# Prova I-Cálculo 2

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Nome:

1. Resolva cada integral abaixo, **apresentando os detalhes da resolução**:

   a. (2,0 pontos) $\displaystyle\int_0^{\pi/2} x^2 \cos(x)\, dx$

   b. (2,5 pontos) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}} dx$

2. (3,0 pontos) Usando o método das frações parciais, determine a integral

$$\int \frac{x^3 + 4x^2 + 7x + 5}{x^2(x^2 + 4x + 5)} dx$$

3. Classifique cada integral imprópria abaixo como convergente ou divergente, **justificando adequadamente sua resposta.**

   a. (2,0 pontos) $\displaystyle\int_1^{\infty} \frac{1 + \cos^2(x)}{x^3 + 2x^2 + 1} dx$

   b. (2,0 pontos) $\displaystyle\int_0^{3} \frac{dx}{x^2 - 5x + 6} dx$

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1) a) $\int_0^{\pi/2} x^2\cos(x)\,dx = x^2\cdot \text{sen}(x)\Big|_0^{\pi/2} - \int_0^{\pi/2} 2x\,\text{sen}(x)\,dx$

$u = x^2 \quad \Big| \quad dv = \cos(x)\,dx$

$du = 2x\,dx \quad \Big| \quad v = \text{sen}(x)$

$u = 2x \quad \Big| \quad dv = \text{sen}(x)\,dx$

$du = 2dx \quad \Big| \quad v = -\cos(x)\,dx$

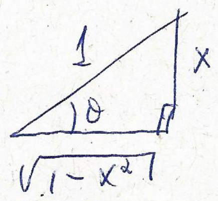
$$\int_0^{\pi/2} x^2\cos(x)\,dx = x^2\cdot\text{sen}(x)\Big|_0^{\pi/2} - \left[-2x\cdot\cos(x)\Big|_0^{\pi/2} + 2\int_0^{\pi/2}\cos(x)\,dx\right]$$

$$\int_0^{\pi/2} x^2\cos(x)\,dx = x^2\cdot\text{sen}(x)\Big|_0^{\pi/2} + 2x\cos(x)\Big|_0^{\pi/2} - 2\,\text{sen}(x)\Big|_0^{\pi/2}$$

$$\int_0^{\pi/2} x^2\cos(x)\,dx = \left(\pi/2\right)^2\cdot 1 - 0 + \left[2\cdot\tfrac{\pi}{2}\cos\left(\tfrac{\pi}{2}\right) - 0\right] - 2\left(\text{sen}\left(\tfrac{\pi}{2}\right) - 0\right) = \frac{\pi^2}{4} - 2$$

b) $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$, $\quad x = \text{sen}(\theta)$, $-\pi/2 < \theta < \pi/2$

$dx = \cos(\theta)\,d\theta$




$\sqrt{1-x^2} = \cos(\theta)$

$$\int \frac{\text{sen}^3(\theta)}{\cos(\theta)}\cdot\cos(\theta)\,d\theta = \int \text{sen}^3(\theta)\,d\theta$$

$$= \int \text{sen}^2(\theta)\cdot\text{sen}(\theta)\,d\theta = \int \left(1-\cos^2(\theta)\right)\text{sen}(\theta)\,d\theta$$

$$= \int \text{sen}(\theta)\,d\theta - \int \cos^2(\theta)\,\text{sen}(\theta)\,d\theta \rightarrow \begin{matrix} u = \cos(\theta) \\ du = -\text{sen}(\theta)\,d\theta \end{matrix}$$

$$= -\cos(\theta) + \frac{\cos^3(\theta)}{3} + C$$

$$= -\sqrt{1-x^2} + \frac{\left(\sqrt{1-x^2}\right)^3}{3} + C$$

2) $$\frac{x^3+4x^2+7x+5}{x^2(x^2+4x+5)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+4x+5} = \frac{A(x^2+4x+5) + Bx(x^2+4x+5) + (Cx+D)x^2}{x^2(x^2+4x+5)}$$

$$(B+C)x^3 + (A+4B+D)x^2 + (4A+5B)x + 5A = x^3+4x^2+7x+5$$

$B+C=1$

$A+4B+D=4$

$4A+5B=7 \leftarrow 5B=3 \Rightarrow \boxed{B=3/5}$

$5A=5 \Rightarrow \boxed{A=1}$

$\frac{3}{5}+C=1$, $\boxed{C=2/5}$

$1+\frac{12}{5}+D=4 \Rightarrow \boxed{D=3/5}$

Logo: $\int \frac{x^3+4x^2+7x+5}{x^2(x^2+4x+5)}\,dx = \int \frac{dx}{x^2} + \frac{3}{5}\int \frac{dx}{x} + \boxed{\int \frac{\frac{2}{5}x+\frac{3}{5}}{x^2+4x+5}\,dx}$ *

$$\int \frac{dx}{x^2} \Downarrow -x^{-1} \qquad \frac{3}{5}\int\frac{dx}{x} \Downarrow \frac{3}{5}\ln|x|$$

$$* = \frac{1}{5}\int \frac{2x+3}{x^2+4x+5}\,dx = \frac{1}{5}\int\left(\frac{2x+4-1}{x^2+4x+5}\right)dx = \frac{1}{5}\int\frac{(2x+4)\,dx}{x^2+4x+5} - \frac{1}{5}\int\frac{dx}{x^2+4x+5}$$

$u = x^2+4x+5$

$du = (2x+4)\,dx$

$$= \frac{1}{5}\int\frac{du}{u} - \frac{1}{5}\int\frac{dx}{(x+2)^2+1}$$

$$= \frac{1}{5}\ln(x^2+4x+5) - \frac{1}{5}\operatorname{arctg}((x+2)) + C$$

$$\int \frac{x^3+4x^2+7x+5}{x^2(x^2+4x+5)}\,dx = -x^{-1} + \frac{3}{5}\ln|x| + \frac{1}{5}\ln(x^2+4x+5) - \frac{1}{5}\operatorname{arctg}(x+2) + C$$

3) a) $1+\cos^2(x) \leq 1+1 = 2 \implies \frac{1+\cos^2(x)}{x^3+2x^2+1} < \frac{2}{x^3}$

$x^3+2x^2+1 > x^3$

$\Downarrow$

$\frac{1}{x^3+2x^2+1} < \frac{1}{x^3}$

Como $\int_1^\infty \frac{2}{x^3}\,dx$ é convergente, pelo teste da comparação a integral dada é também convergente.

b) $x^2-5x+6 = (x-2)(x-3)$

$$\frac{1}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{x^2-5x+6} \implies (A+B)x - 3A - 2B = 1$$

$$\begin{cases} A+B=0 \iff \boxed{B=1} \\ -3A-2B=1 \end{cases}$$

$-A = 1 \implies \boxed{A=-1}$

$$\int_0^3 \frac{1}{x^2-5x+6}\,dx = -\int_0^3\frac{dx}{x-2} + \int_0^3\frac{dx}{x-3}$$

$\Downarrow$

$$\lim_{t\to3^-}\int_0^t \frac{dx}{x-3} = \lim_{t\to3^-} \ln|x-3|\Big|_0^t = \lim_{t\to3^-}(\ln|t-3| - \ln 3)$$

Como $\lim_{t\to3^-}\ln|t-3| = -\infty$, a integral é divergente, o mesmo sendo válido para a $\int_0^3 \frac{dx}{x^2-5x+6}$.