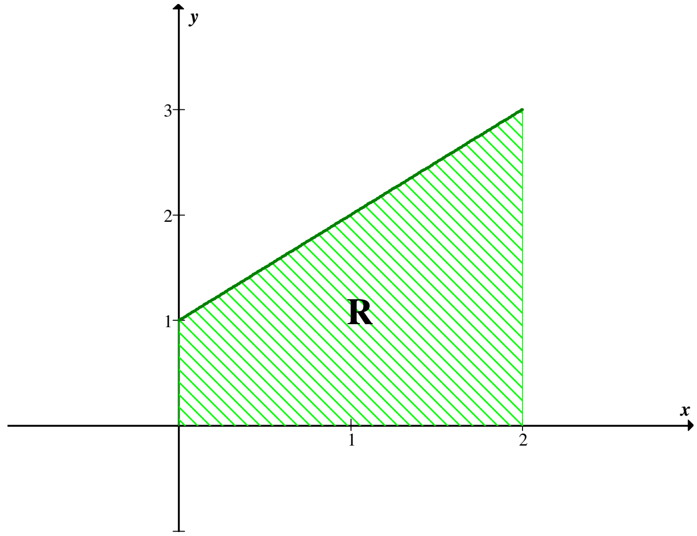
# 8.7 – EXERCÍCIOS – pg. 359

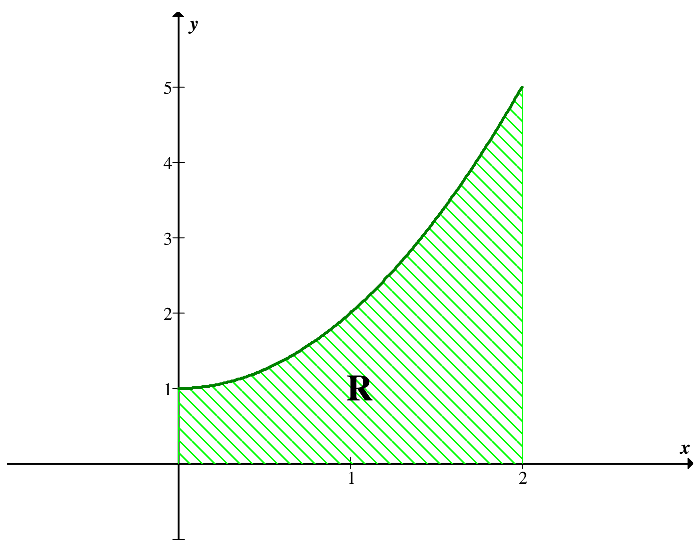
Nos exercícios de 1 a 5, determinar o volume do sólido de revolução gerado pela rotação, em torno do eixo dos *x*, da região R delimitada pelos gráficos das equações dadas.

1. $y = x + 1$, $x = 0$, $x = 2$ e $y = 0$




$$v = \pi \int_0^2 (x+1)^2 dx = \pi \, . \left. \frac{(x+1)^3}{3} \right|_0^2 = \frac{\pi}{3}(27-1) = \frac{26\pi}{3} u.\,v.$$

2. $y = x^2 + 1$, $x = 0$, $x = 2$ e $y = 0$

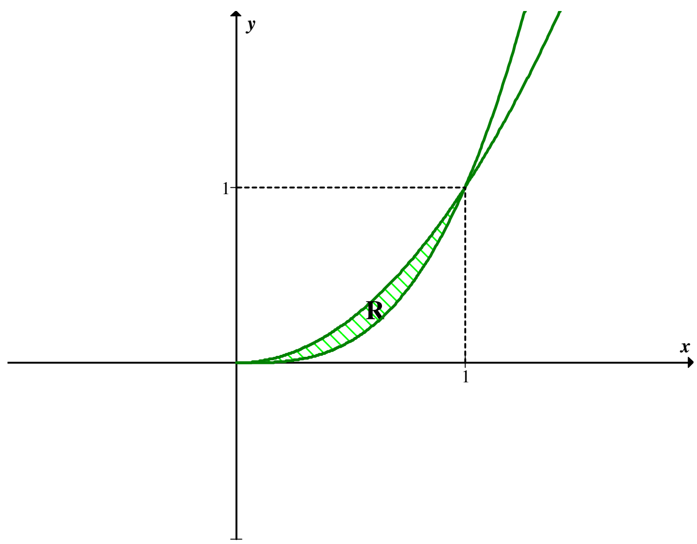


$$v = \pi\int_0^2 (x^2+1)^2\,dx = \int_0^2 (x^4+2x^2+1)dx$$

$$= \pi\left(\frac{x^5}{5} + 2\frac{x^3}{3} + x\right)\Bigg|_0^2$$

$$= \pi\left(\frac{32}{5} + \frac{16}{3} + 2\right) = \frac{206\pi}{15} u.\,v.$$

3. $y = x^2$ e $y = x^3$

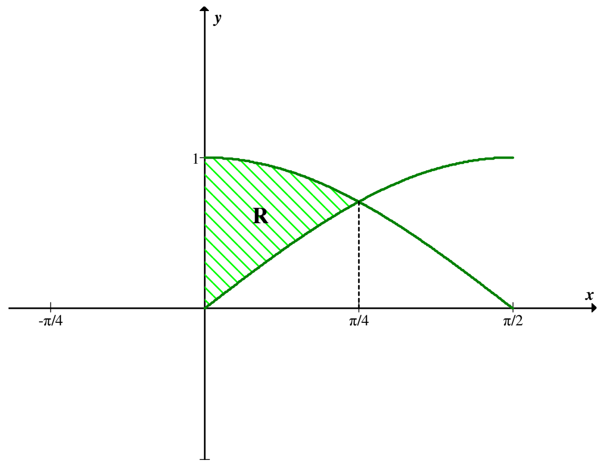



$$v = \pi\int_0^1 \left[(x^2)^2 - (x^3)^2\right]dx$$

$$= \pi\int_0^1 (x^4 - x^6)dx$$

$$= \pi\left(\frac{x^5}{5} - \frac{x^7}{7}\right)\Bigg|_0^1 = \pi\left(\frac{1}{5} - \frac{1}{7}\right) = \frac{2}{35}\pi\, u.\,v.$$

4. $y = \cos x$, $y = sen\ x$, $x = 0$ e $x = \dfrac{\pi}{4}$




$$v = \pi \int_0^{\pi/4} \left(\cos^2 x - sen^2 x\right) dx$$

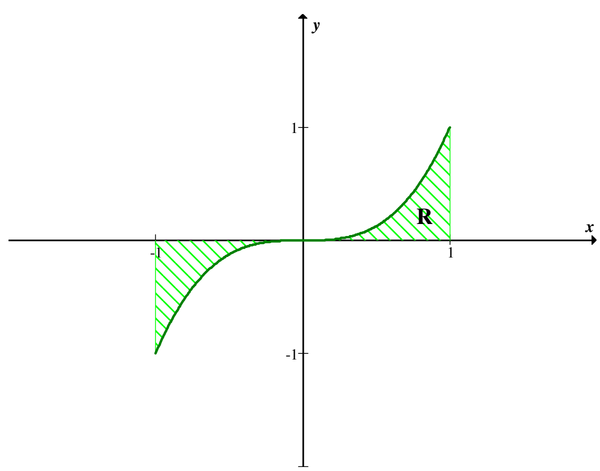
$$= \pi \int_0^{\pi/4} \left(\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}\right) dx$$

$$= \pi \int_0^{\pi/4} \cos 2x \, dx$$

$$= \pi \frac{1}{2} sen\ 2x \Big|_0^{\pi/4}$$

$$= \frac{\pi}{2}\left(sen\ \frac{\pi}{2}\right) = \frac{\pi}{2}\ u.\ v.$$

5. $y = x^3$, $x = -1$, $x = 1$ e $y = 0$



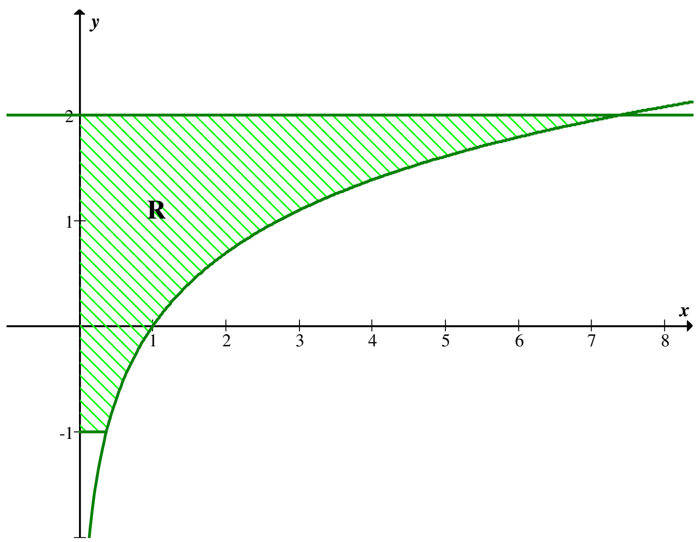
$$v = 2\pi\int_0^1 \left(x^3\right)^2 dx$$

$$= 2\pi\int_0^1 x^6 dx$$

$$= 2\pi \left.\frac{x^7}{7}\right|_0^1 = 2\pi \frac{1}{7} = \frac{2}{7}\pi \ u.\ v.$$

Nos exercícios de 6 a 10 determinar o volume do solído gerado pela rotação, em torno do eixo dos y, da região R, delimitada pelos gráficos das equações dadas.

6. $y = \ln x$, $y = -1$, $y = 2$ e $x = 0$

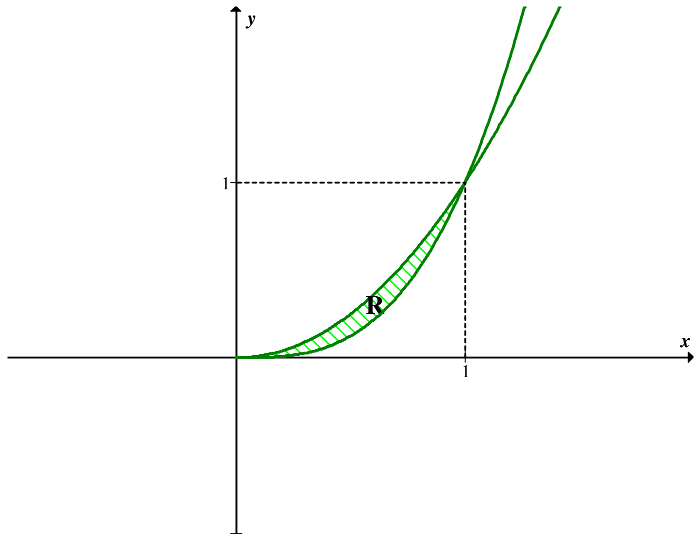



$$v = \pi\int_{-1}^{2} \left(e^y\right)^2 dy$$

$$= \pi\int_{-1}^{2} e^{2y} dy = \pi \left.\frac{1}{2}e^{2y}\right|_{-1}^{2}$$

$$= \frac{\pi}{2}\left(e^4 - e^{-2}\right) = \frac{\pi}{2}\left(e^4 - \frac{1}{e^2}\right) u.v$$

7. $y = x^2$, $y = x^3$



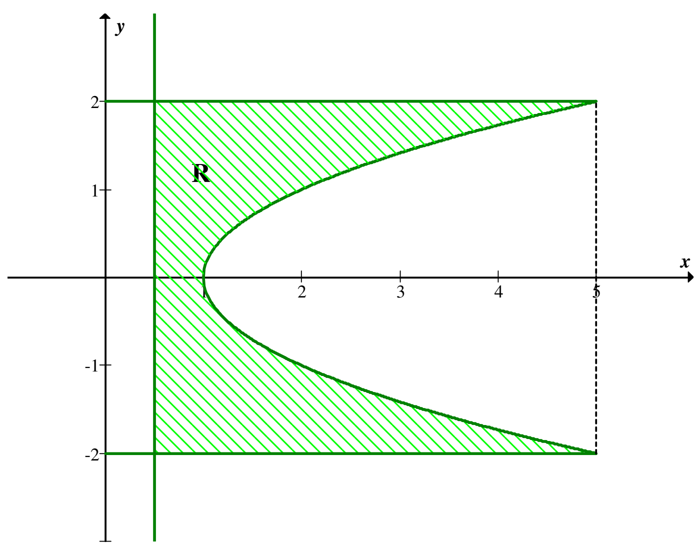

$$v = \pi\int_0^1 \left[\left(y^{1/3}\right)^2 - \left(y^{1/2}\right)^2\right] dy$$

$$= \pi\int_0^1 \left(y^{2/3} - y\right) dy$$

$$= \pi\left(\frac{y^{5/3}}{5/3} - \frac{y^2}{2}\right)\Bigg|_0^1 = \pi\left(\frac{3}{5} - \frac{1}{2}\right)$$

$$= \pi\,\frac{6-5}{10} = \frac{\pi}{10}\,u.\,v.$$

8. $x = y^2 + 1$, $x = \frac{1}{2}$, $y = -2$ e $y = 2$



$$v = 2\pi\int_0^2\left[\left(y^2+1\right)^2-\left(\frac{1}{2}\right)^2\right]dy$$

$$= 2\pi\int_0^2\left(y^4+2y^2+1-\frac{1}{4}\right)dy$$

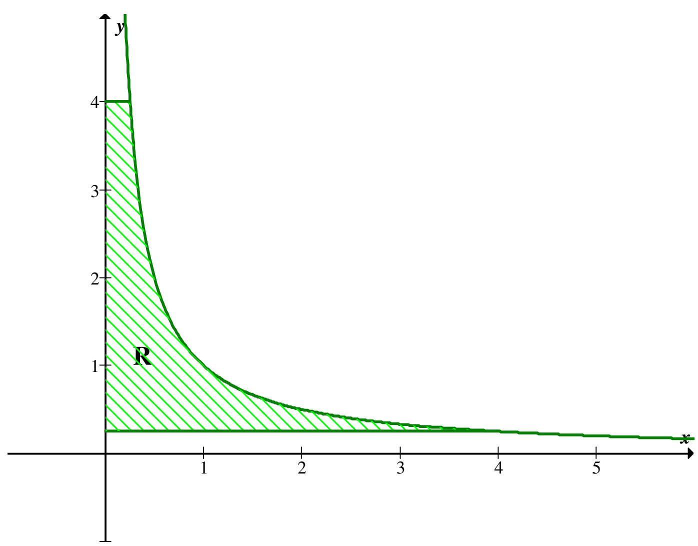
$$= 2\pi\int_0^2\left(y^4+2y^2+\frac{3}{4}\right)dy$$

$$= 2\pi\left(\frac{y^5}{5}+\frac{2}{3}y^3+\frac{3}{4}y\right)\Bigg|_0^2$$

$$= 2\pi\left(\frac{32}{5}+\frac{2}{3}\,.\,8+\frac{3}{4}\,.\,2\right)$$

$$= \frac{397\pi}{15}\ u.\,v.$$

9. $y=\dfrac{1}{x}$, $x=0$, $y=\dfrac{1}{4}$ e $y=4$

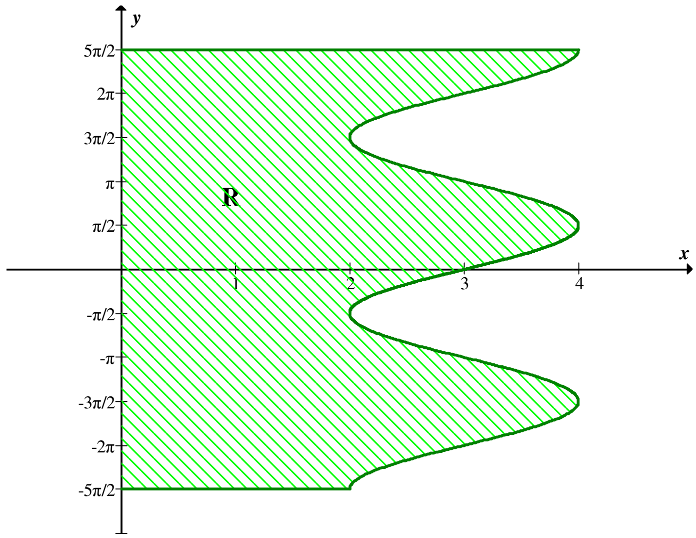



$$v = \pi\int_{1/4}^{4}\left(\frac{1}{y}\right)^2 dy$$

$$= \pi\int_{1/4}^{4}\frac{1}{y^2}\,dy = \pi\,\frac{y^{-1}}{-1}\Bigg|_{1/4}^{4}$$

$$= -\pi\left(\frac{1}{4}-\frac{1}{1/4}\right) = -\pi\left(\frac{1}{4}-4\right) = \frac{15\pi}{4}u.\,v.$$

10. $x = 3 + sen\ y$ , $x = 0$, $y = \dfrac{-5\pi}{2}$ e $y = \dfrac{5\pi}{2}$




$$v = \pi \int_{-5\pi/2}^{5\pi/2} (3 + sen\ y)^2\, dy$$

$$= \pi \int_{-5\pi/2}^{5\pi/2} \left(9 + 6\ sen\ y + sen^2 y\right) dy$$

$$= \pi \left( 9y + 6(-\cos\ y) + \frac{1}{2}\ sen\ y \cos y + \frac{1}{2} y \right)\Bigg|_{-5\pi/2}^{5\pi/2}$$

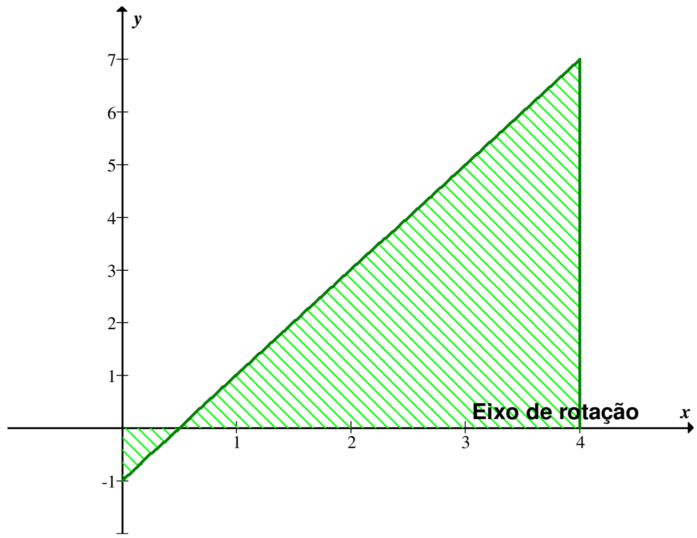
$$= \pi \left( 9 \,.\, \frac{5\pi}{2} + \frac{1}{2} \,.\, \frac{5\pi}{2} - 9 \,.\, \frac{-5\pi}{2} - \frac{1}{2} \frac{-5\pi}{2} \right)$$

$$= \pi \left( \frac{90\pi}{2} + \frac{10\pi}{4} \right)$$

$$= \frac{95\pi^2}{2}\, u.v$$

Nos exercícios de 11 a 16, determinar o volume do sólido de revolução gerado pela rotação das regiões indicadas ao redor dos eixos dados.

11. $y = 2x - 1$, $y = 0$, $x = 0$, $x = 4$ ao redor do eixo dos x



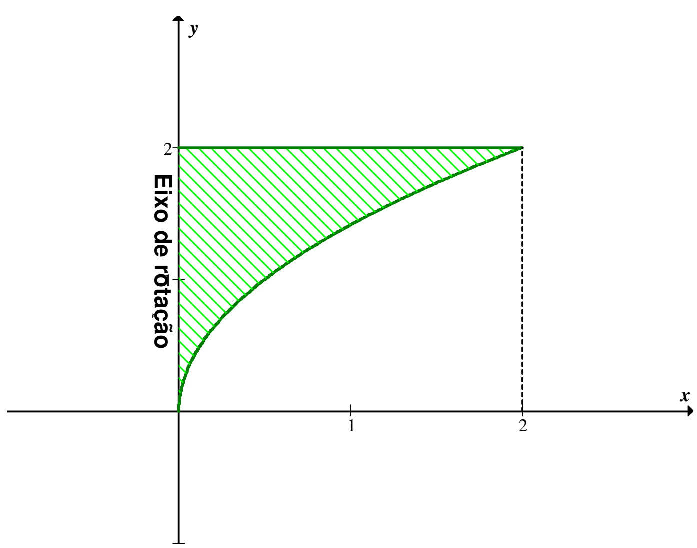

$$v = \pi\int_0^4 (2x-1)^2\,dx$$

$$= \pi\int_0^4 \left(4x^2 - 4x + 1\right)dx$$

$$= \pi\left(4\frac{x^3}{3} - 4\frac{x^2}{2} + x\right)\Bigg|_0^4$$

$$= \frac{172}{3}\pi\, u.\,v$$

12. $y^2 = 2x$, $x = 0$, $y = 0$ e $y = 2$, ao redor do eixo dos y

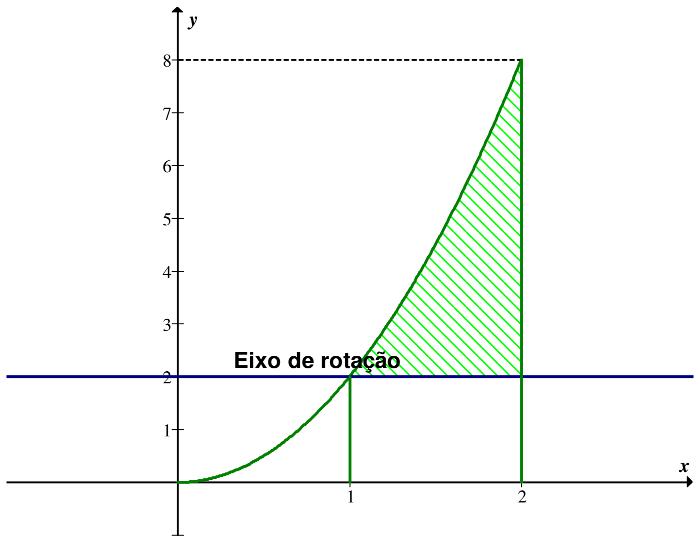


$$v = \pi \int_0^2 \left( \frac{y^2}{2} \right)^2 dy$$

$$= \pi \int_0^2 \frac{y^4}{4} dy$$

$$= \frac{\pi}{4} \frac{y^5}{5} \Bigg|_0^2 = \frac{8\pi}{5} \, u.\, c.$$

13. $y = 2x^2$, $x = 1$, $x = 2$ e $y = 2$, ao redor do eixo $y = 2$



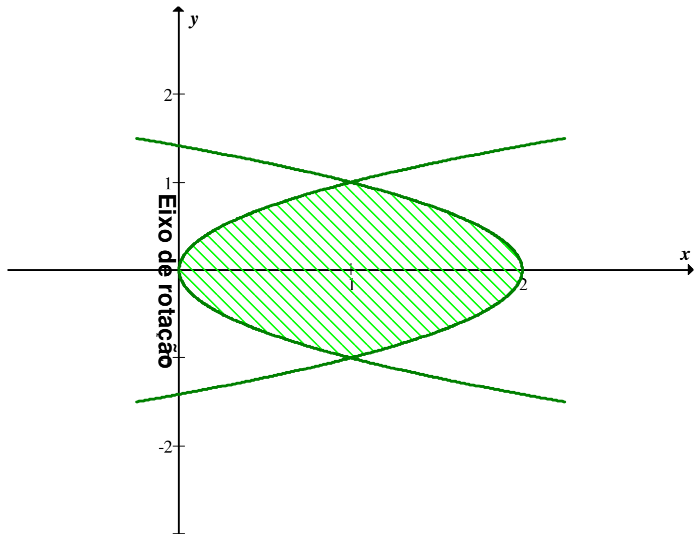

$$v = \pi \int_1^2 \left(2x^2 - 2\right)^2 dx$$

$$= \pi \int_1^2 \left(4x^4 - 8x^2 + 4\right) dx$$

$$= \pi \left( 4\frac{x^5}{5} - 8\frac{x^3}{3} + 4x \right) \Bigg|_1^2$$

$$= \frac{152}{15} \pi \, u.\, c.$$

14. $x = y^2$ e $x = 2 - y^2$; ao redor do eixo dos y




$$v = 2\pi \int_0^1 \left( \left(2 - y^2\right) - \left(y^2\right)^2 \right) dy$$

$$= 2\pi \int_0^1 \left(4 - 4y^2 + y^4 - y^4\right) dy$$

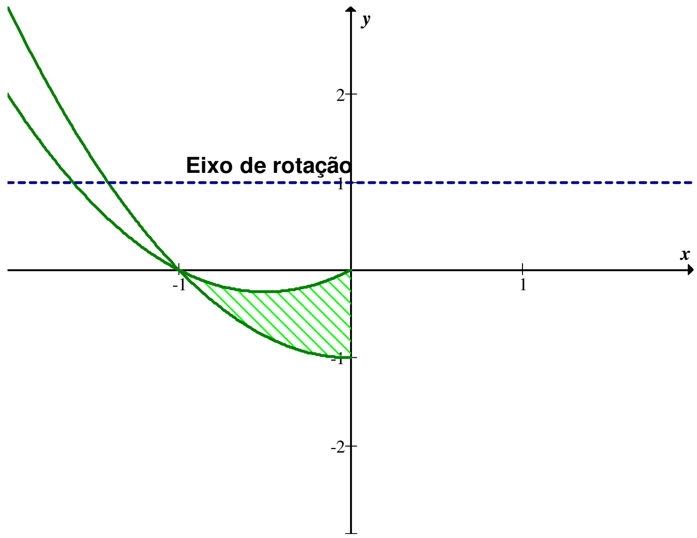
$$= 2\pi \int_0^1 \left(4 - 4y^2\right) dy$$

$$= 2\pi \left( 4y - \frac{4}{3} y^3 \right) \Bigg|_0^1$$

$$= 2\pi \left( 4 - \frac{4}{3} \right)$$

$$= \frac{16}{3} \pi \, u.\, v.$$

15. $y = x + x^2$, $y = x^2 - 1$ e $x = 0$; ao redor do eixo $y = 1$




$$V = +\pi\int_{-1}^{0}\left[\left(x^2-1-1\right)^2-\left(x^2+x-1\right)^2\right]dx$$

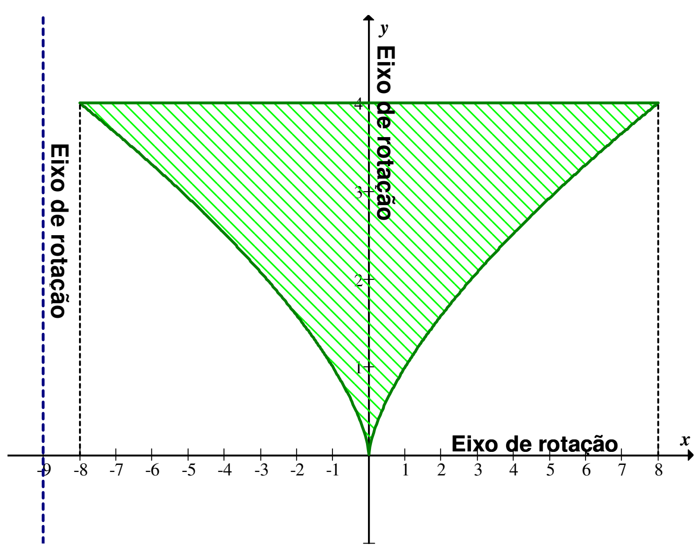
$$= +\pi\int_{-1}^{0}\left(-2x^3-3x^2+2x+3\right)dx$$

$$= +\pi\left[-2\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}+3x\right]\Bigg|_{-1}^{0}$$

$$= -\pi\left(-\frac{1}{2}+1+1-3\right)$$

$$= \frac{3\pi}{2}u.v.$$

16. $y = x^{2/3}$ e $y = 4$; ao redor dos eixos $x = -9$, $y = 0$ e $x = 0$



Eixo $x = -9$

$$V = \pi \int_0^4 \left[ \left( y^{3/2} + 9 \right)^2 - \left( -y^{3/2} + 9 \right)^2 \right] dy$$

$$= \pi \int_0^4 \left[ \left( y^{3/2} \right)^2 + 18 y^{3/2} + 81 - 81 + 18 y^{3/2} - y^3 \right] dy$$

$$= \pi \int_0^4 36 y^{3/2} dy = \pi \, 36 y^{5/2} . \frac{2}{5} \Bigg|_0^4 = \pi . \frac{72}{5} . 32 = \frac{2304}{5} \pi \, u. v$$

Eixo $x = 0$

$$V = \pi \int_0^4 \left( y^{3/2} \right)^2 \, dy$$

$$= \pi \int_0^4 y^3 dy = \pi \frac{y^4}{4} \Bigg|_0^4 = \pi . 64 = 64 \, \pi \, u. v.$$

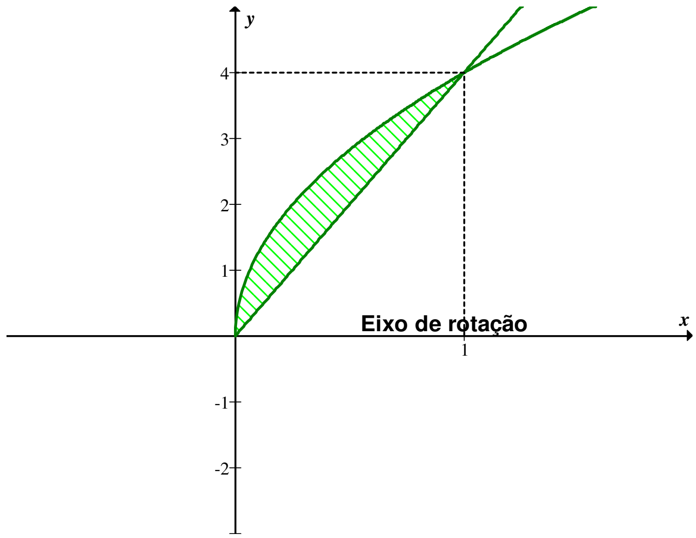
Eixo $y = 0$

$$V = 2\pi \int_0^8 \left( 4^2 - x^{4/3} \right) dy$$

$$= 2\pi \left( 16x - \frac{x^{7/3}}{7/3} \right)_0^8$$

$$= 2\pi \left( 16 . 8 - \frac{3}{7} . 128 \right) = \frac{1024 \, \pi}{7} \, u. v.$$

17. Encontra o volume do sólido gerado pela rotação, em torno do eixo dos *x*, da região limitada por $y^2 = 16x$ e $y = 4x$.



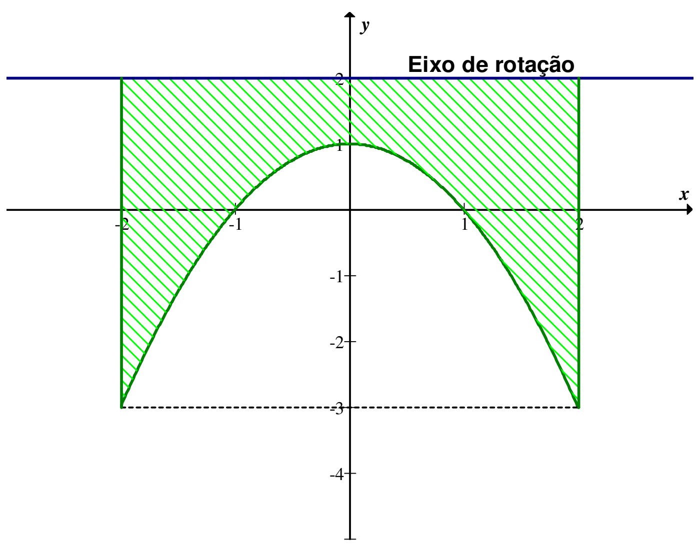

$$V = \pi \int_0^1 \left(16x - 16x^2\right) dx$$

$$= \pi \left( 16\frac{x^2}{2} - 16\frac{x^3}{3} \right) \Bigg|_0^1$$

$$= \pi \left( 8 - \frac{16}{3} \right)$$

$$= \frac{8\pi}{3} \, u.\, v.$$

18. Calcular o volume do sólido gerado pela rotação, em torno da reta $y = 2$, da região limitada por $y = 1 - x^2$, $x = -2$, $x = 2$ e $y = 2$.



$$V = \pi \int_{-2}^{2} \left(1 - x^2 - 2\right)^2 \, dx$$

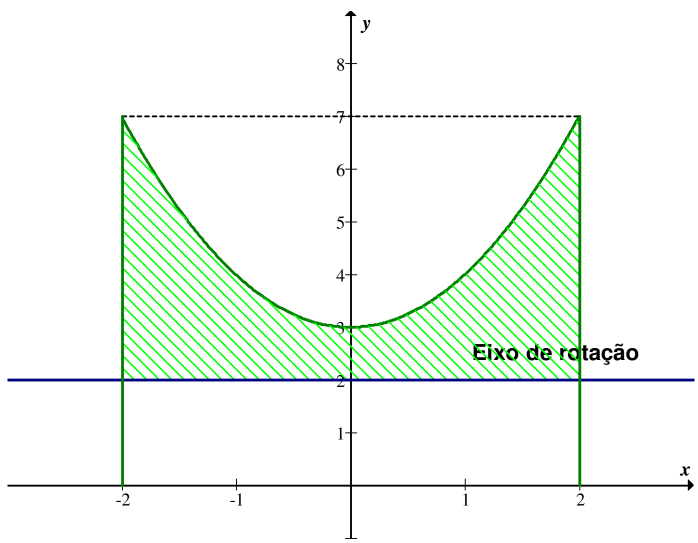
$$= \pi \int_{-2}^{2} \left(-1 - x^2\right)^2 \, dx = \pi \int_{-2}^{2} \left(1 + 2x^2 + x^4\right) dx$$

$$= \pi \left( x + 2\,\frac{x^3}{3} + \frac{x^5}{5} \right)\Bigg|_{-2}^{2}$$

$$= 2\pi \left( 2 + \frac{2}{3}\,.\,8 + \frac{32}{5} \right)$$

$$= \frac{412\pi}{15}\; u.\,v.$$

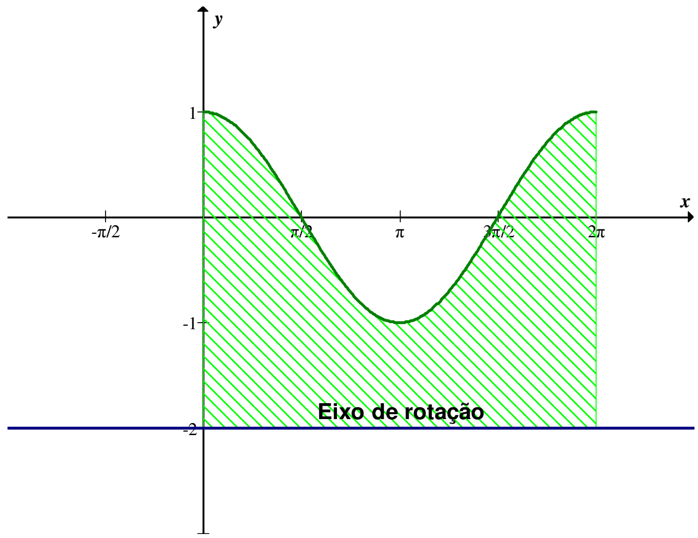
19. Calcular o volume do sólido gerado pela rotação, em torno da reta $y = 2$, da região limitada por $y = 3 + x^2$, $x = -2$, $x = 2$ e $y = 2$.




$$V = 2\pi \int_{0}^{2} \left(3 + x^2 - 2\right)^2 \, dx$$

$$= 2\pi \int_{0}^{2} \left(1 + x^2\right)^2 \, dx = \frac{412\pi}{15}\; u.\,v.$$

20. Determinar o volume do sólido gerado pela rotação, em torno da reta $y = -2$, da região limitada por $y = \cos x$, $y = -2$, $x = 0$ e $x = 2\pi$.




$$V = \pi \int_0^{2\pi} (\cos x + 2)^2 \, dx$$

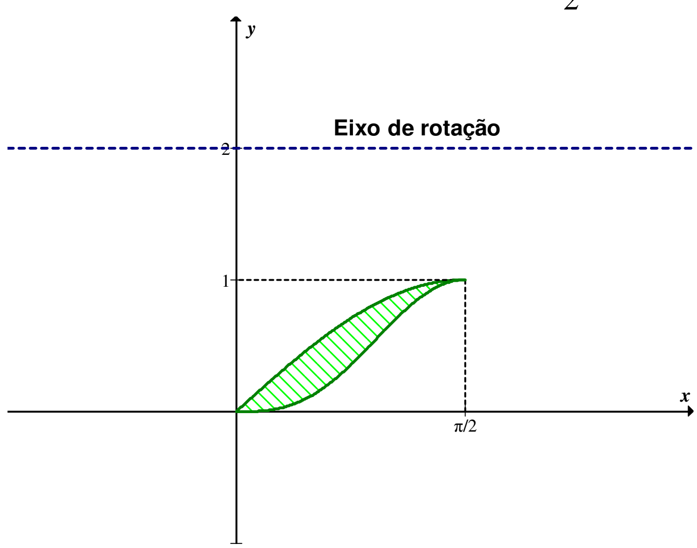
$$= \pi \int_0^{2\pi} \left(\cos^2 x + 2\cos x + 4\right) dx$$

$$= \pi \left( \frac{1}{2}x + \frac{1}{4} sen\ 2x + 2\ sen\ x + 4x \right) \Bigg|_0^{2\pi}$$

$$= \pi \left( \frac{1}{2} \,.\, 2\pi + 4 \,.\, 2\pi \right)$$

$$= \pi(\pi + 8\pi) = 9\pi^2 \ u.v.$$

21. Determinar o volume do sólido gerado pela rotação, em torno da reta $y = 2$, da região entre os gráficos de $y = sen\ x$, $y = sen^3\ x$ de $x = 0$ ate $x = \dfrac{\pi}{2}$.



$$V = \pi \int_0^{\pi/2} \left[ \left( sen^3\, x - 2 \right)^2 - \left( sen\, x - 2 \right)^2 \right] dx$$

$$= \pi \int_0^{\pi/2} \left( sen^6\, x - 4 sen^3\, x - sen^2\, x + 4 senx \right) dx$$

$$= \frac{4}{3}\pi - \frac{3}{32}\pi^2 \; u.\, v.$$

Nos exercícios de 22 a 27, calcular a área da superfície gerada pela rotação do arco de curva dado, em torno do eixo indicado.

22. $y = 2x^3$, $0 \leq x \leq 2$, eixo dos $x$

$$A = 2\pi \int_0^2 2x^3 \sqrt{1 + 36x^4}\, dx$$

$$= 4\pi \,.\, \frac{1}{144} \left( 1 + 36x^4 \right)^{3/2} .\, \frac{2}{3} \Bigg|_0^2$$

$$= \frac{\pi}{54} \left( 577\sqrt{577} - 1 \right) u.\, a.$$

23. $x = \sqrt{y}$, $1 \leq y \leq 4$, eixo dos $y$

$$A = 2\pi \int_1^4 \sqrt{y} \sqrt{1 + \frac{1}{4y}}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y} \sqrt{\frac{4y + 1}{4y}}\, dy$$

$$= 2\pi \int_1^4 \sqrt{y}\, \frac{\left( 1 + 4y \right)^{1/2}}{2\sqrt{y}}\, dy$$

$$= \pi \int_1^4 \left( 1 + 4y \right)^{1/2} dy$$

$$= \frac{\pi}{4} \frac{\left( 1 + 4y \right)^{3/2}}{3/2} \Bigg|_1^4$$

$$= \frac{\pi}{4} \,.\, \frac{2}{3} \left( 17^{3/2} - 5^{3/2} \right)$$

$$= \frac{\pi}{6} \left( 17\sqrt{17} - 5\sqrt{5} \right) u.\, a.$$

24. $y = x^2$, $-2 \le x \le 2$, eixo dos $x$

$$A = 2\pi \int_{2}^{2} x^2 \sqrt{1+4x^2}\,dx$$

$$\int x^2 \sqrt{1+4x^2}\,dx$$

$$u^2 = 4x^2$$
$$u = 2x$$
$$x = \frac{u}{2} \quad \therefore \quad dx = \frac{du}{2}$$

$$\int \frac{u^2}{4} \sqrt{1+u^2} \,.\, \frac{du}{2}$$

$$u = tg\ \theta$$
$$du = \sec^2\theta\ d\theta$$
$$\sqrt{1+u^2} = \sec\theta$$

$$I = \frac{1}{8}\int tg^2\theta \sec\theta . \sec^2\theta\ d\theta$$
$$= \frac{1}{8}\int \sec^3\theta\ tg^2\theta\ d\theta$$
$$= \frac{1}{8}\int \sec^3\theta\left(\sec^2\theta - 1\right) d\theta$$
$$= \frac{1}{8}\int \left(\sec^5\theta - \sec^3\theta\right) d\theta$$

$$= \frac{1}{32}\sec^3\theta\ tg\ \theta - \frac{1}{32}\left(\frac{1}{2}\sec\theta\ tg\ \theta + \frac{1}{2}\ln\left|\sec\theta + tg\ \theta\right|\right) + C$$
$$= \frac{1}{32}\sqrt{1+u^2}^{\,3} . u - \frac{1}{64}\sqrt{1+u^2} . u - \frac{1}{64}\ln\left|\sqrt{1+u^2} + u\right| + C$$
$$= \frac{1}{32}\sqrt{1+4x^2}^{\,3} . 2x - \frac{1}{64}\sqrt{1+4x^2} . 2x - \frac{1}{64}\ln\left|\sqrt{1+4x^2} + 2x\right| + C$$

$$A = 2\pi\left(\frac{1}{32}\sqrt{1+4x^2}^{\,3}\ .\ 2x - \frac{1}{64}\sqrt{1+4x^2}\,.\ 2x - \frac{1}{64}\ln\left|\sqrt{1+4x^2}+2x\right|\right)\Bigg|_{-2}^{2}$$

$$= \pi\left(\frac{17\sqrt{17}}{2} - \frac{17}{4} + \frac{1}{32}\ln\left|\frac{\sqrt{17}-4}{\sqrt{17}+4}\right|\right)$$

$$\cong 53.226$$

25. $y = \frac{1}{2}x$, $0 \le x \le 4$, eixo dos $x$

$$A = 2\pi\int_0^4 \frac{1}{2}x\sqrt{1+\frac{1}{4}}dx$$

$$= 2\pi\ .\ \frac{1}{2}\ .\ \frac{\sqrt{5}}{2}\int_0^4 x\ dx$$

$$= \frac{\sqrt{5}\ \pi}{2}\cdot\frac{x^2}{2}\Bigg|_0^4$$

$$= \frac{\sqrt{5}\ \pi}{2}\cdot\frac{16}{2}$$

$$= 4\sqrt{5}\ \pi\ u.\ a.$$

26. $y = \sqrt{4-x^2}$, $0 \le x \le 1$, eixo dos $x$

$$A = 2\pi\int_0^1 \sqrt{4-x^2}\sqrt{1+\frac{x^2}{4-x^2}}dx$$

$$= 2\pi\int_0^1 \sqrt{4-x^2}\sqrt{\frac{4-x^2+x^2}{4-x^2}}dx$$

$$= 2\pi\int_0^1 2\ dx$$

$$= 4\pi\ x\Big|_0^1$$

$$= 4\ \pi\ u.\ a.$$

27. $y = \sqrt{16-x^2}$, $-3 \le x \le 3$, eixo dos $x$

$$A = 2\pi\int_{-3}^{3}\sqrt{16-x^2}\ .\ \sqrt{1+\frac{x^2}{16-x^2}}dx$$

$$= 2\pi\int_{-3}^{3}\sqrt{16-x^2}\ .\ \sqrt{\frac{16-x^2+x^2}{16-x^2}}dx$$

$$= 2\pi\int_{-3}^{3}\sqrt{16-x^2}\ .\ \frac{4}{\sqrt{16-x^2}}\ dx$$

$$= 2\pi\ .\ 4x\Big|_{-3}^{3}$$

$$= 48\pi\ u.\ a.$$

28. Calcular a área da superfície obtida pela revolução da parábola $y^2 = 8x$, $1 \le x \le 12$, ao redor do eixo dos $x$.

$$A = 2\pi\int_{1}^{12}\sqrt{8x}\ .\ \sqrt{1+\frac{16}{8x}}dx$$

$$= 2\pi\int_{1}^{12}\sqrt{8x}\ .\ \sqrt{\frac{8x+16}{8x}}dx$$

$$= 2\pi\int_{1}^{12}(8x+16.)^{1/2}\ dx$$

$$= 2\pi\ .\ \frac{1}{8}\ .\ \frac{(8x+16.)^{3/2}}{3/2}\Bigg|_{1}^{12}$$

$$= 2\pi\ .\ \frac{1}{8}\ .\ \frac{2}{3}\left(112^{3/2}-24^{3/2}\right)$$

$$= \frac{8\pi}{3}\left[28\sqrt{7}-3\sqrt{6}\right]$$

29. Calcular a área da superfície do cone gerado pela revolução do segmento de reta $y = 4x$, $0 \le x \le 2$:

a) ao redor do eixo dos $x$

$$A = 2\pi\int_{0}^{2}4x\ .\ \sqrt{1+16}dx$$

$$= 2\pi\sqrt{17}\ .\ 4\frac{x^2}{2}\Bigg|_{0}^{2}$$

$$= 16\sqrt{17}\ \pi\ u.\ a.$$

b) ao redor do eixo dos $y$

$$A = 2\pi \int_0^8 \frac{y}{4} \cdot \sqrt{1 + \frac{1}{16}}\, dy$$

$$= 2\pi \cdot \frac{1}{4} \cdot \sqrt{\frac{17}{16}} \cdot \left.\frac{y^2}{2}\right|_0^8$$

$$= 4\sqrt{17}\, \pi\, u.\, a.$$