# 8.4 – EXERCÍCIOS – pg. 344

Nos exercícios de 1 a 14, encontrar o comprimento de arco da curva dada.

1. $y = 5x - 2$, $-2 \le x \le 2$

$$s = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + 5^2}\, dx = \int_{-2}^{2} \sqrt{26}\; dx = \sqrt{26}\; x \Big|_{-2}^{2}$$

$$= \sqrt{26}\;(2 + 2) = 4\sqrt{26}\; u.\,c.$$

2. $y = x^{2/3} - 1$, $1 \le x \le 2$

$$y' = \frac{2}{3} x^{-1/3}$$

$$s = \int_1^2 \sqrt{1 + \frac{4}{9x^{2/3}}}\; dx = \int_1^2 \sqrt{\frac{9x^{2/3} + 4}{9x^{2/3}}}\; dx$$

$$= \int_1^2 \left(9x^{2/3} + 4\right)^{1/2}\; dx\,.\,\frac{1}{3}\,.\,x^{-1/3} dx$$

$$= \frac{1}{3}\cdot\frac{1}{6}\int_1^2 \left(9x^{2/3} + 4\right)^{1/2}.\,6\,.\,x^{-1/3} dx$$

$$= \frac{1}{18}\,.\,\frac{\left(9x^{2/3} + 4^{3/2}\right)}{3/2}\Bigg|_1^2$$

$$= \frac{1}{18}\cdot\frac{2}{3}\left(\left(9x^{2/3} + 4\right)^{3/2} - 13^{3/2}\right) = \frac{1}{27}\left(\left(9\,.\,2^{2/3} + 4\right)^{3/2} - 13\sqrt{3}\right)$$

3. $y = \frac{1}{3}\left(2 + x^2\right)^{3/2}$, $0 \le x \le 3$

$$y' = \frac{1}{3}\cdot\frac{3}{2}\left(2 + x^2\right)^{1/2}.\,2x$$

$$s=\int_0^3 \sqrt{1+x^2\left(2+x^2\right)}dx$$

$$=\int_0^3 \sqrt{1+x^2+x^4}\,dx$$

$$=\int_0^3 \sqrt{\left(x^2+1\right)}\,dx$$

$$=\int_0^3 \left(x^2+1\right)dx=\frac{x^3}{3}+x\Big|_0^3=\frac{3^3}{3}+3=12$$

4. $x^{2/3}+y^{2/3}=2^{2/3}$

$$\begin{cases} x=2\cos^3 t \\ y=2\,sen^3 t \end{cases}$$

$$s=4\int_0^{\pi/2} \sqrt{36\cos^4 t\,sen^2 t+36\,sen^4 t\cos^2 t}\;dt$$

$$=4\int_0^{\pi/2} 6.\sqrt{sen^2 t\cos^2 t\left(\cos^2 t+sen^2 t\right)}\;dt$$

$$=24\int_0^{\pi/2} sen\,t\,.\cos t\;dt=24\,.\,\frac{sen^2 t}{2}\Big|_0^{\pi/2}$$

$$=12\,u.\,c.$$

5. $y=\frac{1}{4}x^4+\frac{1}{8x^2}$, $1\le x\le 2$

$$y'=\frac{1}{4}x^3+\frac{1}{8}\left(-2\right)x^{-3}$$

$$s = \int_1^2 \sqrt{1 + \left(x^3 - \frac{1}{4x^3}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + x^6 - 2x^3 . \frac{1}{4x^3} + \frac{1}{16x6}}\, dx$$

$$= \int_1^2 \sqrt{1 + x^6 - 2x^3 . \frac{1}{16x^6}}\, dx$$

$$= \int_1^2 \sqrt{\frac{8x^6 + 16x^{12} + 1}{16x^6}}\, dx$$

$$= \int_1^2 \frac{1}{4x^3} \sqrt{\left(4x^6 + 1\right)^2}\, dx$$

$$= \int_1^2 \frac{1}{4x^3} \sqrt{\left(4x^6 + 1\right)^2}\, dx$$

$$= \frac{1}{4} \int_1^2 \left(4x^3 + x^{-3}\right) dx$$

$$= \frac{1}{4}\left(4 . \frac{x^4}{4} + \frac{x^{-2}}{2}\right)\Bigg|_1^2$$

$$= \frac{1}{4}\left(2^4 - \frac{1}{2 . 2^2} - 1 + \frac{1}{2}\right)$$

$$= \frac{123}{32}$$

6. $x = \frac{1}{3} y^3 + \frac{1}{4y}$, $1 \le y \le 3$

$$x' = \frac{1}{3} . 3y^2 + \frac{1}{4}(-1) . y^{-2}$$

$$= y^2 - \frac{1}{4y^2}$$

$$= \frac{4y^4 - 1}{4y^2}$$

$$1+\left(\frac{4y^4-1}{4y^2}\right)^2 = 1+\frac{16y^8-8y^4+1}{16y^4} = \frac{16y^4+16y^8-8y^4+1}{16y^4}$$

$$= \frac{16y^8+8y^4+1}{16y^4} = \frac{\left(4y^4-1\right)^2}{16y^4}$$

$$s = \int_1^3 \sqrt{\frac{\left(4y^4+1\right)^2}{16y^4}}\ dy$$

$$= \int_1^3 \frac{4y^4+1}{4y^2}\ dy = \int_1^3 \left(y^2+\frac{1}{4}y^{-2}\right) dy$$

$$= \left(\frac{y^3}{3}+\frac{1}{4}\cdot\frac{y^{-1}}{-1}\right)\Bigg|_1^3$$

$$= \frac{3^3}{3}-\frac{1}{4\,.\,3}-\frac{1}{3}+\frac{1}{4}$$

$$= \frac{53}{6}$$

7. $y=\frac{1}{2}\left(e^x+e^{-x}\right)$ de $(0,1)$ a $\left(1,\frac{e+e^{-1}}{2}\right)$

$$y' = \frac{1}{2}\left(e^x-e^{-x}\right)$$

$$s = \int_0^1 \sqrt{1+\frac{1}{4}\left(e^x-e^{-x}\right)^2}\ dx$$

$$= \int_0^1 \sqrt{1+\frac{1}{4}\left(e^{2x}-2\,.\,e^x\ .\,e^{-x}+e^{-2x}\right)}\,dx$$

$$= \int_0^1 \sqrt{1+\frac{1}{4}\left(e^{2x}-2+e^{-2x}\right)}\ dx$$

$$= \int_0^1 \sqrt{1+\frac{1}{4}e^{2x}-\frac{1}{2}+\frac{1}{4}e^{-2x}}\ \ dx$$

$$= \int_0^1 \sqrt{\frac{4+e^{2x}-2+e^{-2x}}{4}}\ \ dx$$

$$= \int_0^1 \sqrt{2 + e^{2x} + \frac{1}{e^{-2x}}} \ dx$$

$$= \frac{1}{2}\int_0^1 \frac{1}{e^x}\sqrt{\left(e^{2x}+1\right)^2} \ dx$$

$$= \frac{1}{2}\int_0^1 \frac{1}{e^x}\left(e^{2x}+1\right) \ dx$$

$$= \frac{1}{2}\int_0^1 \left(e^x + e^{-x}\right) \ dx$$

$$= \frac{1}{2}\left(e^x + e^{-x}\right)\Big|_0^1$$

$$= \frac{1}{2}\left(e - e^{-1} - 1 + 1\right)$$

$$= sen\, h\, 1$$

8. $y = \ln\, x\,,\ \sqrt{3} \le x \le \sqrt{8}$

$$y' = \frac{1}{x}$$

$$s = \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}} dx = \int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{x^2+1}}{x} dx$$

$$x^2 + 1 = t^2$$

$$x^2 = t^2 - 1$$

$$x = \left(t^2 - 1\right)^{1/2}$$

$$dx = \frac{1}{2}\left(t^2 - 1\right)^{1/2} . \ 2t \ dt$$

$$I = \int \frac{\sqrt{x^2+1}}{x}\, dx = \int \frac{t}{\left(t^2-1\right)^{1/2}} \cdot \frac{t\; dt}{\left(t^2-1\right)^{1/2}}$$

$$= \int \frac{t^2 dt}{t^2-1}$$

$$= \int \left(1 + \frac{1}{t^2-1}\right) dt$$

$$= \int dt + \int \frac{dt}{(t-1)(t+1)}$$

$$= t + \int \frac{1/2}{t-1}\, dt - \int \frac{1/2}{t+1}\, dt$$

$$= t + \frac{1}{2} \ln |t-1| - \frac{1}{2} \ln |t+1| + C$$

$$= t + \frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + C$$

$$= \sqrt{x^2+1} + \frac{1}{2} \ln \left| \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1} \right| + C$$

$$s = \left( \sqrt{x^2+1} + \frac{1}{2} \ln \left| \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1} \right| \right) \Bigg|_{\sqrt{3}}^{\sqrt{8}}$$

$$= \left( 3 + \frac{1}{2} \ln \left| \frac{2}{4} \right| - 2 - \frac{1}{2} \ln \left| \frac{1}{3} \right| \right)$$

$$= 1 + \frac{1}{2} \ln \frac{3}{2}$$

9. $y = 1 - \ln(sen\ x)$, $\frac{\pi}{6} \le x \le \frac{\pi}{4}$

$$y' = -\frac{\cos x}{sen\ x}$$

$$s = \int_{\pi/6}^{\pi/4} \sqrt{1 + \frac{\cos^2 x}{sen^2 x}}\ dx$$

$$= \int_{\pi/6}^{\pi/4} \sqrt{\frac{sen^2 x + \cos^2 x}{sen^2 x}}\ dx = \int_{\pi/6}^{\pi/4} \frac{dx}{sen^x}$$

$$= \int_{\pi/6}^{\pi/4} \cos ec\ x\ dx$$

$$= \ln \left|\cos ec - \cot g\ x\right|\Big|_{\pi/6}^{\pi/4}$$

$$= \ln \left|\frac{2}{\sqrt{2}} - 1\right| - \ln \left|2 - \frac{\sqrt{3}}{\sqrt{3}}\right|$$

$$= \ln \left|\frac{2 - \sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{3} - 1}\right|$$

$$= \ln \left|\frac{2\sqrt{3} - \sqrt{6}}{2\sqrt{6} - \sqrt{2}}\right| = \ln \left|\frac{\sqrt{2} - 1}{2 - \sqrt{3}\ 3}\right| u.\,c.$$

10. $y = \sqrt{x^3}$, de $P_0\ (0, 0)$ ate $P_1\ (4, 8)$

$$y' = \frac{3}{2} x^{1/2}$$

$$s = \int_0^4 \sqrt{1 + \frac{9}{4} x}\ dx$$

$$= \frac{4}{9} \left. \frac{\left(1 + \frac{9}{4} x\right)^{3/2}}{3/2} \right|_0^4$$

$$= \frac{4}{9} \cdot \frac{2}{3} \left[\left(1 + \frac{9}{4} \cdot 4\right)^{3/2} - 1\right]$$

$$= \frac{8}{27} \left(10\sqrt{10} - 1\right) u.\,c.$$

11. $y = 4\sqrt{x^3} + 2$ de $P_0(0, 2)$ ate $P_1(1, 6)$

$$y' = 4.\frac{3}{2}x^{1/2} = 6x^{1/2}$$

$$s = \int_0^1 \sqrt{1+36x}\ dx$$

$$= \frac{1}{36}\left.\frac{(1+36x)^{3/2}}{3/2}\right|_0^1$$

$$= \frac{1}{36}.\frac{2}{3}\left(37\sqrt{37}-1\right)$$

$$= \frac{1}{54}\left(37\sqrt{37}-1\right)u.c.$$

12. $y = 6\left(\sqrt[3]{x^2}-1\right)$ de $P_0(1, 0)$ ate $P_1\left(2\sqrt{2}, 6\right)$

$$y' = 6.\frac{2}{3}x^{-1/3}$$

$$= 4x^{-1/3}$$

$$s = \int_1^{2\sqrt{2}} \sqrt{1+16x^{-2/3}}\ dx$$

$$= \int_1^{2\sqrt{2}} \sqrt{1+\frac{16}{x^{-2/3}}}\ dx$$

$$= \int_1^{2\sqrt{2}} \sqrt{\frac{x^{-2/3}+16}{x^{-2/3}}}\ dx$$

$$= \int_1^{2\sqrt{2}} \left(16+x^{-2/3}\right)^{1/2} x^{-1/3}\ dx$$

$$= \frac{3}{2}\left.\frac{\left(16+x^{-2/3}\right)^{3/2}}{\frac{3}{2}}\right|_{1}^{2\sqrt{2}}$$

$$= \left(16+\left(2\sqrt{2}\right)^{-2/3}\right)^{3/2} - 17^{3/2}$$

$$= 18\sqrt{18} - 17\sqrt{17}$$

$$= 54\sqrt{2} - 17\sqrt{17}\ u.c.$$

13. $(y-1)^2 = (x+4)^3$ de $P_0(-3, 2)$ ate $P_1(0, 9)$

$$y - 1 = (x+4)^{3/2}$$

$$y = 1 + (x+4)^{3/2}$$

$$y' = \frac{3}{2}(x+4)^{1/2}$$

$$s = \int_{3}^{0} \sqrt{1 + \frac{9}{4}(x+4)}\, dx$$

$$= \frac{4}{9}\left.\frac{\left(1 + 9/4\,(x+3)^{3/2}\right)}{3/2}\right|_{-3}^{0}$$

$$= \frac{4}{9} \cdot \frac{2}{3}\left(10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right)$$

$$= \frac{80\sqrt{10} - 13\sqrt{13}}{27}\ .u.c.$$

14. $x^2 = y^3$, de $P_0(0, 0)$ ate $P_1(8,4)$

$$y = x^{2/3}$$

$$y' = \frac{2}{3}x^{-1/3}$$

$$s = \int_0^8 \sqrt{1 + \frac{4}{9} x^{-2/3}}\, dx$$

$$= \int_0^8 \sqrt{\frac{9x^{2/3} + 4}{9x^{2/3}}} dx$$

$$= \int_0^8 \left(9x^{2/3} + 4\right)^{1/2} . 3 . x^{-1/3}\ dx$$

$$u = 9x^{2/3} + 4$$

$$du = 9 . \frac{2}{3} x^{-1/3}$$

$$= \frac{1}{18} . \left.\frac{\left(9x^{2/3} + 4\right)^{3/2}}{3/2}\right|_0^8$$

$$= \frac{1}{18} . \frac{2}{3} \left(40\sqrt{40} - 4\sqrt{4}\right)$$

$$= \frac{1}{27} \left(80\sqrt{10} - 8\right) u.\, c.$$

Nos exercícios de 15 a 21, estabelecer a integral que da o comprimento de arco da curva dada.

15. $y = x^2,\ 0 \le x \le 2$

$y' = 2x$

$$s = \int_0^2 \sqrt{1 + 4x^2}\, dx$$

16. $y = \frac{1}{x}$ de $P_0 \left(\frac{1}{4}, 4\right)$ ate $P_1 \left(4, \frac{1}{4}\right)$

$$y' = \frac{-1}{x^2}$$

$$s=\int_{1/4}^{4}\sqrt{1+\frac{1}{x^4}}dx$$

$$=\int_{1/4}^{4}\sqrt{\frac{x^4+1}{x^4}}dx$$

$$=\int_{1/4}^{4}\frac{\sqrt{x^4+1}}{x^2}dx$$

17. $x^2-y^2=1$ de $P_0\left(3,-2\sqrt{2}\right)$ ate $P_0\left(3,2\sqrt{2}\right)$

$x^2=1+y^2$

$x=\left(1+y^2\right)^{1/2} \quad \Rightarrow \quad x'=\frac{1}{2}\left(1+y^2\right)^{1/2}.\,2y$

$$s=\int_{-2\sqrt{2}}^{2\sqrt{2}}\sqrt{1+\frac{y^2}{1+y^2}}\;dy$$

$$=\int_{-2\sqrt{2}}^{2\sqrt{2}}\sqrt{\frac{1+y^2+y^2}{1+y^2}}\;dy$$

$$=\int_{-2\sqrt{2}}^{2\sqrt{2}}\sqrt{\frac{1+2y^2}{1+y^2}}\;dy$$

18. $y=e^x$, de $P_0(0,1)$ ate $P_1\left(2,e^2\right)$

$y'=e^x$

$$s=\int_0^2\sqrt{1+e^{2x}}dx$$

19. $y=x^2+2x-1,\ 0\le x\le 1$

$y'=2x+2$

$$s = \int_0^1 \sqrt{1 + (2x+2)^2}\, dx$$

$$= \int_0^1 \sqrt{1 + 4x^2 + 8x + 4}\ dx$$

$$= \int_0^1 \sqrt{4x^2 + 8x + 5}\ dx$$

20. $y = \sqrt{x}\,,\ 2 \le x \le 4$

$$y' = \frac{1}{2} x^{-1/2}$$

$$s = \int_2^4 \sqrt{1 + \frac{1}{4x}} dx$$

21. $y = sen\ 3x\,,\ 0 \le x \le 2\pi$

$$y' = 3 \cos 3x$$

$$s = \int_0^{2\pi} \sqrt{1 + 9 \cos^2 3x} dx$$

Nos exercícios de 22 a 29, calcular o comprimento de arco da curva dada na forma paramétrica.

22. $\begin{cases} x = t^3 \\ y = t^2 \end{cases},\ 1 \le t \le 3$

$$s=\int_1^3 \sqrt{9t^4+4t^2}\,dt$$

$$=\int_1^3 t\sqrt{9t^2+4}\;dt$$

$$=\frac{1}{18}\left.\frac{\left(9t^2+4\right)^{3/2}}{3/2}\right|_1^3=\frac{1}{18}\cdot\frac{2}{3}\left(85\sqrt{85}-13\sqrt{13}\right)$$

$$=\frac{1}{27}\left(85\sqrt{85}-13\sqrt{13}\right)u.\,c.$$

23. $\begin{cases} x=2\,(t-sen\;t) \\ y=2(1-\cos t) \end{cases}$, $t\in[0,\pi]$

$x'=2\,(1-\cos t)$
$y'=2\;sen\;t$

$$s=\int_0^\pi \sqrt{4(1-\cos t)^2+4\;sen^2 t}\;dt$$

$$=\int_0^\pi \sqrt{4\left(1-2\cos t+\cos^2 t+sen^2 t\right)}\,dt$$

$$=\int_0^\pi 2\sqrt{2-2\cos t}\;dt$$

$$=\int_0^\pi 2\sqrt{(1-\cos t)}\,dt$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{1-\cos t}\ dt$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{2sen^2 \frac{t}{2}}\ dt$$

$$= 2\sqrt{2}\int_0^{\pi} \sqrt{2}\ sen\ \frac{t}{2} dt$$

$$= -4\ .\ 2\ .\ \cos\frac{t}{2}\bigg|_0^{\pi} = -8\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= -8(0-1)$$

$$= 8\ u.\ c.$$

24. $\begin{cases} x = -sen\ t \\ y = \cos t \end{cases},\ t \in [0, 2\pi]$

$$s = \int_0^{2\pi} \sqrt{\cos^2 t + sen^2 t}\ dt$$

$$= t\Big|_0^{2\pi} = 2\pi$$

25. $\begin{cases} x = t\ sen\ t \\ y = t\ \cos t \end{cases},\ t \in [0, \pi]$

$$s = \int_0^{\pi} \sqrt{t^2 \cos^2 t + 2t \cos t\ sen\ t + sen^2 t + \cos^2 t - 2t\ sen\ t \cos t + t^2 sen^2 t}\ dt$$

$$= \int_0^{\pi} \sqrt{1+t^2}\ dt$$

$$= \frac{t}{2}\sqrt{1+t^2} + \frac{1}{2} \ln \left|t + \sqrt{1+t^2}\right|\Bigg|_0^{\pi}$$

$$= \frac{\pi}{2}\sqrt{1+\pi^2} + \frac{1}{2} \ln \left|\pi + \sqrt{1+\pi^2}\right|$$

26. $\begin{cases} x = 3t + 2 \\ y = t - 1 \end{cases},\ t \in [0, 2]$

$$s=\int_0^2 \sqrt{1+9}\ dt$$

$$=\sqrt{10}\ t\Big|_0^2=2\sqrt{10}$$

27. $\begin{cases} x=\dfrac{1}{3}t^3 \\ y=\dfrac{1}{2}t^2 \end{cases}$ , $0\le t\le 2$

$$\begin{cases} x'=\dfrac{1}{3}.\,3t^2 \\ y'=\dfrac{1}{2}.\,2t \end{cases}$$

$$s=\int_0^2 \sqrt{t^4+t^2}\ dt=\int_0^2 t\sqrt{t^2+1}dt$$

$u=t^2+1$
$du=2t\ dt$

$$s=\int_0^2 t\left(t^2+1\right)^{1/2}dt=\frac{1}{2}\left.\frac{\left(t^2+1\right)^{3/2}}{3/2}\right|_0^2$$

$$=\frac{1}{2}\cdot\frac{2}{3}\left(5^{3/2}-1\right)$$

$$=\frac{1}{3}\left(5\sqrt{5}-1\right)u.\,c.$$

28. $\begin{cases} x=e^t\cos t \\ y=e^t\,sen\ t \end{cases}$ , $1\le t\le 2$

$$\begin{cases} x'=-e^t\,sen\ t+\cos t\ e^t \\ y'=e^t\cos t+sen\ t\ e^t \end{cases}$$

$$s=\int_1^2 \sqrt{e^{2t}(\cos t - sen\ t)^2 + e^{2t}(\cos t + sen\ t)^2}$$

$$= \int_1^2 e^t \sqrt{2}\ dt$$

$$= \sqrt{2}\ e^t \Big|_1^2 = \sqrt{2}\left(e^2 - e\right) u.\,c.$$

29. $\begin{cases} x = 2\cos t + 2t\ sen\ t \\ y = 2\ sen\ t - 2t\cos t \end{cases},\ 0 \le t \le \dfrac{\pi}{2}$

$x' = 2t\cos t$

$y' = 2t\ sen\ t$

$$s = \int_0^{\pi/2} \sqrt{4t^2\cos^2 t + 4t^2 sen^2 t}\ dt$$

$$= \int_0^{\pi/2} 2t\ dt = 2\frac{t^2}{2}\Big|_0^{\pi/2} = \frac{\pi^2}{4}\ u.\,c.$$

30. Achar o comprimento da hipociclóide

$\begin{cases} x = 4\ sen^3 t \\ y = 4\cos 3\ t \end{cases},\ t \in [0, 2\pi]$

$\begin{cases} x' = 4\ .\ 3\ sen^2 t\ .\cos t \\ y' = -4\ .\ 3\cos^2 t\ .\ sen\ t \end{cases}$

$$s = 4\int_0^{\pi/2} \sqrt{12^2\, sen^4 t \cos^2 t + 12 \cos^4 t\, sen^2 t}\; dt$$

$$= 4\int_0^{\pi/2} 12\, sen\, t \cos t\; dt$$

$$= 4\,.\,12\,.\, \frac{sen^2 t}{2}\Big|_0^{\pi/2}$$

$$= \frac{48}{2}(1-0)$$

$$= 24\; u.\,c.$$

31. Achar o comprimento da circunferência.

$$\begin{cases} x = a\cos t \\ y = a\; sen\; t \end{cases},\; t \in [0, 2\pi]$$

$$x' = -a\; sen$$
$$y' = a \cos t$$

$$s = 4\int_0^{\pi/2} \sqrt{a^2 sen^2 t + a^2 \cos^2 t}\; dt$$

$$= 4\int a\; dt = 4\; at\Big|_0^{\pi/2} = 2\; a\pi\; u.\,c.$$

32. Calcular o comprimento da parte da circunferência que está no 1° quadrante

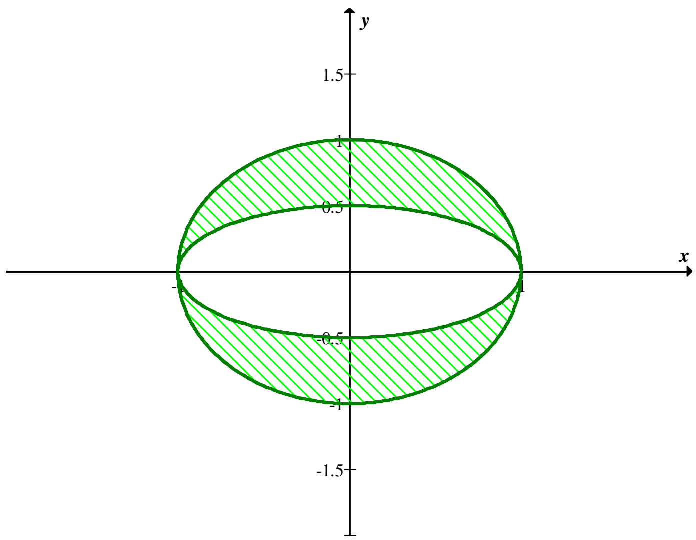
$$\begin{cases} x = 7\cos\dfrac{t}{4} \\[2ex] y = 7\; sen\; \dfrac{t}{4} \end{cases}$$

$$s = \int_0^{2\pi} \sqrt{\left(\frac{7}{4}\right)^2 sen^2\frac{t}{4} + \left(\frac{7}{4}\right)^2 \cos^2\frac{t}{4}}\; dt$$

$$= \int_0^{2\pi} \frac{7}{4} dt = \frac{7}{4} t\Big|_0^{2\pi} = \frac{7}{4}\pi\; u.\,c.$$

Nos exercícios de 33 a 35, calcular a área da região limitada pelas seguintes curvas, dadas na forma paramétrica.

33. $\begin{cases} x = \cos t \\ y = sen\ t \end{cases}$ e $\begin{cases} x = \cos t \\ y = \dfrac{1}{2} sen\ t \end{cases}$

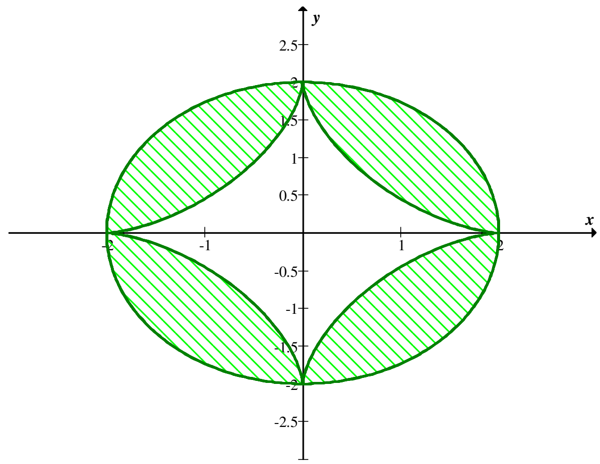



$$A = -4\int_0^{\pi/2} sen\ t\,(-\,sen\ t)\,dt + 4\int_0^{\pi/2} \frac{1}{2} sen\ t\,(-\,sen\ t)\,dt$$

$$= \pi - \frac{1}{2}\pi$$

$$= \frac{1}{2}\pi\ u.c.$$

34. $\begin{cases} x = 2\cos^3 t \\ y = 2\ sen^3 t \end{cases}$ e $\begin{cases} x = 2\cos t \\ y = 2\ sen\ t \end{cases}$




$$A = 4\left[ -\int_0^{\pi/2} 2\ sen\ t\ (-2\ sen\ t)dt - \int_0^{\pi/2} -2\ sen^3 t\ .\ 2\ .\ 3\cos^2 t(-sen\ t)dt \right]$$

$$= 4\left[ 4\ .\ \frac{1}{2}\left( t - \frac{1}{2}\ sen\ 2t \right)\Bigg|_0^{\pi/2} - 12\int_0^{\pi/2} sen^4 t\ \cos^2 t\ dt \right.$$

$$= 4\left[ 2\ .\ \frac{\pi}{2} - 12\int_0^{\pi/2} sen^4 t\ (1 - sen^2 t)\ dt \right]$$

$$= 4\pi - 48\int_0^{\pi/2} (sen^4 t - sen^6 t)\ dt$$

$$= 4\pi + 48\left( -\frac{1}{6}\ sen^5 t\ \cos t + \frac{5}{6}\int sen^4 t\ dt \right) - 48\int sen^4 t\ dt\Bigg|_0^{\pi/2}$$

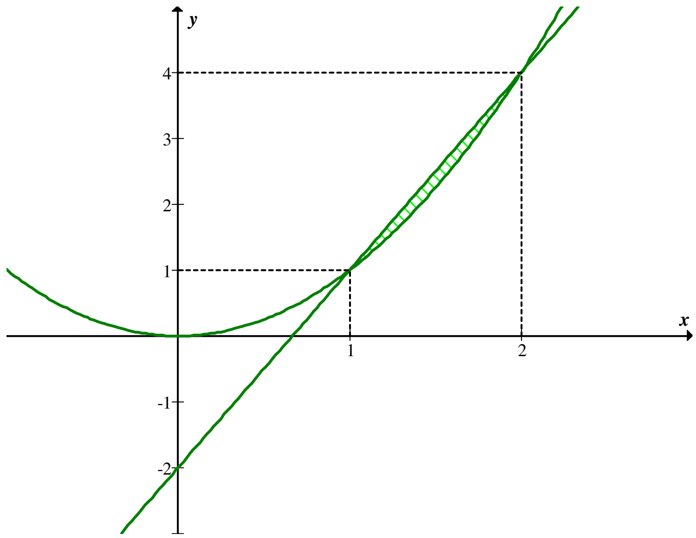
$$= 4\pi - \frac{48}{6}\ sen^5 t\ \cos t\Bigg|_0^{\pi/2} + 48\ \frac{5}{2}\int_0^{\pi/2} sen^4 t\ dt - 48\int_0^{\pi/2} sen^4 t\ dt$$

$$= 4\pi - 8\left( -\frac{1}{4}\ sen^3 t\ \cos t + \frac{3}{4}\int sen^2 t\ dt \right)\Bigg|_0^{\pi/2}$$

$$= 4\pi - 6\left( \frac{1}{2} t \right)\Bigg|_0^{\pi/2}$$

$$= 4\pi - \frac{3\pi}{2} = \frac{8\pi - 3\pi}{2} = \frac{5\pi}{2} u.c.$$

35. $\begin{cases} x = t \\ y = t^2 \end{cases}$ e $\begin{cases} x = 1 + t \\ y = 1 + 3t \end{cases}$




$y = x^2$
$y = 3x - 2$

$x = 2$ e $x = 1$

$(1, 1) \quad \to \quad t = 1$ $\qquad\qquad$ $(1, 1) \quad \to \quad t = 0$
$(2, 4) \quad \to \quad t = 2$ $\qquad\qquad$ $(2, 4) \quad \to \quad t = 1$

$$A_1 = \int_1^2 t^2 . 1 . dt \qquad\qquad A_1 = \int_0^1 (1 + 3 . t) . dt$$

$$= \left. \frac{t^3}{3} \right|_1^2 \qquad\qquad = t + 3 \left. \frac{t^2}{2} \right|_0^1$$

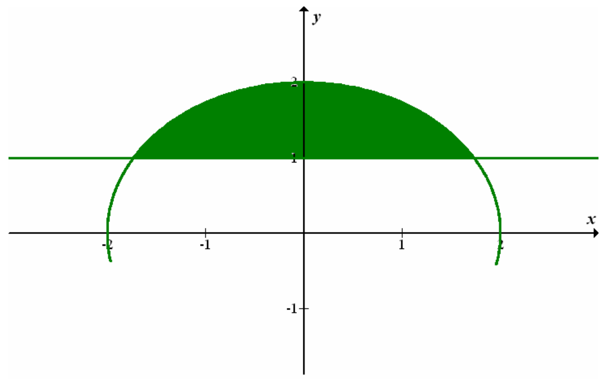
$$= \frac{7}{3} \qquad\qquad = \frac{5}{2}$$

$$A = \frac{5}{2} - \frac{7}{3} = \frac{1}{6} \, u.c.$$

36. Calcular a área da arte da circunferência

$x = 2 \cos t$
$y = 2 \; sen \; t$

que está acima da reta $y = 1$



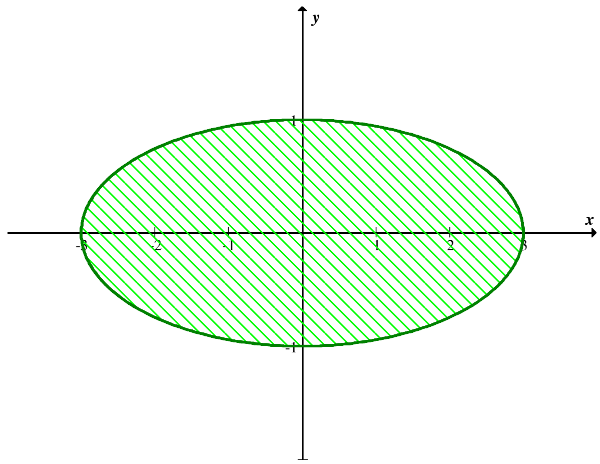

$(0, 2) \quad \to \quad t = \pi/2$

$\left(\sqrt{3}, 1\right) \quad \to \quad 2\cos t = \sqrt{3} \quad \therefore \quad \cos t = \dfrac{\sqrt{3}}{2} \quad \therefore \quad t = \dfrac{\pi}{6}$

$$
\begin{aligned}
A_1 &= \int_{\pi/2}^{\pi/6} 2\, sen\, t\, (-2\, sen\, t)\, dt \\
&= -4\int_{\pi/2}^{\pi/6} \frac{1-\cos 2t}{2}\, dt = -2\left(t - \frac{1}{2}\, sen\, 2t\right)\Bigg|_{\pi/2}^{\pi/6} \\
&= -2\left(\frac{\pi}{6} - \frac{1}{2} sen\, \frac{\pi}{3} - \frac{\pi}{2} + \frac{1}{2} sen\, \pi\right) \\
&= -2\left(\frac{\pi}{6} - \frac{1}{2}\,\frac{\sqrt{3}}{2} - \frac{\pi}{2}\right) \\
&= -\frac{\pi}{3} + \frac{\sqrt{3}}{2} + \pi \\
&= \frac{-2\pi + 3\sqrt{3} + 6\pi}{6} = \frac{4\pi + 3\sqrt{3}}{6}
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{4\pi + 3\sqrt{3}}{6} - \sqrt{3} \\
&= \frac{4\pi + 3\sqrt{3} - 6\sqrt{3}}{6} \\
&= \frac{4\pi - 3\sqrt{3}}{6}\ u.c.
\end{aligned}
$$

37. Calcular a área da região delimitada pela elipse




$$\begin{cases} x = 3\cos t \\ y = sen\ t \end{cases}$$

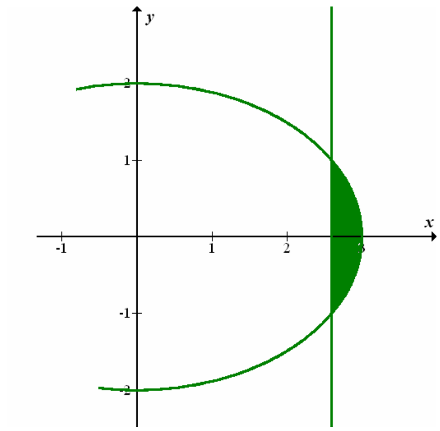
$$A_1 = \int_{\pi/2}^{0} sen\ t\,(-3\ sen\ t)\,dt$$

$$= 3\int_{\pi/2}^{0} sen^2\ t\ \ dt$$

$$= \frac{3\pi}{4}\ u.c.$$

$$A = 4\,.\,\frac{3\pi}{4} = 3\pi\ u.a$$

38. Calcular a área da região limitada à direita pela elipse $\begin{cases} x = 3\cos t \\ y = 2\ sen\ t \end{cases}$ e a esquerda pela reta $x = \dfrac{3\sqrt{3}}{2}$




$$\frac{3\sqrt{3}}{2} = 3\cos t \quad \therefore \quad \frac{\sqrt{3}}{2}\cos t \quad \therefore \quad t = \frac{\pi}{6}$$

$$A_1 = \int_{\frac{\pi}{6}}^{0} 2\, sen\, t\,(-3\, sen\, t)\, dt = 6\int_{\frac{\pi}{6}}^{0} sen^2 t\, dt$$

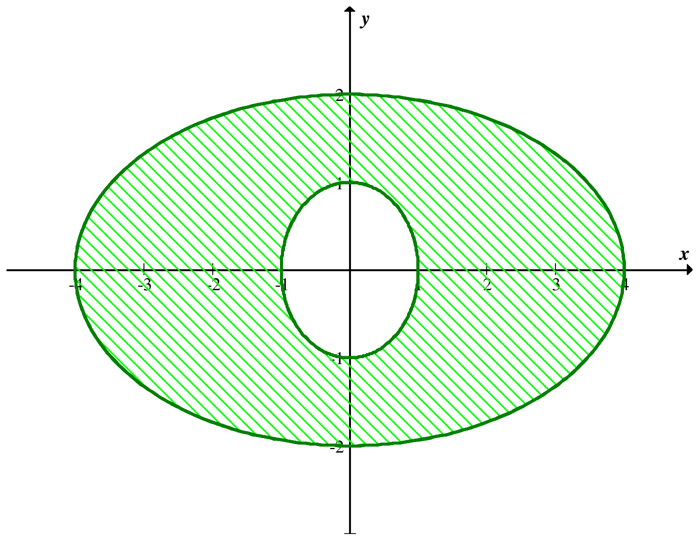
$$= 6\left(\frac{1}{2}t - \frac{1}{4} sen\, 2t\right)\Bigg|_{0}^{\pi/6}$$

$$= 6\left(\frac{1}{2}\cdot\frac{\pi}{6} - \frac{1}{4} sen\,\frac{\pi}{3}\right)$$

$$= \frac{\pi}{2} - \frac{3}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\pi}{2} - \frac{3}{4}\sqrt{3}\ u.\, a.$$

39. Calcular a área da região entre as curvas

$$\begin{cases} x = 4\cos t \\ y = 2\, sen\, t \end{cases} \text{ e } \begin{cases} x = \cos t \\ y = sen\, t \end{cases}$$



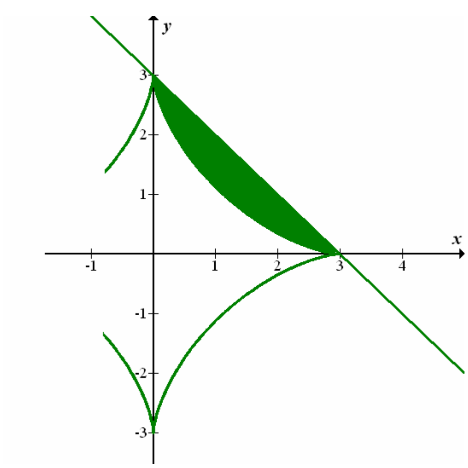

$$A_1 = 4\int_{\pi/2}^{0} 4\ sen\ t(-2\ sen\ t)dt = 8\pi$$

$$A_2 = 4\int_{\pi/2}^{0} -sen\ t\ sen\ t\ dt = -\left(\frac{1}{2}sen\ t\cos t + \frac{1}{2}t\right)\Bigg|_{\pi/2}^{0} = \pi$$

$$A = 8\pi - \pi = 7\pi\ u.\ a.$$

40. Calcular a área entre o arco da hipociclóide $\begin{cases} x = 3\cos^3 t \\ y = 3\ sen^3 t \end{cases}$, $t \in \left[0, \pi/2\right]$

e a reta $x + y = 3$



$$A_1 = \int_{\frac{\pi}{2}}^{0} 3\,sen^3 t\,\left(-3\,.\,3\cos^3 t\,.sen\,t\right)dt$$

$$= 27\int_{\frac{\pi}{2}}^{0} sen^4 t\,\cos^2 t\;dt$$

$$= 27\int_{\frac{\pi}{2}}^{0} sen^4 t\,\left(1 - sen^2 t\right)dt$$

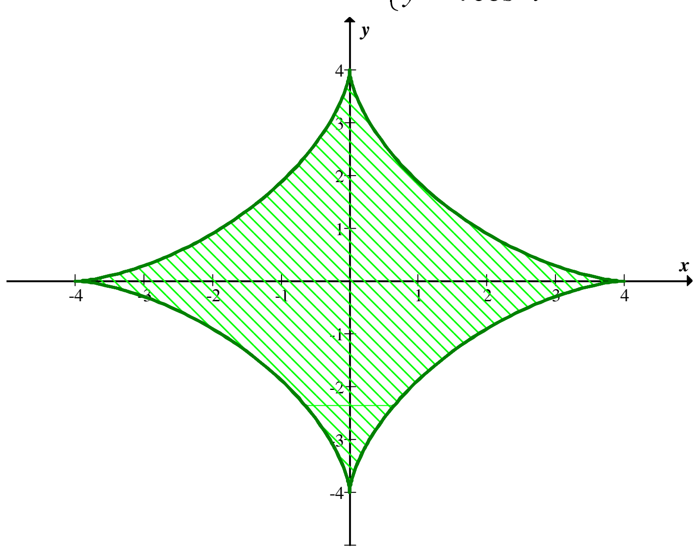
$$= 27\int_{\frac{\pi}{2}}^{0} \left(sen^4 t - sen^6 t\right)dt$$

$$= 27\left[\frac{1}{6}\,sen^5 t\,\cos t - \frac{1}{24}\,sen^3 t\,\cos t - \frac{1}{16}\,sen\,t\,\cos t + \frac{1}{16}t\right]\Bigg|_0^{\pi/2}$$

$$= 27\,.\,\frac{1}{6}\,.\,\frac{\pi}{2} = \frac{27}{32}\pi$$

$$A = \frac{9}{2} - \frac{27}{32}\pi = \frac{144 - 27\pi}{32}\,u.\,a.$$

41. Calcular a área delimitada pela hipociclóide $\begin{cases} x = 4\,sen^3 t \\ y = 4\cos^3 t \end{cases}$



$$A = \int_0^{\pi/2} 4\cos^3 t \,.\, 4 \,.\, 3 \,.\, sen^2 t \,.\, \cos t \; dt$$

$$= \int_0^{\pi/2} \left(48\cos^4 t \; sen^2 t\right) dt$$

$$= \int_0^{\pi/2} 48\cos^4 t\left(1-\cos^2 t\right) dt$$

$$= \int_0^{\pi/2} 48\left(\cos^4 t - \cos^6 t\right) dt$$

$$= \frac{3\pi}{2} \; u.\, a.$$

$$A = 4 \,.\, \frac{3\pi}{2} = 6\pi \; u.\, a$$

42. Calcular a área da região $S$ , hachurada na figura 8.12

$$x = k\left(t - sen\; t\right)$$
$$y = k\left(1-\cos t\right)$$

$$A = \int_0^{2\pi} k\left(1-\cos t\right) k\left(1-\cos t\right) dt$$

$$= k^2 \int_0^{2\pi} \left(1-\cos t\right)^2 dt$$

$$= k^2 \int_0^{2\pi} \left(1 - 2\cos t + \cos^2 t\right) dt$$

$$= k^2 \left( t - 2\; sen\; t + \frac{1}{2}\cos t \; sen\; t + \frac{1}{2} t \right)\Bigg|_0^{2\pi}$$

$$= k^2 \left( 2\pi + \frac{1}{2} \,.\, 2\pi \right)$$

$$= k^2 \,.\, 3\pi$$

$$= 3\pi \; k^2 u.\, a.$$