Prova III - Cálculo 2 - 2019/02

Prof. Luciano Bedin

Nome:

Resolva todas as questões abaixo. **Soluções sem justificativas e/ou cálculos não serão considerados.**

1,5

1. Encontre a solução geral da EDO:

$$y''(t) + y'(t) - 2y(t) = -e^t.$$

2,0

2. Encontre uma família de soluções a um parâmetro para a EDO

$$(y^3 - y^2\text{sen}(x) - x)dx + (3xy^2 + 2y\cos(x))dy = 0.$$

2,0

3. Determine

$$\mathcal{L}\left\{t^2e^{-3t}\cos(2t)\right\}(s).$$

2,0

4. Determine

$$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^3(s^2+1)}\right\}(t).$$

2,5

5. *Usando a transformada de Laplace*, resolva o PVI

$$y'' - 8ty' + 16y = 3, \qquad y(0) = y'(0) = 0.$$

Resolução P3 - 2019/02 - Cálculo II

(1) $r^2 + r - 2 = 0 \to r = 1,\ r = -2$ $\quad y_h(t) = C_1 e^{t} + C_2 e^{-2t}$

$y_p(t) = A t e^{t}$, $\quad y_p' = A[te^{t} + e^{t}]$

$y_p'' = A[te^{t} + 2e^{t}]$

Subst. na EDO: $A[te^{t} + 2e^{t}] + A[te^{t} + e^{t}] - 2Ate^{t} = -e^{t}$

$3Ae^{t} = -e^{t} \Rightarrow A = -1/3$

Conclusão: $\boxed{y(t) = C_1 e^{t} + C_2 e^{-2t} - \frac{1}{3} t e^{t}}$

(2) $M = y^3 - y^2 \operatorname{sen}(x) - x$, $\quad M_y = 3y^2 - 2y \operatorname{sen}(x)$

$N = 3xy^2 + 2y\cos(x)$, $\quad N_x = 3y^2 - 2y\operatorname{sen}(x)$

$M_y = N_x \Longrightarrow$ EDO exata

Então existe $z = f(x,y)$ tal que

$f_x = M \Longrightarrow f_x = y^3 - y^2 \operatorname{sen}(x) - x$

$f_y = N \Longrightarrow f_y = 3xy^2 + 2y\cos(x)$

Logo $f(x,y) = y^3 x + y^2 \cos(x) - \frac{x^2}{2} + h(y)$

e $f_y = 3y^2 x + 2y\cos(x) + h'(y) = 3xy^2 + 2y\cos(x) \Rightarrow h'(y) = 0$

$\Rightarrow h(y) = K$

Conclusão: A sol. implícita da EDO é dada por $y^3x + y^2\cos(x) - \frac{x^2}{2} = \text{Const.}$

(3) $\mathcal{L}\{e^{-3t}\cos(2t)\}(s) = \frac{s+3}{(s+3)^2+4}$, $s > -3$

$$\mathcal{L}\{t\, e^{-3t}\cos(2t)\}(s) = -\frac{d}{ds}\left[\frac{s+3}{(s+3)^2+4}\right]$$

$$= -\left[\frac{1.\left((s+3)^2+4\right) - (s+3).2(s+3)}{\{(s+3)^2+4\}^2}\right]$$

$$= \frac{(s+3)^2-4}{[(s+3)^2+4]^2}$$

$$\mathcal{L}\{t^2 e^{-3t}\cos(2t)\}(s) = -\frac{d}{ds}\left\{\frac{(s+3)^2-4}{[(s+3)^2+4]^2}\right\}$$

$$= -\frac{2(s+3).[(s+3)^2+4]^2 - 2[(s+3)^2+4].2(s+3).[(s+3)^2-4]}{[(s+3)^2+4]^4}$$

$s > -3$

(4) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}(t) = \text{sen}(t)$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{1}{s^2+1}\right\}(t) = \int_0^t \text{sen}(u)\,du = -\cos(u)\Big|_0^t = 1-\cos(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\cdot\frac{1}{s^2+1}\right\}(t) = \int_0^t (1-\cos(u))\,du = t - \text{sen}(u)\Big|_0^t$$

$$= t - \text{sen}(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3}\cdot\frac{1}{s^2+1}\right\}(t) = \int_0^t (u - \text{sen}(u))\,du = \frac{u^2}{2}\Big|_0^t + \cos(u)\Big|_0^t$$

$$= t^2/2 + \cos(t) - 1$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^3(s^2+1)}\right\}(t) = t$$

$$= \left\{\frac{(t-\pi)^2}{2} + \cos(t-\pi) - 1\right\} u_\pi(t)$$

(5) $\mathcal{L}\{t y'\} = -\frac{d}{ds}\mathcal{L}\{y'\} = -\frac{d}{ds}\{sY(s) - y(0)\}$

$$= -sY'(s) - Y(s)$$

$$\mathcal{L}\{y''\} = s^2 Y(s) - s.y(0) - y'(0) = s^2 Y(s)$$

Aplicando $\mathcal{L}$ na EDO:

$$s^2 Y(s) + 8sY'(s) + 8Y(s) + 16Y(s) = \frac{3}{s}$$

$$8sY'(s) + (s^2+24)Y(s) = \frac{3}{s}$$

$$Y'(s) + \left(\frac{s}{8} + \frac{3}{s}\right)Y(s) = \frac{3}{8s^2}$$

$$\mu(s) = e^{\int\left(\frac{s}{8}+\frac{3}{s}\right)ds} = e^{\frac{s^2}{16}} \cdot s^3$$

$$Y(s) = \frac{C}{s^3 e^{s^2/16}} + \frac{1}{s^3 e^{s^2/16}} \cdot \int s^3 \cdot e^{s^2/16} \cdot \frac{3}{8s^2}\, ds$$

$$Y(s) = \frac{C}{s^3 e^{s^2/16}} + \frac{1}{s^3 e^{s^2/16}} \cdot \frac{3}{8} \int s\, e^{s^2/16}\, ds$$

$$Y(s) = \frac{C}{s^3 e^{s^2/16}} + \frac{1}{s^3 e^{s^2/16}} \cdot 3\, e^{s^2/16}$$

$$Y(s) = \frac{C}{s^3 e^{s^2/16}} + \frac{3}{s^3}$$

Para $C=0$ temos $Y(s) = \frac{3}{s^3}$ e

$$y(t) = \mathcal{L}^{-1}\left\{\frac{3}{s^3}\right\}(t) = \frac{3}{2} t^2$$

$y' = 3t$, $y'' = 3 \implies 3 - 8t \cdot (3t) + 16 \cdot \frac{3}{2} t^2$

$= 3$ (satisfaz EDO e tb as condições iniciais)