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Non-Abelian Sugawara construction and the q-deformed $N = 2$ superconformal algebra

E Batista, J F Gomes and I J Lautenschleguer
Instituto de Física Teórica-UNESP, Rua Pamplona 145, 01405-900 São Paulo, SP, Brazil

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Abstract. The construction of a q-deformed $N = 2$ superconformal algebra is proposed in terms of level-1 currents of the $\mathcal{U}_q(\widehat{su}(2))$ quantum affine Lie algebra and a single real Fermi field. In particular, it suggests the expression for the q-deformed energy–momentum tensor in the Sugawara form. Its constituents generate two isomorphic quadratic algebraic structures. The generalization to $\mathcal{U}_q(\widehat{su}(N+1))$ is also proposed.

1. Introduction

It has become quite well known that conformal invariance uncovers a common structure among many important field theoretical models in two dimensions. The physics content of such models may be extracted by use of the representation theory of the underlying conformal algebra. String theory is among the many important examples where the energy–momentum tensor is written in the Sugawara form, i.e. it is bilinear in the conserved currents.

Quantum groups has also played an important role in the sense that a class of perturbed systems may be cast within a deformed algebraic structure providing, by representation theory, the physics content of the model. This algebraic structure is characterized by a deformation parameter q. In particular, a q-deformed version of the Veneziano model was proposed in [4] and [6] by replacing the ordinary oscillators by q-deformed ones in the operator formalism of Fubini and Veneziano. The deformed model (when $q \neq 1$) was shown to lead to nonlinear Regge trajectories. It is hoped that a conformal structure arises in terms of a q-deformed Kac–Moody algebra such that the usual Sugawara construction is smoothly recovered in the limit $q \to 1$. Also along these lines, a q-deformed proposal for the Nambu action was discussed by de Vega and Sanchez in [7]. Their action shows a non-local character and the canonical energy–momentum tensor was proposed bilinearly in terms of q-oscillators.

In this paper we study how a q-conformal structure may be constructed in terms of the oscillators proposed by Frenkel and Jing [10]. They have constructed vertex operators for simply laced Lie algebras satisfying a level-1 q-deformed Kac–Moody algebra under the operator product expansion (OPE).

An interesting feature of a q-deformed field theory is that the OPE turns out to be less divergent than the $q = 1$ theory. This fact shows up since poles are smeared out in a symmetric manner in terms of the deformation parameter q, indicating a non-local structure. The short-distance behaviour of the operators require a consistent definition of normal ordering. In order to produce the correct analytic structure in terms of poles we shall consider the deformation parameter restricted to a pure phase.

In section 2 we discuss and review the Abelian q-Sugawara construction and the $N = 1$ q-superconformal algebra obtained from a level-1 q-deformed $H_q(\infty)$ infinite Heisenberg algebra and a single real Fermi field. In section 3 we discuss the vertex operator construction of Frenkel and Jing for $\mathcal{U}_q(\widehat{su}(2))$ (in the appendix we derive many useful identities using the q-Taylor expansion).

In section 4, guided by the basic principle of decomposing the most divergent poles into a product of simple poles shifted in a symmetric manner in terms of the deformation parameter q, together with the closure of algebra, we were led to define a family of super charge generators $G^{\pm}_{\alpha}$ in terms of the vertex operators of $\mathcal{U}_q(\widehat{su}(2))$ and a single real Fermi field. We show that this construction leads to a q-deformed $N = 2$ superconformal algebra. In particular, the OPE of the two super charge generators define the energy–momentum tensor which is shown to decompose into commuting bosonic and fermionic couterparts, as preditcted by Goddard and Schwimmer [13]. The fermionic counterpart of the energy–momentum tensor defines a q-deformed Virasoro algebra as in [15]. The bosonic energy–momentum tensor, however, is no longer bilinear in the q-oscillators and does not yield a closed algebra, but it can be written as a sum of two components, each of which generates closed algebras of quadratic type. Similar structures have recently been constructed by Frenkel and Reshetikhin [9] using a Wakimoto realization for the $\mathcal{U}_q(\widehat{sl}(2))$ currents. Finally, we generalize, in section 5, the results for other simply laced q-deformed affine Lie algebras.

2. The Abelian Sugawara construction and the *q*-deformed *N* = 1 superconformal algebra

An Abelian q-deformed Kac–Moody algebra can be constructed from the usual undeformed level-1 $U(1)$ current algebra:

$$H(z)H(w) = \frac{1}{(z-w)^2} + \text{regular terms} \tag{1}$$

by replacing the double pole into a product of two simple poles symmetrically displaced in terms of a deformation parameter q, i.e.

$$H(z)H(w) = \frac{1}{(z-qw)(z-wq^{-1})} + \text{regular terms}\,. \tag{2}$$

The Abelian field $H(z)$ can be Laurent expanded in terms of a q-deformed infinite Heisenberg algebra $H_q(\infty)$

$$H(z) = \sum_{n\in\mathbb{Z}} a_n z^{-n-1} \tag{3}$$

where from (2) we find

$$[a_n, a_m] = [n]\,\delta_{m+n,0} \tag{4}$$

where $[n] = (q^n - q^{-n})/(q - q^{-1})$. Because the non-local character of the pole structure, to obtain a consistent definition of normal ordering, we shall be assuming the deformation parameter to be a pure phase, i.e. $q = \mathrm{e}^{\mathrm{i}\epsilon}$, $\epsilon \in \mathbb{R}$. The Sugawara construction implies that the energy—momentum tensor can be written as

$$T(z) = \tfrac{1}{2} : H(z)H(z) : \tag{5}$$

where the colons (:) denote normal ordering in the sense that the positive oscillator modes are moved to the right of those with negative modes. A classical version of (5) was proposed in [5], while normal ordering was later introduced in [14] with the effect of generating central terms in the q-Virasoro algebra. From equation (2) the expression

$$T(z)H(w) = \frac{1}{w(q-q^{-1})}\left(\frac{H(wq)}{z-wq} - \frac{H(wq^{-1})}{z-wq^{-1}}\right) + \text{regular terms} \qquad |z| > |w| \tag{6}$$

can be evaluated straightforwardly.

The shift in the arguments of $H(w)$ on the right-hand side requires an extra index for the closure of the algebra of the energy—momentum tensor (5). We define

$$T^{\alpha}(z) = \tfrac{1}{2} : H(zq^{\frac{1}{2}\alpha})H(zq^{-\frac{1}{2}\alpha}) : \qquad \alpha \in \mathbb{Z}\,. \tag{7}$$

It then follows from (6) that these operators satisfy a deformation of the Virasoro algebra with unit central charge:

$$\begin{aligned}
T^{\alpha}(z)T^{\beta}(w) &= \frac{1}{wq^{\frac{1}{2}(\beta+\alpha)}(q-q^{-1})}\left(\frac{T^{\beta-\alpha+1}(wq^{\frac{1}{2}(-\alpha+1)})}{z-wq^{\frac{1}{2}(\beta-\alpha)+1}} - \frac{T^{\beta-\alpha-1}(wq^{\frac{1}{2}(-\alpha-1)})}{z-wq^{\frac{1}{2}(\beta-\alpha)-1}}\right)\\
&\quad + \frac{1}{wq^{\frac{1}{2}(-\beta+\alpha)}(q-q^{-1})}\left(\frac{T^{-\beta-\alpha+1}(wq^{\frac{1}{2}(-\alpha+1)})}{z-wq^{\frac{1}{2}(-\beta-\alpha)+1}} - \frac{T^{-\beta-\alpha-1}(wq^{\frac{1}{2}(-\alpha-1)})}{z-wq^{\frac{1}{2}(-\beta-\alpha)-1}}\right)\\
&\quad + \frac{1}{wq^{\frac{1}{2}(\beta-\alpha)}(q-q^{-1})}\left(\frac{T^{\beta+\alpha+1}(wq^{\frac{1}{2}(\alpha+1)})}{z-wq^{\frac{1}{2}(\beta+\alpha)+1}} - \frac{T^{\beta+\alpha-1}(wq^{\frac{1}{2}(\alpha-1)})}{z-wq^{\frac{1}{2}(\beta+\alpha)-1}}\right)\\
&\quad + \frac{1}{wq^{\frac{1}{2}(-\beta-\alpha)}(q-q^{-1})}\left(\frac{T^{-\beta+\alpha+1}(wq^{\frac{1}{2}(\alpha+1)})}{z-wq^{\frac{1}{2}(-\beta+\alpha)+1}} - \frac{T^{-\beta+\alpha-1}(wq^{\frac{1}{2}(\alpha-1)})}{z-wq^{\frac{1}{2}(-\beta+\alpha)-1}}\right)\\
&\quad + \frac{1}{4(z-wq^{\frac{1}{2}(\beta-\alpha)+1})(z-wq^{\frac{1}{2}(\beta-\alpha)-1})(z-wq^{\frac{1}{2}(-\beta+\alpha)+1})(z-wq^{\frac{1}{2}(-\beta+\alpha)-1})}\\
&\quad + \frac{1}{4(z-wq^{\frac{1}{2}(-\beta-\alpha)+1})(z-wq^{\frac{1}{2}(-\beta-\alpha)-1})(z-wq^{\frac{1}{2}(\beta+\alpha)+1})(z-wq^{\frac{1}{2}(\beta+\alpha)-1})}\\
&\qquad |z| > |w|\,.
\end{aligned} \tag{8}$$

A construction of a supercharge generator was proposed in [5] by introducing a real Fermi field $\psi(z)$

$$\psi(z)\psi(w) =: \psi(z)\psi(w) : + \frac{1}{(z-w)} \qquad |z| > |w|\,. \tag{9}$$

Since the triple pole produced by the product of two supercharge generators in the $q = 1$ case should be replaced by the product of three simple poles

$$\frac{1}{(z-w)^3} \to \frac{1}{(z-wq^{-1})(z-w)(z-wq)}$$

we realize this by the product of the bosonic current (2) and the fermion (9), i.e. $G(z) = H(z)\psi(z)$. The closure condition of the algebra requires a family of supercharge generators to be defined as

$$G^{\alpha}(z) = H(zq^{\frac{1}{2}\alpha})\psi(zq^{-\frac{1}{2}\alpha})\,. \tag{10}$$

Their algebra is given by

$$G^{\alpha}(z)G^{\beta}(w) = \frac{2q^{\frac{1}{2}\alpha}T^{\beta+\alpha}(wq^{\frac{1}{2}\alpha})}{z-wq^{\frac{1}{2}(-\beta+\alpha)}} + \frac{[\beta-\alpha+1]L^{\beta-\alpha+1}(wq^{\frac{1}{2}(-\alpha+1)})}{q^{\frac{1}{2}(\beta-1)+\alpha}(z-wq^{\frac{1}{2}(\beta-\alpha)+1})}$$
$$-\frac{[\beta-\alpha-1]L^{\beta-\alpha-1}(wq^{\frac{1}{2}(-\alpha-1)})}{q^{\frac{1}{2}(\beta+1)+\alpha}(z-wq^{\frac{1}{2}(\beta-\alpha)-1})}$$
$$+\frac{q^{-\frac{1}{2}\alpha}}{(z-wq^{\frac{1}{2}(-\beta+\alpha)})(z-wq^{\frac{1}{2}(\beta-\alpha)+1})(z-wq^{\frac{1}{2}(\beta-\alpha)+1})} \qquad |z|>|w|\,. \tag{11}$$

The operators $L^{\alpha}(z)$ are the fermionic part of the energy–momentum tensor

$$L^{\alpha}(z) = \frac{1}{[\alpha](q-q^{-1})} :\psi(zq^{\frac{1}{2}\alpha})\psi(zq^{-\frac{1}{2}\alpha}): \tag{12}$$

and satisfy a q-deformed Virasoro algebra (see [15])

$$L^{\alpha}(z)L^{\beta}(w) = \frac{1}{[\alpha][\beta]w(q-q^{-1})}\left(\frac{[\alpha+\beta]L^{\alpha+\beta}(wq^{\frac{1}{2}\alpha})}{q^{\frac{1}{2}(\beta-\alpha)}(z-wq^{\frac{1}{2}(\beta+\alpha)})}\right.$$
$$+\frac{[-\alpha-\beta]L^{-\alpha-\beta}(wq^{-\frac{1}{2}\alpha})}{q^{\frac{1}{2}(-\beta+\alpha)}(z-wq^{\frac{1}{2}(-\beta-\alpha)})} - \frac{[\alpha-\beta]L^{\alpha-\beta}(wq^{\frac{1}{2}\alpha})}{q^{\frac{1}{2}(-\beta-\alpha)}(z-wq^{\frac{1}{2}(-\beta+\alpha)})}$$
$$-\frac{[-\alpha+\beta]L^{-\alpha+\beta}(wq^{-\frac{1}{2}-\alpha})}{q^{\frac{1}{2}(\beta+\alpha)}(z-wq^{\frac{1}{2}(\beta-\alpha)})} + \frac{1}{(z-wq^{\frac{1}{2}(-\beta-\alpha)})(z-wq^{\frac{1}{2}(\beta+\alpha)})}$$
$$\left.-\frac{1}{(z-wq^{\frac{1}{2}(-\beta+\alpha)})(z-wq^{\frac{1}{2}(\beta-\alpha)})}\right) \qquad |z|>|w|\,. \tag{13}$$

This algebra is not isomorphic to the bosonic one given in (8), but in the limit $q\to 1$, these generators become the usual fermionic energy–momentum tensor (see [16], for instance)

$$L(z) = \tfrac{1}{2} :\partial_z\psi(z)\psi(z):$$

satisfying the usual Virasoro algebra with $c=\frac{1}{2}$.

The closure of the $N=1$ super conformal algebra is achieved with the following OPE relations:

$$T^{\alpha}(z)G^{\beta}(w) = \frac{1}{2wq^{\frac{1}{2}\beta}(q-q^{-1})}\left(\frac{q^{-\frac{1}{2}\alpha}G^{\beta-\alpha+1}(wq^{\frac{1}{2}(-\alpha+1)})}{z-wq^{\frac{1}{2}(\beta-\alpha)+1}}\right.$$
$$-\frac{q^{-\frac{1}{2}\alpha}G^{\beta-\alpha-1}(wq^{\frac{1}{2}(-\alpha-1)})}{z-wq^{\frac{1}{2}(\beta-\alpha)-1}} + \frac{q^{\frac{1}{2}\alpha}G^{\beta+\alpha+1}(wq^{\frac{1}{2}(\alpha+1)})}{z-wq^{\frac{1}{2}(\beta+\alpha)+1}}$$
$$\left.-\frac{q^{\frac{1}{2}\alpha}G^{\beta+\alpha-1}(wq^{\frac{1}{2}(\alpha-1)})}{z-wq^{\frac{1}{2}(\beta+\alpha)-1}}\right) \qquad |z|>|w| \tag{14}$$

$$L^{\alpha}(z)G^{\beta}(w) = \frac{1}{[\alpha]wq^{-\frac{1}{2}\beta}(q-q^{-1})}\left(\frac{G^{\beta-\alpha}(wq^{\frac{1}{2}\alpha})}{z-wq^{\frac{1}{2}(-\beta+\alpha)}} - \frac{G^{\beta+\alpha}(wq^{-\frac{1}{2}\alpha})}{z-wq^{\frac{1}{2}(-\beta-\alpha)}}\right)$$
$$|z|>|w|\,. \tag{15}$$

In the limit $q \to 1$ we recover the usual undeformed $N = 1$ superconformal algebra, namely

$$G(z)G(w) = \frac{1}{(z-w)^3} + \frac{2T(w)}{(z-w)}$$

$$T(z)G(w) = \frac{3G(w)}{2(z-w)^2} + \frac{G'(w)}{(z-w)}$$

$$T(z)T(w) = \frac{2T(w)}{(z-w)^2} + \frac{T'(z)}{(z-w)} + \frac{3}{4(z-w)^4}$$

where $T = T_{\text{bosonic}} + L_{\text{fermionic}}$.

The deformation is therefore responsible for introducing an additional integer index into the algebraic structure. Similar structures have already been found in the context of two loop Kac–Moody algebras and with the conformal affine Toda models (see [1, 8], for instance). We should also point out that the decomposition of higher-order poles into the product of simple poles does not alter the merororphic character of the theory. In fact, a consequence of this is the splitting of the bosonic and fermionic parts of the total energy—momentum tensor (see [13]).

3. Deformed $\mathcal{U}_q(\widehat{su}(2))$ Kac–Moody algebras and vertex operators

Consider a level-1 $\mathcal{U}_q(\widehat{su}(2))$ construction in terms of vertex operators proposed by Frenkel and Jing [10] using the q-deformed oscillators

$$[\alpha_n, \alpha_m] = \frac{[2n][n]}{2n}\,\delta_{m+n,0} \tag{16}$$

together with the undeformed Heisenberg algebra

$$[Q, P] = i\,. \tag{17}$$

Define a Fubini–Veneziano field

$$\xi^{\pm}(z) = Q - iP\ln z + i\sum_{n<0}\frac{\alpha_n}{[n]}(zq^{\mp\frac{1}{2}})^{-n} + i\sum_{n>0}\frac{\alpha_n}{[n]}(zq^{\pm\frac{1}{2}})^{-n} \tag{18}$$

and then write the generators for $\mathcal{U}_q(\widehat{su}(2))$ as

$$E^{\pm}(z) =: \exp\left\{\pm\sqrt{2}\xi^{\pm}(z)\right\}: \qquad H(z) = \sum_{n\in\mathbb{Z}}\alpha_n z^{-n-1} \tag{19}$$

where the colons (:) denote normal ordering in the sense that positive oscillator modes are moved to the right of those with negative modes and P to the right of Q. The OPE for vertices like (19) is obtained using the Baker–Campbell–Haussdorff formula. In particular, we find

$$E^{+}(z)E^{-}(w) = \frac{D(z,w)}{(z-wq)(z-wq^{-1})} \qquad |z| > |w| \tag{20}$$

where

$$D(z,w) = \left(\frac{z}{w}\right)^{\sqrt{2}P}\exp\left\{-\sqrt{2}\sum_{n<0}\frac{\alpha_n}{[n]}((zq^{-\frac{1}{2}})^{-n} - (wq^{\frac{1}{2}})^{-n})\right\}$$

$$\times\exp\left\{-\sqrt{2}\sum_{n>0}\frac{\alpha_n}{[n]}((zq^{\frac{1}{2}})^{-n} - (wq^{-\frac{1}{2}})^{-n})\right\}. \tag{21}$$

The non-local structure of the poles in (20) requires an expansion of the numerator in q-Taylor series (see the appendix). In order to get a symmetric result we split the numerator in two equal terms and expand each one separately around the diferent poles. The OPE may be written as

$$E^+(z)E^-(w) \sim \frac{1}{w(q-q^{-1})}\left\{\frac{\Psi(wq^{\frac{1}{2}})}{z-wq} - \frac{\Phi(wq^{-\frac{1}{2}})}{z-wq^{-1}}\right\} \qquad |z| > |w| \tag{22}$$

where

$$\Psi(z) = q^{\sqrt{2}P}\exp\left\{\sqrt{2}(q-q^{-1})\sum_{n>0}\alpha_n z^{-n}\right\} \tag{23}$$

and

$$\Phi(z) = q^{-\sqrt{2}P}\exp\left\{-\sqrt{2}(q-q^{-1})\sum_{n<0}\alpha_n z^{-n}\right\} \tag{24}$$

the symbol $\sim$ standing for equality up to regular terms. The currents $E^{\pm}(z)$ are the generating functions of the Drinfeld generators for the q-deformed algebra $\mathcal{U}_q(\widehat{su}(2))$, $E^{\pm}(z) = \sum_{n\in\mathbb{Z}} E_n^{\pm} z^{-n-1}$ and the combination $(\Psi(z) - \Phi(z))/(\sqrt{2}z(q-q^{-1}))$ correspond to the q-deformed analogue of the undeformed Cartan sub algebra current $H(z)$ of $\widehat{su}(2)$. It should be noted from (22) that the deformation naturally splits the positive and negative oscillator modes according to positive and negative powers of q. The full $\mathcal{U}_q(\widehat{su}(2))$ is obtained from (22), together with

$$\begin{aligned}
\Psi(z)\Phi(w) &= \frac{(z-wq^3)(z-wq^{-3})}{(z-wq)(z-wq^{-1})}\Phi(w)\Psi(z) \\
\Psi(z)E^{\pm}(w) &= q^{\pm 2}\frac{(z-wq^{\mp\frac{5}{2}})}{(z-wq^{\pm\frac{3}{2}})}E^{\pm}(w)\Psi(z) \\
E^{\pm}(z)\Phi(w) &= q^{\pm 2}\frac{(z-wq^{\mp\frac{5}{2}})}{(z-wq^{\pm\frac{3}{2}})}\Phi(w)E^{\pm}(z) \\
E^{\pm}(z)E^{\pm}(w) &= \frac{(zq^{\pm 2}-w)}{(z-wq^{\pm 2})}E^{\pm}(w)E^{\pm}(z) \qquad |z| > |w| .
\end{aligned} \tag{25}$$

The regular part of the OPE in (22) is important in defining the energy–momentum tensor in Sugawara form. Here we introduce the notation ${}^{\times}_{\times}E^+(w)E^-(w){}^{\times}_{\times}$ for the remaining regular terms after taking the equal-point limit, i.e.

$${}^{\times}_{\times}E^+(w)E^-(w){}^{\times}_{\times} = \lim_{z\to w}\left\{E^+(z)E^-(w) - \frac{1}{w(q-q^{-1})}\left(\frac{\Psi(wq^{\frac{1}{2}})}{z-wq} - \frac{\Phi(wq^{-\frac{1}{2}})}{z-wq^{-1}}\right)\right\} . \tag{26}$$

Due to the non-local character of the q-Taylor expansion, it consists of an infinite number of terms which vanish when $q \to 1$. The first-order term is given by

$$\begin{aligned}
{}^{\times}_{\times}E^+(w)E^-(w){}^{\times}_{\times} = \frac{1}{2[2]w^2(q-q^{-1})^2}&\Big\{[q^{-3}(A^1(wq^2)-1) + (q-q^{-1})]\Psi(wq^{\frac{1}{2}}) \\
&+\Phi(wq^{-\frac{1}{2}})[q^3(B^1(wq^{-2})-1) - (q-q^{-1})]\Big\} + \mathcal{O}(q-q^{-1})
\end{aligned} \tag{27}$$

where the composite fields A^α and B^α are defined as

$$A^\alpha(w) = \Phi^{-1}(wq^{-\frac{1}{2}\alpha})\Psi(wq^{\frac{1}{2}\alpha}) \tag{28}$$

$$B^\alpha(w) = \Phi(wq^{-\frac{1}{2}\alpha})\Psi^{-1}(wq^{\frac{1}{2}\alpha})\,. \tag{29}$$

They are obtained by direct calculation, as explained in the appendix. We also prove that the higher power terms in $(q-q^{-1})$ appearing in the regular part of $E^+(z)E^-(w)$ only depend on either $D(wq^{2k+1}, w)$ or $D(wq^{-2k-1}, w)$, with their general structure given by

$$\begin{aligned} D(wq^{2k+1}, w) &=: A^1(wq^2)A^1(wq^4)\ldots A^1(wq^{2k}) : \Psi(wq^{\frac{1}{2}}) \\ D(wq^{-2k-1}, w) &= \Phi(wq^{-\frac{1}{2}}) : B^1(wq^{-2})B^1(wq^{-4})\ldots B^1(wq^{-2k}) : \,. \end{aligned} \tag{30}$$

It should be noted that all non-vanishing terms constituting the regular part of $E^+(z)E^-(z)$ are functionals of Ψ and Φ and, henceforth, nonlinear functionals of the Cartan subalgebra current $H(z)$. This fact suggests a generalization of the q-analogue of the quantum equivalence theorem which establishes, in particular, the equality of the energy—momentum tensor constructed from a level-1 simply laced current algebra $\widehat{\mathfrak{g}}$ and its Cartan subalgebra (see Goddard and Olive [12]).

4. $N = 2$ q-superconformal algebra

The construction of the $N = 2$ superconformal algebra requires two supercharges, $G^\pm(z)$. The most singular term of their OPE is proporcional to a triple pole, as we have seen in section 2 for the $N = 1$ case. As we have argued before, the triple pole is expected to be replaced by a product of simple poles symmetrically displaced in terms of the deformation parameter q. Following the same reasoning, we define the deformed supercharges as

$$G^\pm(z) = E^\pm(z)\psi(z) \tag{31}$$

where $E^\pm(z)$ denote the currents related to $\mathcal{U}_q(\widehat{su}(2))$ step operators, and $\psi(z)$ is the Fermi field defined by expression (9). In particular, for level-1 representations, we realize them as the vertex operators (19). The OPE for the two supercharges yields

$$\begin{aligned} G^+(z)G^-(w) = &\left\{\frac{1}{(z-w)(z-wq)} + \frac{q^{\frac{1}{2}}L^1(wq^{\frac{1}{2}})}{(z-wq)}\right. \\ &\left. +\frac{1}{2}\frac{(q^{-3}(A^1(wq^2)-1)+(q-q^{-1}))}{[2]w(q-q^{-1})(z-w)} + \mathcal{O}(q-q^{-1})\right\}\frac{\Psi(wq^{\frac{1}{2}})}{w(q-q^{-1})} \\ &-\frac{\Phi(wq^{-\frac{1}{2}})}{w(q-q^{-1})}\left\{\frac{1}{(z-w)(z-wq^{-1})} + \frac{q^{-\frac{1}{2}}L^{-1}(wq^{-\frac{1}{2}})}{(z-wq^{-1})}\right. \\ &\left. +\frac{1}{2}\frac{(q^3(B^1(wq^{-2})-1)-(q-q^{-1}))}{[2]w(q-q^{-1})(z-w)} + \mathcal{O}(q-q^{-1})\right\} \end{aligned} \tag{32}$$

where L^α is the fermionic energy–momentum tensor defined in (12), A^α and B^α are defined in (28), (29). Again, the shift in the arguments of currents in the right-hand side requires an additional index to label an infinite family of supercharge generators, we therefore define

$$G^\pm_\alpha(z) = E^\pm(zq^{\pm\frac{1}{2}\alpha})\psi(zq^{\mp\frac{1}{2}\alpha})\,. \tag{33}$$

The OPE relations for the supercharge generators is given by

$$
\begin{aligned}
G^+_\alpha(z)G^-_\beta(w) = {} & \frac{{}^\times_\times E^+(wq^{\frac12\beta+\alpha})E^-(wq^{-\frac12\beta}){}^\times_\times}{(zq^{-\frac12\alpha}-wq^{\frac12\beta})} \\
& + \frac{[1-\alpha-\beta]q^{\frac12(1+\beta-\alpha)}L^{1-\alpha-\beta}(wq^{\frac12(1-\alpha)})\Psi(wq^{\frac12(1-\beta)})}{zq^{\frac12\alpha}-wq^{-\frac12\beta+1}} \\
& + \frac{[1+\alpha+\beta]q^{\frac12(1+\beta+\alpha)}\Phi(wq^{\frac12(-1-\beta)})L^{1+\alpha+\beta}(wq^{\frac12(1+\alpha)})}{zq^{\frac12\alpha}-wq^{-\frac{\beta}{2}-1}} \\
& + \frac{1}{wq^{-\frac12\beta}(q-q^{-1})(zq^{-\frac12\alpha}-wq^{\frac12\beta})} \\
& \times\left\{\frac{\Psi(wq^{\frac12(1-\beta)})}{zq^{\frac12\alpha}-wq^{-\frac12\beta+1}}-\frac{\Phi(wq^{\frac12(-1-\beta)})}{zq^{\frac12\alpha}-wq^{-\frac12\beta-1}}\right\}
\end{aligned}
\tag{34}
$$

where

$$
\begin{aligned}
{}^\times_\times E^+(wq^{\frac12\beta+\alpha})E^-(wq^{-\frac12\beta}){}^\times_\times = {} & \frac{1}{2[2]w^2q^{-\beta}(q-q^{-1})^2}\Big\{\big[q^{-3}(A^1(wq^{-\frac12\beta+2})-1) \\
& +(q-q^{-1})\big]\Psi(wq^{\frac12(-\beta+1)})+\Phi(wq^{\frac12(-\beta-1)})\big[q^3(B^1(wq^{-\frac12\beta-2})-1) \\
& -(q-q^{-1})\big]\Big\}+\mathcal{O}(q-q^{-1})\,.
\end{aligned}
\tag{35}
$$

In the classical limit ($q\to1$) these generators $G^\pm_\alpha$ become the usual supercharges of $N=2$ superconformal algebra, with $G^\pm(z)=E^\pm(z)\psi(z)$ obeying the OPE relation (see [17]):

$$
G^+(z)G^-(w)=\frac{1}{(z-w)^3}+\frac{\sqrt2H(w)}{(z-w)^2}+\frac{2T(w)+\sqrt2H'(w)}{(z-w)}
$$

where $T(w)=T_{\mathrm{bosonic}}(w)+L_{\mathrm{fermionic}}(w)$ is the total energy–momentum tensor, as in $N=1$ case. Note that, in order not to generate spurious poles in the OPE of $G^\pm(z)G^\pm(w)$, instead of using a single real Fermi field $\psi(z)$ multiplying both $E^\pm$, we could have introduced a pair of independent Fermi fields, $\psi^\pm(z)=(\psi_1(z)\pm\mathrm{i}\psi_2(z))/\sqrt2$ multiplying $E^\pm$ respectively. The fermion fields $\psi_{1,2}$ do obey the OPE relations (9) independently. Analysing expressions (34) and (35), we can formulate an expression for the bosonic energy–momentum tensor taking the coefficient of simple poles in the OPE, bearing in mind the correct classical limit to be

$$
T(z)=\tfrac12:H^2(z):\,.
$$

We therefore define a family of operators labeled by two indices

$$
T^{\alpha;\beta}(z)=\frac{1}{2[2]z^2(q-q^{-1})^2}\left\{A^\alpha(zq^\beta)+B^\alpha(zq^\beta)-2\right\}
\tag{36}
$$

with A^α and B^α defined by expressions (28), (29). The algebra of the q-deformed energy–momentum tensor (36) does not close; however, its components A^α and B^α define a quadratic algebra, namely

$$
\begin{aligned}
A^\alpha(z)A^\beta(w) &= f(zq^{\frac{1}{2}\alpha}; wq^{-\frac{1}{2}\beta}) : A^\alpha(z)A^\beta(w) : \\
B^\alpha(z)B^\beta(w) &= f(zq^{\frac{1}{2}\alpha}; wq^{-\frac{1}{2}\beta}) : B^\alpha(z)B^\beta(w) : \\
A^\alpha(z)B^\beta(w) &= f^{-1}(zq^{\frac{1}{2}\alpha}; wq^{-\frac{1}{2}\beta}) : A^\alpha(z)B^\beta(w) : \\
B^\alpha(z)A^\beta(w) &= f^{-1}(zq^{\frac{1}{2}\alpha}; wq^{-\frac{1}{2}\beta}) : B^\alpha(z)A^\beta(w) : \\
A^\alpha(z)G^\pm_\beta(w) &= q^{\pm 2}\frac{(zq^{\frac{1}{2}\alpha} - wq^{\pm(\frac{1}{2}\beta-\frac{5}{2})})}{(zq^{\frac{1}{2}\alpha} - wq^{\pm(\frac{1}{2}\beta+\frac{3}{2})})} : A^\alpha(z)G^\pm_\beta(w) : \\
B^\alpha(z)G^\pm_\beta(w) &= q^{\mp 2}\frac{(zq^{\frac{1}{2}\alpha} - wq^{\pm(\frac{1}{2}\beta+\frac{3}{2})})}{(zq^{\frac{1}{2}\alpha} - wq^{\pm(\frac{1}{2}\beta-\frac{5}{2})})} : B^\alpha(z)G^\pm_\beta(w) : \\
L^\alpha(z)G^\pm_\beta(w) &= \frac{1}{[\alpha]w(q-q^{-1})}\left\{\frac{G^{\pm(\pm\beta-\alpha)}(wq^{\frac{1}{2}\alpha})}{z - wq^{\mp\frac{1}{2}\beta+\frac{1}{2}\alpha}} - \frac{G^{\pm(\pm\beta+\alpha)}(wq^{-\frac{1}{2}\alpha})}{z - wq^{\mp\frac{1}{2}\beta-\frac{1}{2}\alpha}}\right\}
\end{aligned}
\tag{37}
$$

where the analytic structure is given by

$$
f(z;w) = \frac{(z-wq)(z-wq^{-1})}{(z-wq^3)(z-wq^{-3})}
$$

and $L^\alpha(z)$ is the fermionic energy–momentum tensor as defined by expression (12).

The higher-order terms in the q-Taylor expansion of the OPE (see the appendix) lead us to a more general class of q-deformed energy–momentum tensors in Sugawara form. In fact, the multi-index family of operators is given by

$$
\begin{aligned}
T^{\alpha_1,\beta_1;\ldots;\alpha_n,\beta_n}(z) = \frac{1}{2n^2[2]z^2(q-q^{-1})^2}\Big\{&: A^{\alpha_1}(zq^{\beta_1})\cdots A^{\alpha_n}(zq^{\beta_n}) : \\
&+ : B^{\alpha_1}(zq^{\beta_1})\cdots B^{\alpha_n}(zq^{\beta_n}) : -2\Big\}
\end{aligned}
\tag{38}
$$

where the colons (:) indicate that all Φ's are placed to the left of the Ψ's. As before, the algebra of the multi index energy–momentum tensor (38) does not close; however, their constituents

$$
A^{\alpha_1,\beta_1;\ldots;\alpha_n,\beta_n}(z) =: A^{\alpha_1}(zq^{\beta_1})\cdots A^{\alpha_n}(zq^{\beta_n}) : \tag{39}
$$

and

$$
B^{\alpha_1,\beta_1;\ldots;\alpha_n,\beta_n}(z) =: B^{\alpha_1}(zq^{\beta_1})\cdots B^{\alpha_n}(zq^{\beta_n}) : \tag{40}
$$

close two isomorphic OPE algebras of the type

$$
\begin{aligned}
&A^{\alpha_1,\beta_1;\ldots;\alpha_n,\beta_n}(z)A^{\gamma_1,\delta_1;\ldots;\gamma_m,\delta_m}(w) \\
&\quad = \prod_{i=1}^{n}\prod_{j=1}^{m} f(zq^{\beta_i+\frac{1}{2}\alpha_i}; wq^{\delta_j-\frac{1}{2}\gamma_j}) : A^{\alpha_1,\beta_1;\ldots;\alpha_n,\beta_n}(z)A^{\gamma_1,\delta_1;\ldots;\gamma_m,\delta_m}(w) : .
\end{aligned}
\tag{41}
$$

Note that the OPE $G^+(z)G^-(w)$ naturally produces terms of the type (39) and (40), because $D(wq^{2k+1}, w) = A^{1,2;1,4;\ldots;1,2k}(w)\Psi(wq^{\frac{1}{2}})$ and $D(wq^{-2k-1}, w) = \Phi(wq^{-\frac{1}{2}})B^{1,-2;1,-4;\ldots;1,-2k}(w)$ given in (30). These are particular cases when $\alpha_i = 1$, $\beta_i = 2i$ in the case of A, and $\beta_i = -2i$ in the case of B, leading to a remarkable cancellation of poles in (41) in which the $2mn$ poles result in $2m$ only.

5. The Sugawara construction and $\mathcal{U}_q(\widehat{su}(N+1))$ vertex operators

The vertex operator construction for an affine simply laced Lie algebra $\mathcal{U}_q(\widehat{su}(N+1))$, requires N independent copies of q-oscillators satisfying

$$[\alpha_n^i, \alpha_m^j] = \frac{[K_{ij}n][n]}{2n}\delta_{m+n,0} \qquad i = 1, \ldots, N \tag{42}$$

where K_{ij} is the Cartan matrix of the underlying Lie algebra $su(N+1)$, and zero modes P^i and Q^i satisfy the usual commutation relations

$$[Q^i, P^j] = i\delta^{ij}\,.$$

These commutation relations become, in the limit $q \to 1$, the usual ones for the modes of the Cartan sub-algebra of the undeformed $\widehat{su}(N+1)$ affine Lie algebra in the Chevalley basis, i.e.

$$[\alpha_n^i, \alpha_m^j] = K_{ij}n\,\delta_{m+n,0}\,.$$

Let β_i, $i = 1, \ldots, N$ be the simple roots of g algebra. For each simple root we define vertex operators as indicated in [10]:

$$\begin{aligned}
E_i^{\pm}(z) &= \exp\left\{\mp\sqrt{2}\sum_{n<0}\frac{\alpha_n^i}{[n]}(zq^{\mp\frac{1}{2}})^{-n}\right\}\exp\left\{\mp\sqrt{2}\sum_{n>0}\frac{\alpha_n^i}{[n]}(zq^{\pm\frac{1}{2}})^{-n}\right\}\mathrm{e}^{\pm\mathrm{i}\sqrt{2}Q^i}z^{\pm\sqrt{2}P^i}\\
\Psi_i(z) &= q^{\sqrt{2}P^i}\exp\left\{\sqrt{2}(q-q^{-1})\sum_{n>0}\alpha_n^i z^{-n}\right\}\\
\Phi_i(z) &= q^{-\sqrt{2}P^i}\exp\left\{-\sqrt{2}(q-q^{-1})\sum_{n<0}\alpha_n^i z^{-n}\right\}
\end{aligned}\tag{43}$$

satisfying the OPE relations

$$\begin{aligned}
E_i^+(z)E_i^-(w) &\sim \frac{1}{w(q-q^{-1})}\left\{\frac{\Psi_i(wq^{\frac{1}{2}})}{z-wq^1} - \frac{\Phi_i(wq^{-\frac{1}{2}})}{z-wq^{-1}}\right\}\\
E_i^-(z)E_i^+(w) &\sim \frac{1}{w(q-q^{-1})}\left\{\frac{\Phi_i(wq^{\frac{1}{2}})}{z-wq^1} - \frac{\Psi_i(wq^{-\frac{1}{2}})}{z-wq^{-1}}\right\}\\
E_i^+(z)E_j^-(w) &= \frac{(z-w)}{z} : E_i^+(z)E_j^-(w) : \qquad i = j \pm 1\\
\Psi_i(z)E_i^{\pm}(w) &= q^{\pm 2}\frac{(z-wq^{\mp\frac{5}{2}})}{(z-wq^{\pm\frac{3}{2}})}E_i^{\pm}(w)\Psi_i(z)\\
\Psi_i(z)E_j^{\pm}(w) &= \frac{(z-wq^{\pm\frac{1}{2}})}{(z-wq^{\mp\frac{3}{2}})}E_j^{\pm}(w)\Psi_i(z) \qquad i = j \pm 1\\
E_i^{\pm}(z)\Phi_i(w) &= q^{\pm 2}\frac{(z-wq^{\mp\frac{5}{2}})}{(z-wq^{\pm\frac{3}{2}})}\Phi_i(w)E_i^{\pm}(z)\\
E_i^{\pm}(z)\Phi_j(w) &= \frac{(z-wq^{\pm\frac{1}{2}})}{(z-wq^{\mp\frac{3}{2}})}\Phi_j(w)E_i^{\pm}(z) \qquad i = j \pm 1\\
\Psi_i(z)\Phi_i(w) &= \frac{(z-wq^3)(z-wq^{-3})}{(z-wq)(z-wq^{-1})}\Phi_i(w)\Psi_i(z)\\
\Psi_i(z)\Phi_j(w) &= \frac{(z-w)^2}{(z-wq^2)(z-wq^{-2})}\Phi_j(w)\Psi_i(z) \qquad i = j \pm 1\,.
\end{aligned}\tag{44}$$

For a positive root $\beta = \beta_i + \beta_{i+1} + \cdots + \beta_{i+s},\ 0 \leqslant s \leqslant N - i$, we consider the normal ordered products [10]

$$\begin{aligned} E^+_\beta(z) &=: E^+_i(zq^i)E^+_{i+1}(zq^{i+1}) \cdots E^+_{i+s}(zq^{i+s}) : \\ E^-_{-\beta}(z) &=: E^-_{i+s}(zq^{i+s})E^-_{i+s-1}(zq^{i+s-1}) \cdots E^-_i(zq^i) : \\ \Psi_\beta(z) &= \Psi_i(zq^i)\Psi_{i+1}(zq^{i+1}) \cdots \Psi_{i+s}(zq^{i+s}) \\ \Phi_\beta(z) &= \Phi_i(zq^i)\Phi_{i+1}(zq^{i+1}) \cdots \Phi_{i+s}(zq^{i+s}) . \end{aligned} \tag{45}$$

The OPE relations for these operators are obtained using rules (44). In particular the OPE of $E^+(z)E^-(w)$ we find

$$\begin{aligned} \zeta(q,z)E^+_\beta(z)E^-_{-\beta}(w) &= \frac{1}{w(q-q^{-1})}\left\{\frac{\Psi_\beta(wq^{\frac{1}{2}})}{z-wq^1} - \frac{\Phi_\beta(wq^{-\frac{1}{2}})}{z-wq^{-1}}\right\} \\ &+\frac{1}{2[2]w^2q^2(q-q^{-1})^2}\Big\{[q^{-3}(A^1_\beta(wq^2)-1)\ +(q-q^{-1})]\Psi_\beta(wq^{\frac{1}{2}}) \\ &+\Phi_\beta(wq^{-\frac{1}{2}})[q^3(B^1_\beta(wq^{-2})-1)-(q-q^{-1})]\Big\} + \mathcal{O}(q-q^{-1}) \end{aligned} \tag{46}$$

where

$$\zeta(q,z) = z^{2(s-1)}q^{i(2s-1)+s(s+1)}$$

is a normalizer term introduced only to cancel factors apearing in the OPEs of vertices associated with neighbouring roots. The fields A^k_β and B^k_β are defined as

$$\begin{aligned} A^k_\beta(z) &=: A^k_i(zq^i)A^k_{i+1}(zq^{i+1}) \cdots A^k_{i+s}(zq^{i+s}) : \\ B^k_\beta(z) &=: B^k_i(zq^i)B^k_{i+1}(zq^{i+1}) \cdots B^k_{i+s}(zq^{i+s}) : . \end{aligned} \tag{47}$$

The energy–momentum tensor associated to $\mathcal{U}_q(\widehat{su}(N+1))$ is therefore proposed to be

$$\begin{aligned} T^{k,l}(z) &= \frac{1}{2(2+N)[2]z^2(q-q^{-1})^2}\Big\{\sum_{i=1}^{N}[A^k_i(zq^l) + B^k_i(zq^l) - 2] \\ &+2\sum_{\beta>0}[A^k_\beta(zq^l) + B^k_\beta(zq^l) - 2]\Big\} \end{aligned} \tag{48}$$

providing a closed quadratic algebra for its constituents A^k_β and B^k_β which can be obtained using the primitive OPE relations in (44). In the limit $q \to 1$, we recover the usual Sugawara construction, namely [12]

$$T(z) = \frac{1}{2(1+h)}\left\{\sum_{i=1}^{N-1} {}^\times_\times H^2_i(z){}^\times_\times + \sum_{\beta>0} {}^\times_\times E^\beta(z)E^{-\beta}(z) + E^{-\beta}(z)E^\beta(z){}^\times_\times\right\} \tag{49}$$

where h is the dual Coxeter number (for the algebra $su(N+1)$ we have $h = N+1$), and the crosses indicate normal ordering for the modes of $su(N+1)$ currents. For level-1 simply laced algebras, the contribution of terms quadratic in step operators in the energy–momentum tensor is known to be proportional to terms dependent on the Cartan sub-algebra, see [12]

$${}^\times_\times E^\beta(z)E^{-\beta}(z) + E^{-\beta}(z)E^\beta(z){}^\times_\times = {}^\times_\times(\beta.H(z))^2{}^\times_\times .$$

6. Conclusion and outlook

A q-deformed version of the $N = 2$ superconformal algebra was proposed in terms of level-1 representations of $\mathcal{U}_q(\widehat{su}(2))$ Kac–Moody algebra and a single real Fermi field. This construction hints the form for the q-analogue for the Sugawara energy–momentum tensor possessing an exponential dependence on the Cartan subalgebra generators. An interesting point to be further investigated concerns the construction of an action from which such energy–momentum tensor could be obtained using canonical methods following the line of [7].

The study of the representations of such algebraic structure also deserves to be further developed. In particular, identities involving conformal embeddings [2] as well as coset constructions [11] may be obtained from representation theory.

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Appendix: q-Taylor series expansions

In this appendix we review some properties of the q-Taylor expansion, which is obtained from the usual Taylor series by adding and subtracting terms. This ensures the same content of the series in classical analysis, with the additional advantage of providing a non-local expansion. An analytic function $f(z)$ may be written near $z = \omega$ as either

$$f(z) = \sum_{k=0}^{\infty} \frac{\partial_q^{2k} f(wq)}{[2k]!}(z-w)_q^{2k} + \sum_{k=0}^{\infty} \frac{\partial_q^{2k+1} f(w)}{[2k+1]!}(z-wq)_q^{2k+1} \tag{A1}$$

or

$$f(z) = \sum_{k=0}^{\infty} \frac{\partial_q^{2k} f(wq^{-1})}{[2k]!}(z-w)_q^{2k} + \sum_{k=0}^{\infty} \frac{\partial_q^{2k+1} f(w)}{[2k+1]!}(z-wq^{-1})_q^{2k+1} \tag{A2}$$

where $[n]! = [n][n-1]!$, the symbol ∂_q denotes the q-derivative defined as

$$\partial_q f(z) = \frac{f(zq) - f(zq^{-1})}{z(q-q^{-1})} \tag{A3}$$

and

$$(z-w)_q^n = \prod_{k=1}^{n}(z - wq^{n-2k+1}) = \sum_{k=0}^{n} \frac{[n]!}{[k]![n-k]!} z^k (-w)^{n-k} \tag{A4}$$

is the q-binomial. These expansions display a non-local character and become useful in finding regular parts of OPE in q-deformed meromorphic field theories like those presented in this paper.

From equation (A3) we propose a general closed expression for the nth q-derivative

$$\partial_q^n f(z) = \frac{1}{z^n(q-q^{-1})^n} \sum_{\sigma_0=\pm1} \sum_{\sigma_1=\pm1} \cdots \sum_{\sigma_{n-1}=\pm1} \left(\prod_{i=0}^{n-1} \sigma_i q^{-i\sigma_i} \right) f\left(zq^{\sum_{i=0}^{n-1} \sigma_i} \right) \tag{A5}$$

which can easily be proved by induction.

A crucial observation from (A5) is that the nth-order q-derivative is proportional to the original function with the argument shifted by some power of the deformation parameter. The n sums over $\sigma_i = \pm 1$, $i = 0, \ldots, n-1$ leads to a shifted argument proportional to $q^{n-2\alpha}$. Henceforth, the parity of the power of q is always the same as the order of the derivative. On the other hand, odd derivative terms in the q-Taylor expansion (A1, A2) are evaluated in $z = w$ while the even ones in $z = wq$ or $z = wq^{-1}$. In either case, the expansion of $f(z)$ in (A1), (A2) consist of linear combination of terms like $f(wq^{2k+1})$, $k \in \mathbb{Z}$.

We now apply this result to calculate the regular part of the OPE $E^+(z)E^-(w)$ in (20). The above argument implies that the right-hand side of (20) is a linear combination of $D(wq^{2k+1}, w)$. The peculiar form of $D(z, w)$ yields a dependence entirely in terms of the Cartan subalgebra current $H(z)$. To be more specific, the following identities can be obtained by direct calculation:

$$\begin{aligned} D(wq^{2k+1}, w) &= \left(\prod_{i=0}^{k-1} \Phi^{-1}(wq^{-\frac{1}{2}+2(k-i)})\right)\left(\prod_{j=0}^{k} \Psi(wq^{\frac{1}{2}+2(k-j)})\right) \\ &=: A^1(wq^2)A^1(wq^4)\cdots A^1(wq^{2k}) : \Psi(wq^{\frac{1}{2}}) \end{aligned} \tag{A6}$$

and

$$\begin{aligned} D(wq^{-(2k+1)}, w) &= \left(\prod_{i=0}^{k} \Phi(wq^{-\frac{1}{2}-2i})\right)\left(\prod_{j=0}^{k-1} \Psi^{-1}(wq^{\frac{1}{2}-2j})\right) \\ &= \Phi(wq^{-\frac{1}{2}}) : B^1(wq^{-2})B^1(wq^{-4})\cdots B^1(wq^{-2k}) : . \end{aligned} \tag{A7}$$

We should also point out that due to the non-locality, the expansion of the regular part of $E^+(z)E^-(w)$ does not truncate when equal point limit is taken. It is, in fact composed of infinite number of terms which are classified in powers of $(q - q^{-1})$.

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