Cálculo B- Prova 3

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Nome:

1. Determine os pontos da superfície $z^2 = xy + 1$ que estão mais próximos da origem.

2. Detemine o volume da região delimitada pelos cilindros $z = x^2$, $y = x^2$ e pelos planos $z = 0$ e $y = 4$.

3. Uma broca cilíndrica de raio r_1 é usada para fazer um furo no centro de uma esfera de raio r_2 (evidentemente, $r_1 < r_2$). Determine o volume do sólido em formato de anel restante.

4. Considere a integral $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$.

   a. Escreva-a em coordenadas cilíndricas.

   b. Escreva-a em coordenadas esféricas.

5. Calcule $\int\int_R e^{\frac{x+y}{x-y}} dA$, onde R é a região trapezoidal com vértices em $(1,0)$, $(2,0)$, $(0,-2)$ e $(0,-1)$.

6. Represente a região que está dentro de ambas as curvas $r = 3 + 2\,\mathrm{sen}(\theta)$ e $r = 2$ e determine a área dessa região.

Resolução Prova III – Cálculo B:

(1) $z^2 = xy+1$

$$d^2 = x^2+y^2+z^2 = x^2+y^2+xy+1 \Rightarrow f(x,y) = x^2+y^2+xy+1$$

$$\frac{\partial f}{\partial x} = 2x+y, \quad \frac{\partial f}{\partial y} = 2y+x$$

Pontos críticos: $\begin{cases} 2x+y=0 \\ 2y+x=0 \end{cases} \Rightarrow x=0, \; y=0$

$$f_{xx}=2, \quad f_{yy}=2, \quad f_{xy}=f_{yx}=1$$

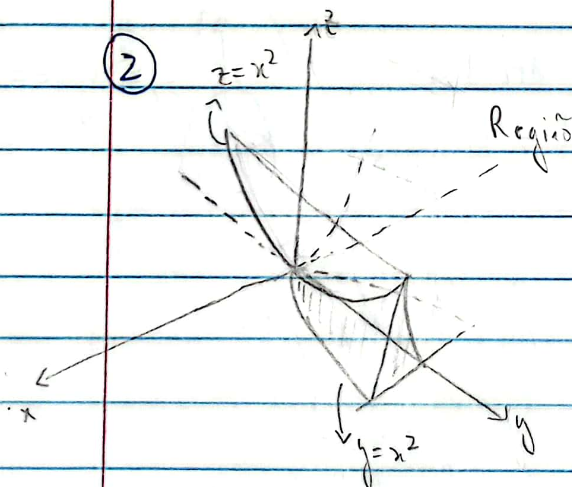
$$D = f_{xx}\cdot f_{yy} - (f_{xy})^2 = 4-1 = 3>0$$

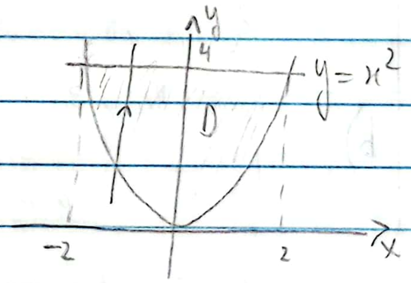
$f_{xx}(0,0) = 2>0 \Rightarrow (0,0)$ é ponto de mínimo de f.

A geometria do problema garante que esse é o ponto de mínimo absoluto, já que ele é o único ponto crítico.

Logo os pontos $(0,0,1)$ e $(0,0,-1)$ são os pontos procurados.

(2)



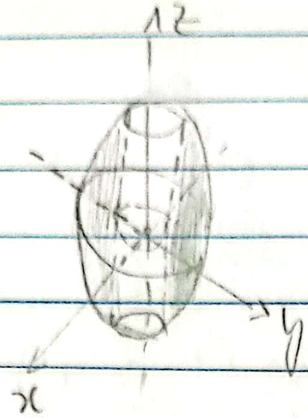


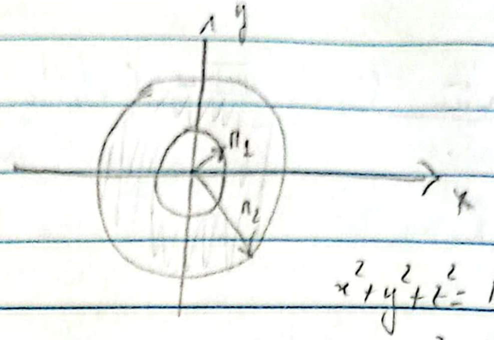
$$V = \iint_D x^2 \, dA = \int_{-2}^{2}\int_{x^2}^{4} x^2 \, dy \, dx$$

$$= \int_{-2}^{2} x^2 \cdot y \Big|_{x^2}^{4} dx = \int_{-2}^{2} (4x^2 - x^4)\, dx = \left(\frac{4x^3}{3} - \frac{x^5}{5}\right)\Big|_{-2}^{2} = \left(\frac{4\cdot 8}{3} - \frac{32}{5}\right) - \left(\frac{-32}{3} + \frac{32}{5}\right)$$

$$= \frac{64}{3} - \frac{64}{5} = 64\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{128}{15}$$

(3)





$x^2+y^2+z^2 = r_2^2$

$z = \sqrt{r_2^2 - r^2}$

$$V = 2\int_0^{2\pi}\int_{r_1}^{r_2}\int_0^{\sqrt{r_2^2-r^2}} r \, dz \, dr \, d\theta$$

$$V = 2\int_0^{2\pi}\int_{r_1}^{r_2} r \cdot z \Big|_0^{\sqrt{r_2^2-r^2}} dr \, d\theta = 2\int_0^{2\pi}\int_{r_1}^{r_2} r \cdot \sqrt{r_2^2-r^2} \, dr \, d\theta$$

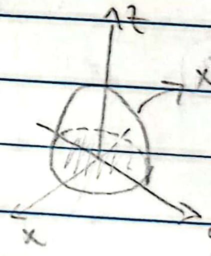
$$= 2\int_0^{2\pi} -\frac{1}{2} \cdot \frac{(r_2^2-r^2)^{3/2}}{3/2} \Big|_{r_1}^{r_2} d\theta = -\frac{1}{3} \cdot 4\pi \cdot (r_2^2-r^2)^{3/2} \Big|_{r_1}^{r_2}$$

$$= -\frac{4\pi}{3} \cdot \left[0 - (r_2^2 - r_1^2)^{3/2} \right]$$

$$V = \frac{4\pi}{3} \cdot (r_2^2 - r_1^2)^{3/2}$$

(4) a)

$$\int_0^{2\pi}\int_0^3\int_0^{\sqrt{9-r^2}} z\sqrt{r^2+z^2} \, r \, dz \, dr \, d\theta$$

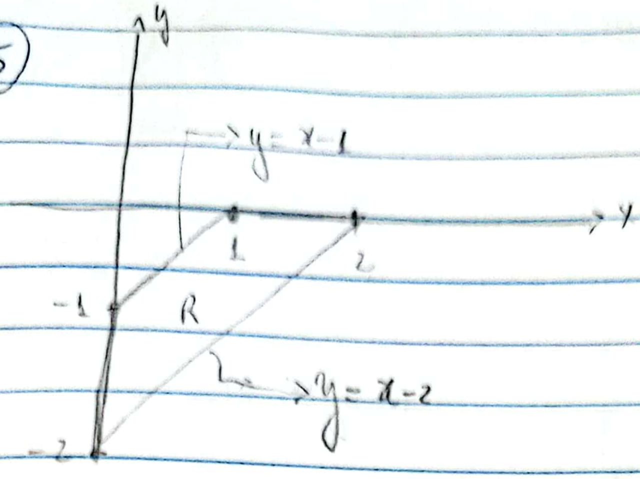


b)

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^3 \rho\cos\phi \cdot \rho \cdot \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/2}\int_0^3 \rho^4 \cos\phi \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

⑤



$u = x + y \quad , \quad v = x - y$

- $y = x - 2 \Rightarrow x - y = 2 \Rightarrow v = 2$
- $y = x - 1 \Rightarrow x - y = 1 \Rightarrow v = 1$

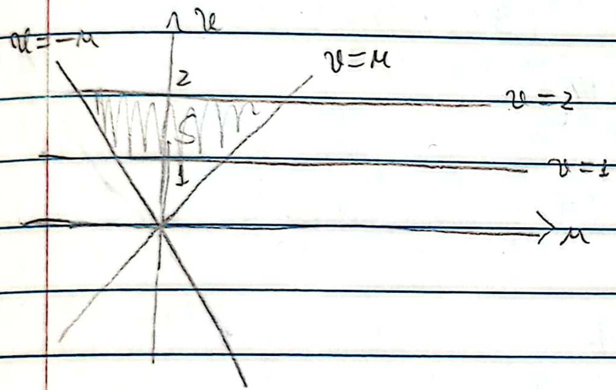
- $x = 0 \quad , \quad -1 \leq y \leq -2$

$u = y \, , \; v = -y$

$u = -v$

- $y = 0 \, , \quad 1 \leq x \leq 2$

$u = x \, , \; v = x \Rightarrow u = v$



$u = x + y$, $v = x - y \Rightarrow 2x = u + v$, $\boxed{x = \frac{u+v}{2}}$

$\Downarrow$

$2y = u - v \Rightarrow \boxed{y = \frac{u-v}{2}}$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} = -1/4 - 1/4 = -1/2$$

$$\iint_R e^{\frac{x+y}{x-y}} \, dA = \iint_S e^{\frac{u}{v}} \cdot \frac{1}{2} \, dA = \int_1^2 \int_{-v}^{v} e^{\frac{u}{v}} \cdot \frac{1}{2} \, du \, dv$$

$$= \frac{1}{2} \int_1^2 v \, e^{\frac{u}{v}} \Big|_{-v}^{v} dv = \frac{1}{2} \int_1^2 v \cdot (e - e^{-1}) \, dv = \frac{e - e^{-1}}{2} \cdot \frac{v^2}{2} \Big|_1^2$$

$$= \frac{e - e^{-1}}{2} \cdot \left[2 - \frac{1}{2} \right]$$

$$= \frac{3}{4} \left(e - e^{-1} \right)$$

6) $r = 3 + 2\,\text{sen}\,\theta$, $r = 2$

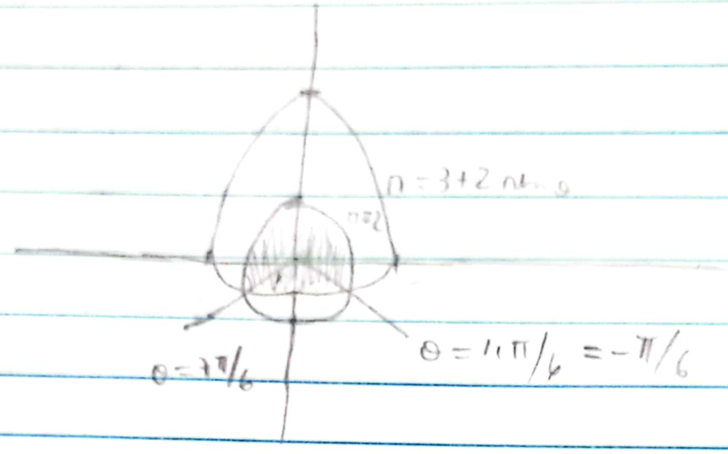
$\hookrightarrow r = \frac{3}{2}(1 + \text{sen}\,\theta)$

Pts de intersecção

$2 = 3 + 2\,\text{sen}\,\theta$

$\text{sen}\,\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}$, $\theta = \frac{11\pi}{6}$



$$A = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} 2^2\,d\theta + \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (3 + 2\,\text{sen}\,\theta)^2\,d\theta$$

$$A = \frac{1}{2}\cdot 4\cdot\left(\frac{7\pi}{6} + \pi/6\right) + \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (9 + 12\,\text{sen}\,\theta + 4\,\text{sen}^2\theta)\,d\theta$$

$$A = \frac{8\pi}{3} + \frac{9}{2}\cdot\frac{4\pi}{6} - 6\cos\theta\Big|_{7\pi/6}^{11\pi/6} + 2\int_{7\pi/6}^{11\pi/6}\left(\frac{1 - \cos(2\theta)}{2}\right)d\theta$$

$$= \frac{8\pi}{3} + 3\pi - 6\left[+\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] + \left(\frac{11\pi}{6} - \frac{7\pi}{6}\right) - \frac{1}{2}\,\text{sen}(2\theta)\Big|_{7\pi/6}^{\frac{11\pi}{6}}$$

$$= \frac{17\pi}{3} - 6\sqrt{3} + \frac{4\pi}{6} - \frac{1}{2}\left[\text{sen}\left(\frac{11\pi}{3}\right) - \text{sen}\left(\frac{7\pi}{3}\right)\right]$$

$$= \frac{19\pi}{3} - 6\sqrt{3} - \frac{1}{2}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right] = \frac{19\pi}{3} - 6\sqrt{3} + \frac{\sqrt{3}}{2} = \frac{19\pi}{3} - \frac{11\sqrt{3}}{2}$$