Prova III - Cálculo C - 2017/01

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Nome:

Resolva todas as questões abaixo. **Soluções sem justificativas e/ou cálculos não serão consideradas.**

2,0

1. Encontre a solução geral da EDO:

$$y''(x) + 9y(x) = 9\sec^2(3x), \quad 0 < x < \pi/6.$$

2,5

2. Encontre uma família de soluções a um parâmetro para a EDO.

$$2yxe^{y+x^2}dx + (2e^{y+x^2} + ye^{y+x^2} + 2)dy = 0.$$

3,0

3. *Usando a transformada de Laplace*, resolva o PVI.

$$y''(t) + 2y'(t) + 2y(t) = f(t), \quad y(0) = 0,\ y'(0) = 0,$$

sendo

$$f(t) = \begin{cases} 0, & 0 < t < \pi \\ e^t \cos(t), & \pi < t < 2\pi \\ 0, & t > 2\pi \end{cases}.$$

2,5

4. Calcule

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}(2s+3)}{s^2+6s+10}\right\}(t).$$

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Resolução P3 - Cálculo C - 2017/01

1) $y''(x) + 9y(x) = 9\sec^2(3x), \quad 0 < x < \pi/6$

$\rightsquigarrow y'' + 9y = 0 \Rightarrow \lambda^2 + 9 = 0 \Rightarrow y_1(x) = \cos(3x), \quad y_2(x) = \text{sen}(3x)$

$W(y_1, y_2) = \begin{vmatrix} \cos(3x) & \text{sen}(3x) \\ -3\text{sen}(3x) & 3\cos(3x) \end{vmatrix} = 3$

$\rightarrow$ Sol. particular: $y_p = u_1 y_1 + u_2 y_2$

$u_1' = \begin{vmatrix} 0 & \text{sen}(3x) \\ 9\sec^2(3x) & 3\cos(3x) \end{vmatrix} / 3 \Rightarrow u_1' = \dfrac{-9\sec^2(3x)\cdot\text{sen}(3x)}{3} = \dfrac{-3\cdot\text{sen}(3x)}{\cos^2(3x)}$

$u_1 = \dfrac{3\cdot\cos^{-1}(3x)}{3\cdot(-1)} = -\sec(3x)$

$u_2' = \begin{vmatrix} \cos(3x) & 0 \\ -3\text{sen}(3x) & 9\sec^2(3x) \end{vmatrix} / 3 = \dfrac{9\sec^2(3x)\cdot\cos(3x)}{3} = 3\cdot\dfrac{1}{\cos(3x)} = 3\cdot\sec(3x)$

$u_2 = \ln|\sec(x) + tg(x)|$

$\rightsquigarrow$ Sol. geral: $y(x) = C_1\cos(3x) + C_2\text{sen}(3x) - \sec(3x)\cdot\cos(3x)$
$+ \text{sen}(3x)\ln|\sec(x) + tg(x)|$

2) $M(x,y) = 2yxe^{y+x^2} \Rightarrow M_y = 2x\left[ye^{y+x^2} + e^{y+x^2}\right]$

$N(x,y) = 2e^{x^2+y} + ye^{x^2+y} + 2 \Rightarrow N_x = 4xe^{x^2+y} + 2xye^{x^2+y}$

$+\dfrac{(M_y - N_x)}{M} = \dfrac{+2xe^{y+x^2}}{2yxe^{y+x^2}} = \dfrac{1}{y} \Rightarrow \dfrac{1}{\beta}\dfrac{d\beta}{dy} = \dfrac{1}{y} \Rightarrow \ln|\beta| = \ln|y|$

(EDO quase-exata) $\quad \boxed{\beta = y}$

Multiplicando por y: $\underbrace{2y^2xe^{y+x^2}}_{P}dx + \underbrace{(2ye^{y+x^2} + y^2e^{y+x^2} + 2y)}_{Q}dy = 0$

$P_y = 4yx e^{y+x^2} + 2y^2 x e^{y+x^2}$, $Q_x = 4xy e^{y+x^2} + 2xy^2 e^{y+x^2}$

$P_y = Q_x \Rightarrow$ EDO exata

$f_x = 2y^2 x e^{y+x^2}$, $f_y = 2y e^{y+x^2} + y^2 e^{y+x^2} + 2y$

$\int f_x \, dx = y^2 \int 2x e^y \cdot e^{x^2} dx = y^2 \cdot e^y \int 2x e^{x^2} dx$, $u = x^2$, $du = 2x\,dx$

$f(x,y) = y^2 e^y \cdot e^{x^2} + h(y)$

$f_y = 2y e^{y+x^2} + y^2 e^{y+x^2} + h'(y) = 2y e^{y+x^2} + y^2 e^{y+x^2} + 2y$

$h'(y) = 2y \Rightarrow h(y) = y^2 + C$

Logo $f(x,y) = y^2 e^{y+x^2} + y^2 + C$

Sol. da EDO: $y^2 e^{y+x^2} + y^2 = \text{const}$

3 $f(t) = (u_\pi(t) - u_{2\pi}(t)) e^t \cos(t) = u_\pi(t) e^{t-\pi} \cdot e^{\pi} \cdot \cos(t-\pi+\pi)$
$- u_{2\pi}(t) e^{t-2\pi} \cdot e^{2\pi} \cos(t-2\pi+2\pi)$

$f(t) = -e^{\pi} u_\pi(t) e^{t-\pi} \cos(t-\pi) - e^{2\pi} u_{2\pi}(t) e^{t-2\pi} \cos(t-2\pi)$

$F(s) = \mathcal{L}\{f(t)\}(s) = -e^{\pi} \cdot e^{-\pi s} \cdot \dfrac{s-1}{(s-1)^2+1} - e^{2\pi} e^{-2\pi s} \left(\dfrac{s-1}{(s-1)^2+1}\right)$

Aplicando $\mathcal{L}$ na EDO: $s^2 Y(s) + 2s Y(s) + 2Y(s) = F(s)$

$Y(s) = \dfrac{1}{(s^2+2s+2)} \cdot F(s) \Rightarrow y(t) = \mathcal{L}^{-1}\left\{\dfrac{F(s)}{s^2+2s+2}\right\}(t)$

$$\rightsquigarrow \mathcal{L}^{-1}\left\{\left(\frac{s-1}{(s-1)^2+1}\right)\cdot\left(\frac{1}{(s+1)^2+1}\right)\right\}(t) = (f*g)(t), \text{ sendo}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+1}\right\}(t) = e^{t}\cos(t), \quad g(t) = e^{-t}\,\text{sen}(t)$$

$$(f*g)(t) = \int_0^t f(u)\,g(t-u)\,dt = \int_0^t e^{u}\cos(u)\cdot e^{-(t-u)}\cdot\text{sen}(t-u)\,du$$

$$= e^{-t}\int_0^t e^{2u}\cos(u)\cdot\text{sen}(t-u)\,du$$

$$\text{sen}\,a\cdot\cos b = \frac{1}{2}\left[\text{sen}(a+b)+\text{sen}(a-b)\right]$$

$$(f*g)(t) = e^{-t}\int_0^t e^{2u}\cdot\frac{1}{2}\left[\text{sen}(t)+\text{sen}(t-2u)\right]du$$

$$= \frac{1}{2}e^{-t}\,\text{sen}(t)\int_0^t e^{2u}\,du + \frac{1}{2}\int_0^t e^{-(t-2u)}\cdot\text{sen}(t-2u)\,du$$

$$= \frac{1}{2}e^{-t}\,\text{sen}(t)\cdot\frac{1}{2}e^{2u}\Big|_0^t + \frac{1}{2}\int_0^t e^{-(t-2u)}\,\text{sen}(t-2u)\,du$$

$y = t-2u$, $dy = -2\,du$

$$= \frac{1}{4}e^{-t}\,\text{sen}(t)\,(e^{2t}-1) + \frac{1}{2}\left(\frac{-1}{2}\right)\int_t^{-t} e^{-y}\,\text{sen}(y)\cdot dy$$

$$\int e^{-y}\,\text{sen}(y)\,dy = -\cos y\cdot e^{-y} - \int \cos y\cdot e^{-y}\,dy$$

$u = e^{-y}$ $\quad dv = \text{sen}\,y\,dy$

$du = -e^{-y}\,dy$ $\quad v = -\cos y$

$u = e^{-y}$ $\quad dv = \cos y\,dy$

$du = -e^{-y}\,dy$ $\quad v = \text{sen}(y)\,dy$

$$\int e^{-y}\,\text{sen}(y)\,dy = -\cos y\cdot e^{-y} - \left[e^{-y}\,\text{sen}(y) + \int e^{-y}\,\text{sen}(y)\,dy\right] + C$$

$$\int_t^{-t} e^{-y}\,\text{sen}(y) = \frac{1}{2}\left[-\cos y\, e^{-y} - e^{-y}\,\text{sen}(y)\right] = -\frac{1}{2}\left[\begin{array}{l} e^{+t}(\cos(-t)+\text{sen}(-t)) \\ -e^{-t}(\cos(t)+\text{sen}(t)) \end{array}\right]$$

$$\int_t^{-t} e^{y}\,\text{sen}(y)\,dy = -\frac{1}{2}\left[e^{t}\cos(t) - e^{t}\,\text{sen}(t) - e^{-t}\cos(t) - e^{-t}\,\text{sen}(t)\right]$$

$$(f*g)(t) = \frac{1}{4} e^{-t} \operatorname{sen}(t)\left(e^{2t} - 1\right) - \frac{1}{4} \cdot \left(\frac{-1}{2}\right)\left(e^{t}\cos(t) - e^{t}\operatorname{sen}(t) - e^{-t}\cos(t) - e^{-t}\operatorname{sen}(t)\right)$$

$$(f*g)(t) = e^{-t}\left[-\frac{1}{4}\operatorname{sen}(t) - \frac{1}{8}\operatorname{sen}(t) - \frac{1}{8}\cos(t)\right] + e^{t}\left[\frac{1}{4}\operatorname{sen}(t) + \frac{1}{8}\cos(t) - \frac{1}{8}\operatorname{sen}(t)\right]$$

$$(f*g)(t) = -e^{-t}\left[\frac{1}{8}\cos(t) + \frac{3}{8}\operatorname{sen}(t)\right] + e^{t}\left[\frac{1}{8}\cos(t) + \frac{1}{8}\operatorname{sen}(t)\right]$$

Assim

$$y(t) = -e^{\pi} u_{\pi}(t)\left\{-e^{-(t-\pi)}\left[\frac{1}{8}\cos(t-\pi) + \frac{3}{8}\operatorname{sen}(t-\pi)\right] + e^{t-\pi}\left[\frac{1}{8}\cos(t-\pi) + \frac{1}{8}\operatorname{sen}(t-\pi)\right]\right\}$$

$$-e^{2\pi} u_{2\pi}(t)\left\{e^{-(t-2\pi)}\left[\frac{1}{8}\cos(t) + \frac{3}{8}\operatorname{sen}(t)\right] + e^{t-2\pi}\cdot\left[\frac{1}{8}\cos(t) + \frac{1}{8}\operatorname{sen}(t)\right]\right\}$$

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$$\frac{2s+3}{s^2+6s+10} = \frac{2s+3}{(s+3)^2+1} = 2\left(\frac{s+3/2}{(s+3)^2+1}\right) = 2\left(\frac{s+3}{(s+3)^2+1} - \frac{3/2}{(s+3)^2+1}\right)$$

$$\frac{2s+3}{s^2+6s+10} = 2\left(\frac{s+3}{(s+3)^2+1}\right) - 3\left(\frac{1}{(s+3)^2+1}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{2s+3}{s^2+6s+10}\right\}(t) = 2e^{-3t}\cos(t) - 3e^{-3t}\operatorname{sen}(t)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}(2s+3)}{s^2+6s+10}\right\}(t) = 2u_1(t)e^{-3(t-1)}\cos(t-1)$$

$$-3u_1(t)e^{-3(t-1)}\operatorname{sen}(t-1)$$