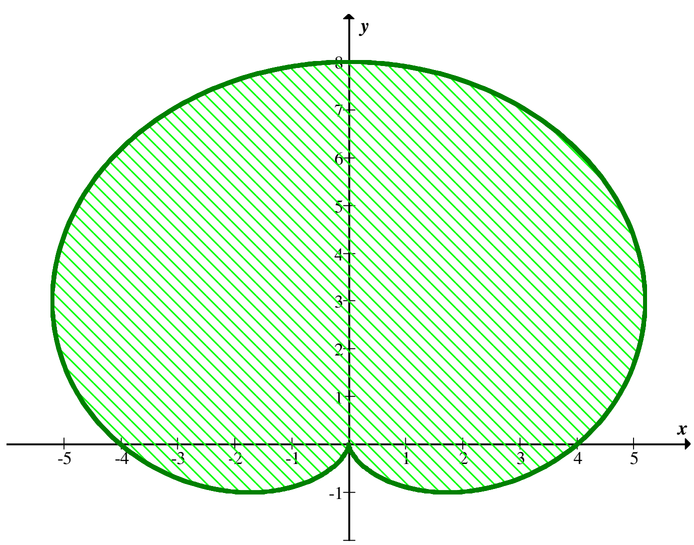
$$= \int_0^{\pi} 16\left(1 - 2\cos\theta + \cos^2\theta\right) d\theta$$

$$= 16\left(\theta - 2\,sen\,\theta + \frac{1}{2}\theta + \frac{1}{4}sen\,2\theta\right)\Bigg|_0^{\pi}$$

$$= 24\pi\; u.\,a.$$

.

55. $r = 4(1 + sen\,\theta)$



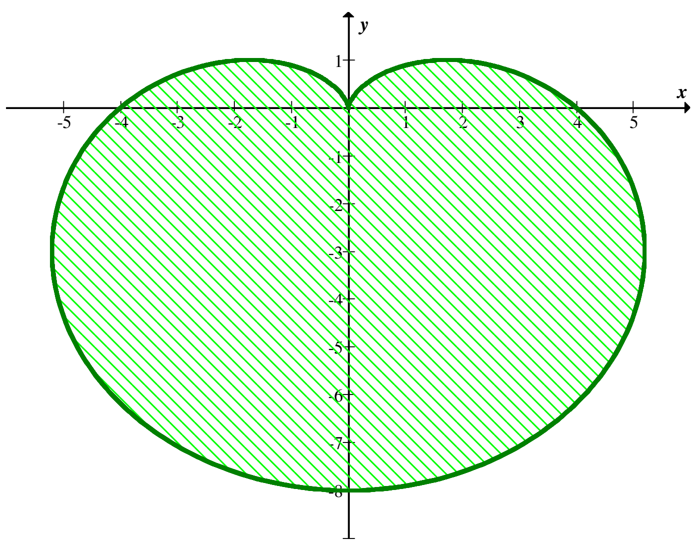
$$A = 2 \cdot \frac{1}{2} \int_{-\pi/2}^{\pi/2} 16(1 + sen\ \theta)^2 d\theta$$

$$= \int_{-\pi/2}^{\pi/2} 16\left(1 + 2\ sen\ \theta + sen^2\ \theta\right) d\theta$$

$$= 16\left(\theta + 2\,(-\cos\theta) + \frac{1}{2}\theta - \frac{1}{4} sen\ 2\theta \right)\Bigg|_{-\pi/2}^{\pi/2}$$

$$= 24\pi\ u.\ a.$$

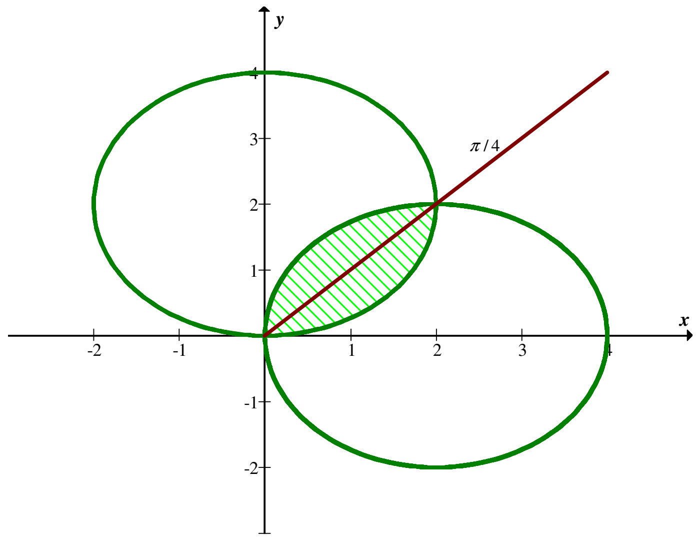
56. . $r = 4(1 - sen\ \theta)$



$$A = 2 \cdot \frac{1}{2} \int_{-\pi/2}^{\pi/2} 16(1 - sen\ \theta)^2 d\theta$$

$$= 24\pi\ u.\ a.$$

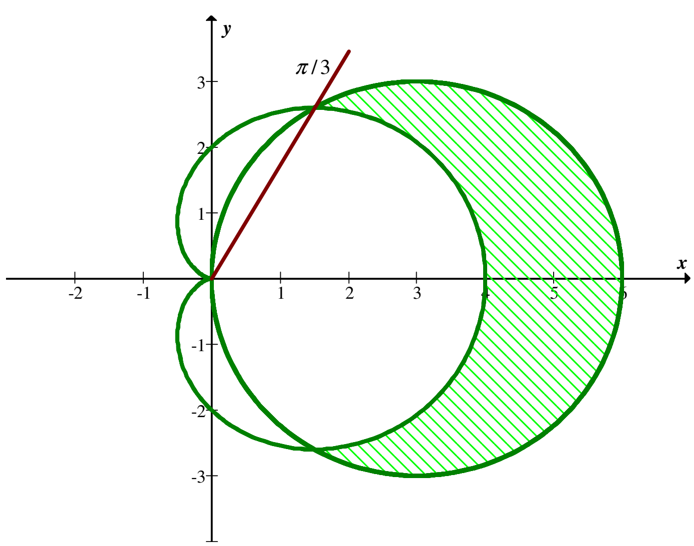
57. Encontrar a área da intersecção entre $r = 2a\cos\theta$ e $r = 2a\ sen\ \theta$



$$2a\cos\theta = 2a\ sen\ \theta$$
$$\cos\theta = sen\ \theta$$
$$\theta = \frac{\pi}{4}$$

$$A = 2\,.\,\frac{1}{2}\int_{\pi/4}^{\pi/2} 4a^2\cos^2\theta\, d\theta$$
$$= \frac{a^2(\pi-2)}{2}\,u.\,a$$

58. Encontrar a área interior ao círculo $r = 6\cos\theta$ e exterior a $r = 2\,(1+\cos\theta)$



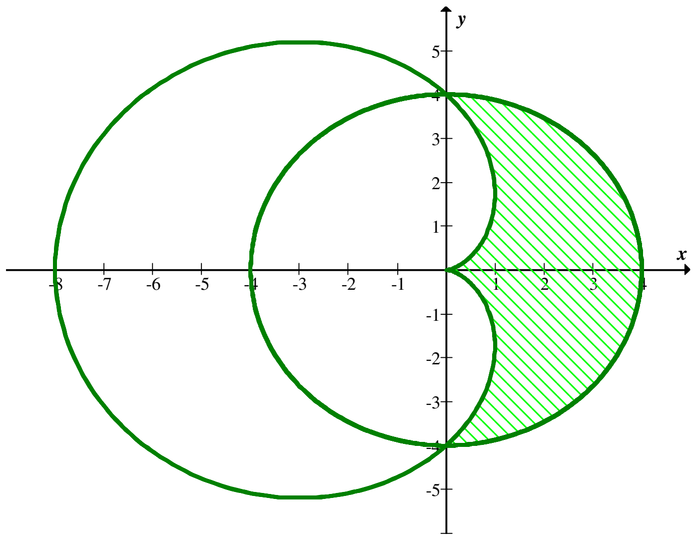
$$6\cos\theta = 2 + 2\cos\theta$$
$$4\cos\theta = 2$$
$$\cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$$

$$A_1 = 2\,.\,\frac{1}{2}\int_0^{\pi/3} 36\cos^2\theta\, d\theta$$
$$= \int_0^{\pi/3} 36\cos^2\theta\, d\theta$$
$$= 36\left(\frac{1}{2}\theta + \frac{1}{4}\,sen\,2\theta\right)\Big|_0^{\pi/3}$$
$$= 18\,.\,\frac{\pi}{3} + 9\,.\,\frac{\sqrt{3}}{2} = 6\pi + \frac{9\sqrt{3}}{2}$$

$$A_2 = 2.\frac{1}{2}\int_0^{\pi/3} 4(1+\cos\theta)^2\, d\theta$$
$$= \int_0^{\pi/3} 4\left(1 + 2\cos\theta + \cos^2\theta\right) d\theta$$
$$= 4\left(\theta + 2\,sen\,\theta + \frac{1}{2}\,sen\,2\theta\right)\Big|_0^{\pi/3}$$
$$= \frac{4\pi}{3} + 4\sqrt{3} + \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$$

$$A = A_1 - A_2$$
$$= 6\pi + \frac{9\sqrt{3}}{2} - \frac{4\pi}{3} + 4\sqrt{3} + \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$$
$$= 4\pi\; u.\,a.$$

59. Encontrar a área interna ao círculo $r = 4$ e exterior à cardióide $r = 4(1-\cos\theta)$

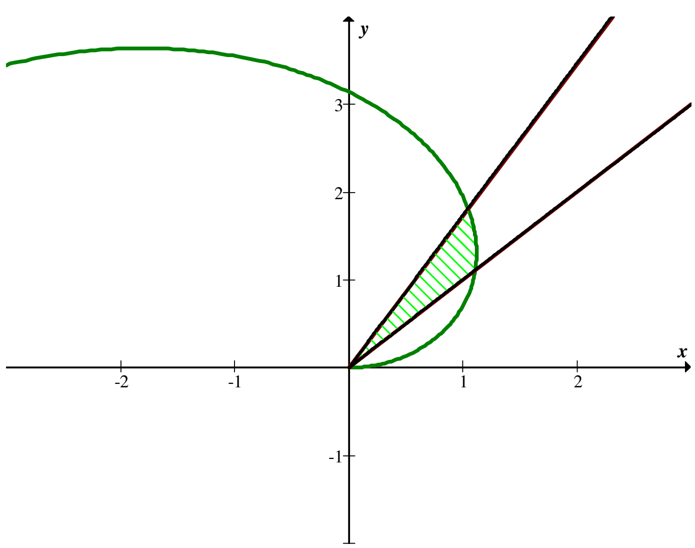


$$A = 2.\frac{1}{2}\int_0^{\pi/2}\left[16 - 16(1-\cos\theta)^2\right]d\theta$$

$$= \int_0^{\pi/2}\left(16 - 16 + 32\cos\theta - 16\cos^2\theta\right)d\theta$$

$$= 32 - 4\pi u.a$$

60. Encontrar a área da região do 1° quadrante delimitada pelo primeiro laço da espiral $r = 2\theta$, $\theta \geq 0$ e pelas retas $\theta = \dfrac{\pi}{4}$ e $\theta = \dfrac{\pi}{3}$

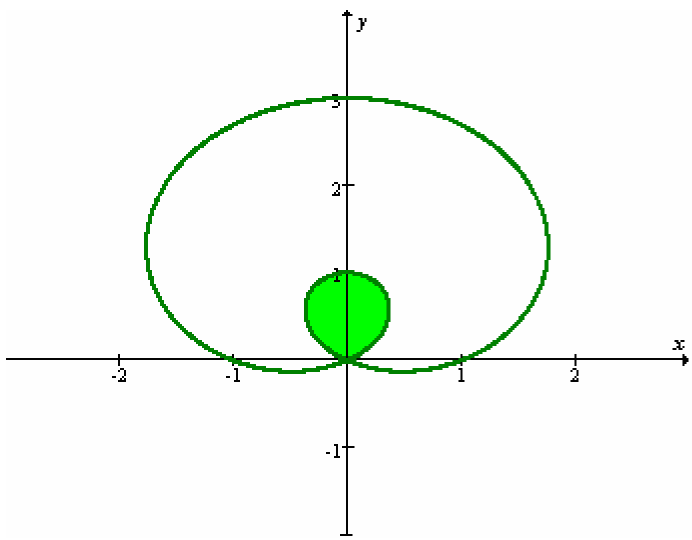


$$A = \frac{1}{2}\int_{\pi/4}^{\pi/3} 4\,\theta^2\ d\theta$$

$$= 2\,.\,\frac{\theta^3}{3}\Bigg|_{\pi/4}^{\pi/3}$$

$$= \frac{37\pi^3}{2592}\,u.\,a.$$

61. Encontrar a área da região delimitada pelo laço interno da limaçon $r = 1 + 2\ sen\ \theta$.



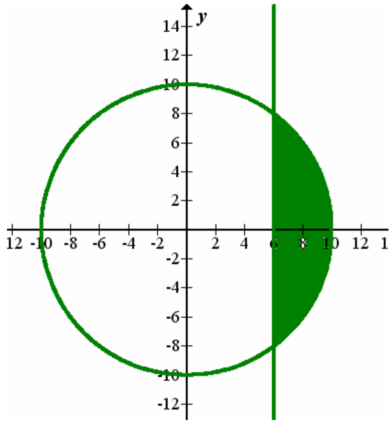
$$A = 2\,.\,\frac{1}{2}\int_{7\pi/6}^{3\pi/2} (1 + 2\ sen\ \theta)^2\ d\theta$$

$$= \int_{7\pi/6}^{3\pi/2} \left(1 + 4\ sen\ \theta + 4\ sen^2\ \theta\right) d\theta$$

$$= \theta - 4\cos\theta + 4\left(\frac{1}{2}\,\theta - \frac{1}{4}\,sen\ 2\theta\right)\Bigg|_{7\pi/6}^{3\pi/2}$$

$$= \pi - \frac{3\sqrt{3}}{2}\,u.\,a.$$

62. Encontrar a área da região interior ao círculo $r = 10$ e a direita da reta $r\cos\theta = 6$.



$10\cos\theta = 6$

$\cos\theta = \dfrac{3}{5} \quad \therefore \quad \theta = arc\cos\dfrac{3}{5}$

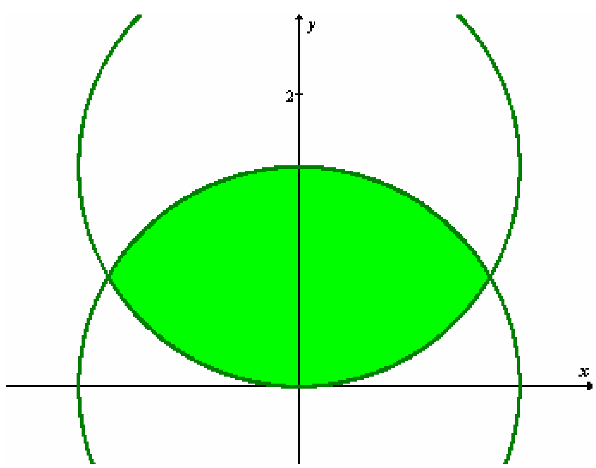
$$A = 2.\frac{1}{2}\int_0^{\arccos\frac{3}{5}}\left[(10)^2 - \left(\frac{6}{\cos\theta}\right)^2\right]d\theta$$

$$= \left(100\theta - 36tg\,\theta\right)\Big|_0^{\arccos\frac{3}{5}}$$

$$100\arccos\frac{3}{5} - 48u.a.$$

63. Calcular a área da região interior às duas curvas:

a) $2r = 3$ e $r = 3\,sen\,\theta$



$$\frac{3}{2} = 3\ sen\ \theta$$

$$sen\ \theta = \frac{1}{2} \Rightarrow \quad \theta = \frac{\pi}{6}$$

$$A_1 = \frac{1}{2}\int_0^{\pi/6} 9\ sen^2\theta\ \ d\theta$$

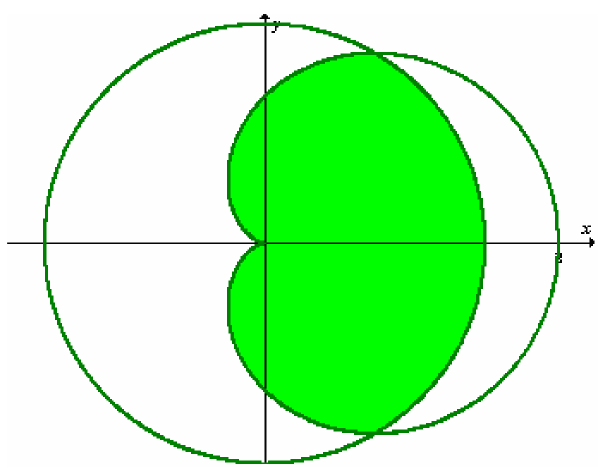
$$= \frac{1}{2}\,.\,9\left(\frac{1}{2}\theta - \frac{1}{4}\ sen\ 2\theta\right)\Bigg|_0^{\pi/6}$$

$$= \frac{9}{2}\left(\frac{1}{2}\cdot\frac{\pi}{6} - \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right) = \frac{9\pi}{24} - \frac{9\sqrt{3}}{16}$$

$$A_2 = \frac{1}{2}\int_{\pi/6}^{\pi/2} \frac{9}{4}\ d\theta = \frac{9\pi}{24}$$

$$A = 2\left(A_1 + A_2\right) = \frac{3\pi}{2} - \frac{9\sqrt{3}}{8}\ u.\,a.$$

b) $2r = 3$ e $r = 1 + \cos\theta$



$$1 + \cos\theta = \frac{3}{2}$$

$$\cos\theta = \frac{1}{2} \quad \therefore \quad \theta = \pi/3$$

$$A_1 = 2 \cdot \frac{1}{2} \int_0^{\pi/3} 9/4 \, d\theta = \frac{3\pi}{4}$$

$$A_2 = 2 \cdot \frac{1}{2} \int_{\pi/3}^{\pi} (1 + \cos\theta)^2 \, d\theta$$

$$= \frac{24\pi - 27\sqrt{3}}{24}$$

$$A = A_1 + A_2 = \frac{14\pi - 9\sqrt{3}}{8} \; u.\, a.$$