Cálculo B- Prova 1

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Nome:

1. Escreva uma integral (sem calcular) para o volume do sólido gerado pela rotação de cada região abaixo em torno da reta especificada:

   a. Região delimitada pelas curvas $x = y^2 - 3y + 4$ e $x = -y^2 + 7y - 8$; rotação em torno da reta $y = -1$.

   b. Região delimitada por $y = x^2(1 - x)$ e o eixo x, para $0 \leq x \leq 1$; rotação em torno do eixo y.

2. a. Esboce as curvas de nível da função $f(x, y) = x^2 - y^2$.

   b. Esboce as superfícies de nível da função $f(x, y, z) = x^2 + 3y^2 + 5z^2$.

3. Determine o limite, se existir, ou mostre que o limite não existe

   a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$.

   b. $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$.

4. Determine as derivadas parciais indicadas:

   a. $f(x, y) = x(x^2 + y^2)^{-3/2} e^{\sin(x^2 y)}$, f_x, f_y.

   b. $w = f(x, y, z) = xyz^2 \tan(yz)$, $\frac{\partial^2 w}{\partial x \partial y}$, $\frac{\partial^4 w}{\partial z^2 \partial y \partial x}$.

5. Uma faixa de 3 mm de largura é pintada ao redor de um retângulo de dimensões 1 metro por 2 metros. Utilize diferenciais para aproximar a área pintada na faixa.

6. Suponha que $z = f(x, y)$, onde $x = g(s, t)$ e $y = h(s, t)$. Mostre que

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial t}\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2 + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial t^2} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial t^2}.$$

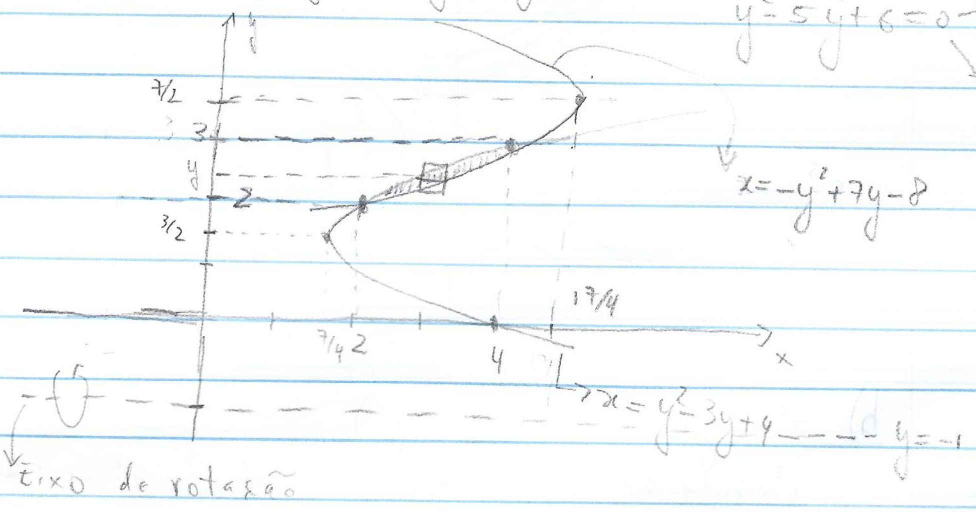
Prova II: Cálculo B

① a) $x = y^2 - 3y + 4$, $x = -y^2 + 7y - 8$

Ponto de interseção: $y^2 - 3y + 4 = -y^2 + 7y - 8 \Rightarrow 2y^2 - 10y + 12 = 0$

$y^2 - 5y + 6 = 0 \rightarrow y = 2$
$\rightarrow y = 3$

$P_1(2, 2)$
$P_2(4, 3)$

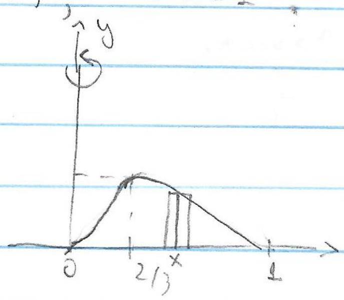


Método das Cascas Cilíndricas:

$$V = 2\pi \int_2^3 (y+1) \cdot ((-y^2 + 7y - 8) - (y^2 - 3y + 4))\, dy$$

$$\boxed{V = 2\pi \int_2^3 (y+1) \cdot (-2y^2 + 10y - 12)\, dy}$$

b) $y = x^2(1-x)$, $0 \leq x \leq 1$



Cascas cilíndricas:

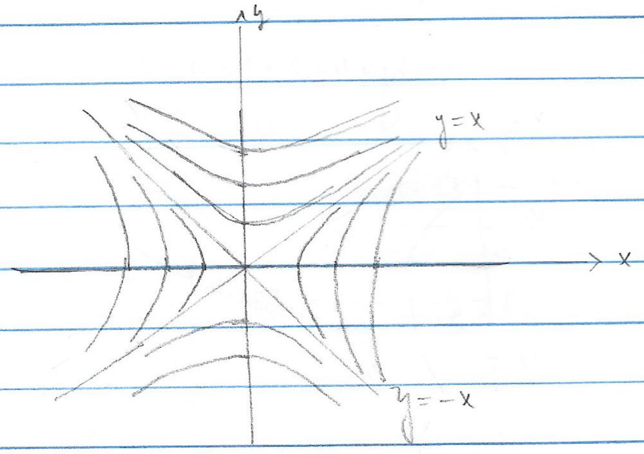
$$V = 2\pi \int_0^1 x \cdot [x^2(1-x)]\, dx$$

$$V = 2\pi \int_0^1 x^3(1-x)\, dx$$

(2) a) $x^2-y^2=k$, com $k>0$ hipérboles com vértice no eixo x

$x^2-y^2=k$, com $k<0$, " " " " " y.

$x^2-y^2=0 \Rightarrow y=x$ e $y=-x$

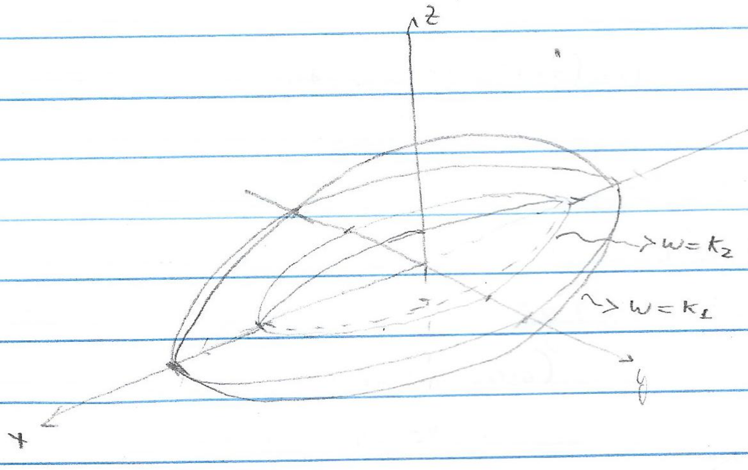


b) $x^2+3y^2+5z^2=0 \Rightarrow (0,0,0)$

$x^2+3y^2+5z^2=k^2$, $\Rightarrow$ elipsóide: $\dfrac{x^2}{k^2}+\dfrac{y^2}{\frac{k^2}{3}}+\dfrac{z^2}{\frac{k^2}{5}}=1$

Superfícies de nível são elipsóides centrados na origem

Por exemplo se $k^2=15 \Rightarrow \dfrac{x^2}{15}+\dfrac{y^2}{5}+\dfrac{z^2}{3}=1$



③ a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^8}$

Escolhendo o caminho $x=0$ e $y\to 0^+ \Rightarrow \lim\limits_{y\to 0} \dfrac{0}{y^8} = 0$

Escolhendo o caminho $x=y^4$, $y\to 0 \Rightarrow \lim\limits_{y\to 0} \dfrac{y^8}{2y^8} = 1/2$

Logo, o limite não existe

b) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+xz^2+yz^2}{x^2+y^2+z^4}$

Escolhendo o caminho $x=z=0$, $y\to 0$, tem

$$\lim_{y\to 0} \frac{0}{y^2} = 0$$

Escolhendo o caminho $x=0$ e $y=z^2$, $z\to 0$, tem

$$\lim_{z\to 0} \frac{z^4}{2z^4} = 1/2$$

Logo, o limite não existe.

④ a) $f(x,y) = x(x^2+y^2)^{-3/2} e^{\operatorname{sen}(x^2y)}$

$f_x(x,y) = (x^2+y^2)^{-3/2} \cdot e^{\operatorname{sen}(x^2y)} + x \cdot 2x \cdot \left(-\frac{3}{2}\right)(x^2+y^2)^{-5/2} e^{\operatorname{sen}(x^2y)}$

$+ x(x^2+y^2)^{-3/2} e^{\operatorname{sen}(x^2y)} \cos(x^2y) \cdot 2xy$

$f_x(x,y) = (x^2+y^2)^{-5/2} \cdot e^{\operatorname{sen}(x^2y)} \cdot \left[x^2+y^2-3x^2+2x^2y\cos(x^2y)(x^2+y^2)\right]$

$f_x(x,y) = (x^2+y^2)^{-5/2} e^{\operatorname{sen}(x^2y)} \left[y^2-2x^2+2x^2y\cos(x^2y)(x^2+y^2)\right]$

$$f_y(x,y) = x \cdot 2y \cdot \left(-\tfrac{3}{2}\right)(x^2+y^2)^{-5/2}\, e^{\operatorname{sen}(x^2y)} + x(x^2+y^2)^{-3/2}\, e^{\operatorname{sen}(x^2y)} \cdot \cos(x^2y)\, x^2$$

$$f_y(x,y) = x\,(x^2+y^2)^{-5/2}\, e^{\operatorname{sen}(x^2y)} \left[-3y + (x^2+y^2)x^2 \cos(x^2y)\right]$$

b) $w = f(x,y,z) = xyz^2 \tan(yz)$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial}{\partial x}\left[\frac{\partial w}{\partial y}\right] = \frac{\partial}{\partial x}\left[xz^2 \tan(yz) + xyz^2 \sec^2(yz)\cdot z\right]$$

$$= z^2 \tan(yz) + yz^3 \sec^2(yz)$$

$$\frac{\partial^4 w}{\partial z^2 \partial y \partial x} = \frac{\partial^3}{\partial z^2 \partial y}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial^3}{\partial z^2 \partial y}\left(yz^2 \tan(yz)\right)$$

$$= \frac{\partial^2}{\partial z^2}\left(z^2 \tan(yz) + yz^2 \sec^2(yz)\cdot z\right) = \frac{\partial^2}{\partial z^2}\left[z^2 \tan(yz) + yz^3 \sec^2(yz)\right]$$

$$= \frac{\partial}{\partial z}\left[2z\tan(yz) + z^2\sec^2(yz)\cdot y + 3yz^2\sec^2(yz) + yz^3\, 2\sec(yz)\cdot\sec(yz)\cdot\tan(yz)\cdot y\right]$$

$$= \frac{\partial}{\partial z}\left[2z\tan(yz) + 4z^2y\sec^2(yz) + 2y^2z^3\sec^2(yz)\tan(yz)\right]$$

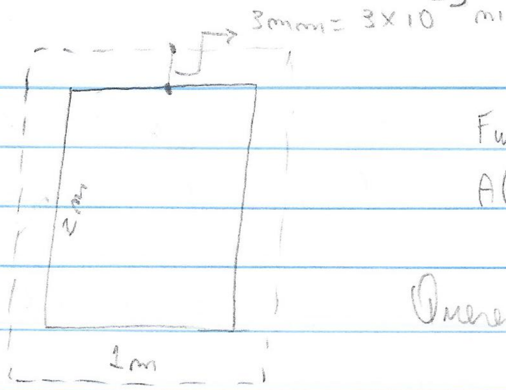
$$= \left[2\tan(yz) + 2z\sec^2(yz)\cdot y\right] + \left[8zy\sec^2(yz) + 4z^2y\cdot 2\sec(yz)\sec(yz)\tan(yz)\,y\right]$$

$$+ \left[6y^2z^2\sec^2(yz)\tan(yz) + 2y^2z^3\cdot 2\sec(yz)\cdot\sec(yz)\tan^2(yz)\cdot y + 2y^2z^3\sec^2(yz)\sec^2(yz)\cdot y\right]$$

$$= 2\tan(yz) + 10zy\sec^2(yz) + 14y^2z^2\sec^2(yz)\tan(yz) + 4y^3z^3\sec^2(yz)\tan^2(yz) + 4y^3z^3\sec^4(yz)$$

//

(5)



Função área de um retângulo de dimensões x×y:

$A(x,y) = x \cdot y \longrightarrow A_y(x,y) = x$

$\qquad\qquad\qquad\searrow A_x(x,y) = y$

Queremos estimar:

$$\Delta A = A(1+6\times10^{-3},\ 2+6\times10^{-3}) - A(1,2)$$

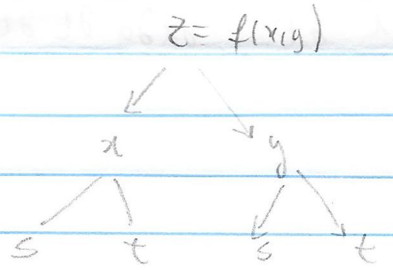
$$dA \approx \Delta A \Rightarrow dA = A_x(1,2)\cdot \Delta x + A_y(1,2)\cdot \Delta y$$

$$dA = A_x(1,2)\cdot 6\times10^{-3} + A_y(1,2)\cdot 6\times10^{-3}$$

$$dA = 2\cdot 6\times10^{-3} + 6\times10^{-3} = 18\times10^{-3}\ m^2$$

Logo a área da faixa é aproximadamente $0,018\ m^2$

(6)



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} \quad \text{(Regra da cadeia)}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t}\left[\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}\right]$$, usando a regra do produto, temos

$$(*)\quad \frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial x}\right)\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial t^2} + \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial y}\right)\cdot\frac{\partial y}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial^2 y}{\partial t^2}$$

Usando novamente a Regra da Cadeia, temos $\longrightarrow$

$$\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\cdot\frac{\partial x}{\partial t}+\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\cdot\frac{\partial y}{\partial t}=\frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial t}+\frac{\partial^2 z}{\partial y\partial x}\frac{\partial y}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\cdot\frac{\partial x}{\partial t}+\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\cdot\frac{\partial y}{\partial t}=\frac{\partial^2 z}{\partial x\partial y}\frac{\partial x}{\partial t}+\frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial t}$$

Substituindo em (*), obtemos:

$$\frac{\partial^2 z}{\partial t^2}=\left(\frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial x}{\partial t}+\frac{\partial^2 z}{\partial y\partial x}\frac{\partial y}{\partial t}\right)\frac{\partial x}{\partial t}+\frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial t^2}+\left(\frac{\partial^2 z}{\partial x\partial y}\frac{\partial x}{\partial t}+\frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial t}\right)\cdot\frac{\partial y}{\partial t}+$$

$$+\frac{\partial z}{\partial y}\cdot\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 z}{\partial t^2}=\frac{\partial^2 z}{\partial x^2}\cdot\left(\frac{\partial x}{\partial t}\right)^2+\frac{\partial^2 z}{\partial y\partial x}\frac{\partial y}{\partial t}\frac{\partial x}{\partial t}+\frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial t^2}+\frac{\partial^2 z}{\partial x\partial y}\frac{\partial x}{\partial t}\frac{\partial y}{\partial t}+$$

$$+\frac{\partial^2 z}{\partial y^2}\cdot\left(\frac{\partial y}{\partial t}\right)^2+\frac{\partial z}{\partial y}\cdot\frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 z}{\partial t^2}=\frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2+2\frac{\partial^2 z}{\partial y\partial x}\frac{\partial y}{\partial t}\frac{\partial x}{\partial t}+\frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2+\frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial t^2}+\frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial t^2}}$$