

Lista 6

Integración función racionales de seno 0 e coseno 01

Ejercicios, (Leitnadel Pg. 569)

$$1. \int \frac{3 dx}{8+7\cos x}$$

$$z = \frac{1}{2}x \rightarrow dx = \frac{2dz}{1+z^2}$$

$$= \int \frac{3 \frac{2dz}{1+z^2}}{8+7 \frac{1-z^2}{1+z^2}}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{6dz}{\frac{8+8z^2+7-7z^2}{1+z^2}} = \int \frac{6dz}{z^2+15}$$

$$= 6 \int \frac{dz}{z^2+\sqrt{15}^2}$$

$$= 6 \frac{1}{\sqrt{15}} \arctan \frac{z}{\sqrt{15}} + C$$

$$= \frac{6}{\sqrt{15}} \arctan \left(\frac{1}{\sqrt{15}} \arctan \frac{x}{2} \right) + C //$$

$$2. \int \frac{dx}{1+\sin x}$$

$$z = \frac{1}{2}x \rightarrow dx = \frac{2dz}{1+z^2}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$= \int \frac{2dz}{\frac{1+z^2}{1+z^2} + \frac{2z}{1+z^2}} = \int \frac{2dz}{z^2+z+1}$$

$$= \int \frac{2dz}{(z+1/2)^2 + 3/4}$$

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$$= \int \frac{2 dz}{(z+1)^2} = 2 \frac{1}{-(z+1)} + C$$

$$= -\frac{2}{z+1} + C$$

$$\int \frac{dx}{1+\sin x} = -\frac{2}{\sqrt{z}+1} + C$$

3. $\int \frac{3 dx}{7+8\cos x}$ $z = \sqrt{z} \rightarrow dx = \frac{2dz}{1+z^2}$

$$= \int \frac{3 \frac{2dz}{1+z^2}}{7+8 \frac{1-z^2}{1+z^2}}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{\frac{6dz}{1+z^2}}{\frac{7+7z^2+8-z^2}{1+z^2}} = \int \frac{6dz}{15-z^2}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$= 6 \int \frac{dz}{\sqrt{15-z^2}}$$

$$= \frac{6}{2\sqrt{15}} \ln \left| \frac{3+\sqrt{15}}{3-\sqrt{15}} \right| + C$$

$$\int \frac{3 dx}{7+8\cos x} = \frac{3}{\sqrt{15}} \ln \left| \frac{\sqrt{z}+1+\sqrt{15}}{\sqrt{z}+1-\sqrt{15}} \right| + C$$

$$4_0 \int \frac{dx}{\sin x - \cos x + 2} \quad z = \tan \frac{x}{2}, \quad dx = \frac{2dz}{1+z^2}$$

$$= \int \frac{2dz}{1+z^2} \quad \sin x = \frac{2z}{1+z^2}$$

$$\frac{2z}{1+z^2} - \frac{1-z^2}{1+z^2} + 2 \quad \cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{2dz}{\frac{2z}{1+z^2} - \frac{1-z^2}{1+z^2} + 2}$$

$$= \int \frac{2dz}{3z^2 + 2z + 1}$$

$$= \int \frac{2dz}{3(z^2 + \frac{2z}{3} + \frac{1}{3})} = \frac{2}{3} \int \frac{dz}{z^2 + \frac{2z}{3} + \frac{1}{3}}$$

$$\frac{1}{q} + \frac{1}{q} = \frac{1}{3} \quad = \frac{2}{3} \int \frac{dz}{(z + \frac{1}{3})^2 + \frac{2}{9}} \quad z + \frac{1}{3} = u$$

$$x = \frac{1}{3} - \frac{1}{q}$$

$$= \frac{2}{3}$$

$$= \frac{2}{3} \int \frac{du}{u^2 + \frac{2}{9}}$$

$$= \frac{2}{3} \int \frac{du}{u^2 + (\frac{\sqrt{2}}{3})^2}$$

$$= \frac{2}{3} \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1} \frac{u}{\frac{\sqrt{2}}{3}}$$

$$= \sqrt{2} \tan^{-1} \frac{3u}{\sqrt{2}}$$

$$= \sqrt{2} \tan^{-1} \frac{3z+1}{\sqrt{2}} + C \rightarrow$$

$$= \sqrt{2} \log^{-1} \frac{3z+1}{2} + C$$

$$\int \frac{dx}{\sin x - \cos x + 2} = \sqrt{2} \log^{-1} \left(\frac{3 + \sqrt{2}x}{2} \right) + C$$

$$5. \int \frac{dx}{\sin x + \sqrt{2}x}$$

$$z = \sqrt{2}x \rightarrow dx = \frac{dz}{\sqrt{2}}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{\frac{dz}{\sqrt{2}}}{\frac{2z}{1+z^2} + \frac{2z}{1-z^2}}$$

$$= \int \frac{\frac{dz}{\sqrt{2}}}{\frac{2z(1-z^2) + 2z(1+z^2)}{(1+z^2)(1-z^2)}}$$

$$\sin x = \frac{2z}{1+z^2}$$

$$= \int \frac{2(1-z^2)}{2z(1-z^2) + 2z(1+z^2)} dz$$

$$= \int \frac{2(1-z^2)}{2z - 2z^3 + 2z + 2z^3} dz$$

$$= \int \frac{2(1-z^2)}{4z} dz = \frac{1}{2} \int \frac{(1-z^2)}{z} dz$$

$$= \frac{1}{2} \int \frac{dz}{z} = \frac{1}{2} \int \frac{dz}{z}$$

$$= \frac{1}{2} \ln|z| - \frac{1}{2} \frac{z^2}{2} + C \rightarrow$$

$$= \frac{1}{4} \ln |z| - \frac{z^2}{4} + C$$

$$\int \frac{dx}{\sin x + \frac{1}{2}x} = \frac{1}{2} \ln \left| \frac{1}{2}x \right| - \frac{1}{4} \frac{1}{2}x^2 + C$$

$$6. \int \frac{dx}{5+4\cos x} \quad \theta = \frac{x}{2}$$

$$= \int \frac{2d\theta}{5+4\frac{1-\theta^2}{1+\theta^2}} = \int \frac{2d\theta}{\frac{5+5\theta^2+4-4\theta^2}{1+\theta^2}}$$

$$= \int \frac{2d\theta}{\theta^2+9}$$

$$= 2 \int \frac{d\theta}{\theta^2+3^2}$$

$$= 2 \frac{1}{3} \tan^{-1} \frac{\theta}{3} + C$$

$$\int \frac{dx}{5+4\cos x} = \frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan \frac{x}{2} \right) + C$$

$$7. \int \frac{dx}{3-5\sin x}$$

$$= \int \frac{2dz}{\frac{3-5\frac{2z}{1+z^2}}{1+z^2}} = \int \frac{2dz}{\frac{3+3z^2-10z}{1+z^2}}$$

$$= \int \frac{2dz}{3z^2-10z+3}$$

$$= \int \frac{2dz}{3(z^2-\frac{10}{3}z+1)}$$

$$= \frac{2}{3} \int \frac{dz}{z^2-\frac{10}{3}z+1}$$

$$= \frac{2}{3} \int \frac{dz}{(z-\frac{5}{3})^2 - \frac{16}{9}} \quad ; \quad z-\frac{5}{3} = u$$

$$= \frac{2}{3} \int \frac{du}{u^2 - (\frac{4}{3})^2}$$

$$= \frac{2}{3} \left(\frac{1}{\frac{24}{3}} \ln \left| \frac{u-\frac{4}{3}}{u+\frac{4}{3}} \right| \right)$$

$$= \frac{2}{8} \left(\frac{8}{8} \ln \left| \frac{z-\frac{5}{3}-\frac{4}{3}}{z-\frac{5}{3}+\frac{4}{3}} \right| \right) + C$$

$$= \frac{1}{4} \ln \left| \frac{z-3}{z-\frac{1}{3}} \right| + C \quad \rightarrow$$

$$\frac{25}{9} + x = 1$$

$$x = 1 - \frac{25}{9}$$

$$= \frac{9-25}{9} = -\frac{16}{9}$$

$$\int \frac{du}{u^2 - a^2} =$$

$$= \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right|$$

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$$\int \frac{dx}{3-5\sin x} = \frac{1}{4} \ln \left| \frac{t - \frac{1}{3} - 5}{t - \frac{1}{3} - \frac{1}{3}} \right|$$

8. $\int \frac{dx}{4\sin x - 3\cos x}$

$$= \int \frac{2dz}{1+z^2}$$

$$\frac{4z^2 - 3}{1+z^2} = \frac{1-3z^2}{1+z^2}$$

$$= \int \frac{2dz}{1+z^2}$$

$$\frac{8z}{1+z^2} + \frac{-3+3z^2}{1+z^2}$$

$$= \int \frac{2dz}{8z - 3 + 3z^2}$$

$$= \int \frac{2dz}{3z^2 + 8z - 3}$$

$$= \frac{2}{3} \int \frac{dz}{z^2 + \frac{8}{3}z - 1}$$

$$\frac{16+k}{9} = -1$$

$$k = -1 - \frac{16}{9}$$

$$= -\frac{25}{9}$$

$$= \frac{2}{3} \int \frac{dz}{\left(z + \frac{4}{3}\right)^2 - \frac{25}{9}} \rightarrow$$

$$= \frac{1}{3} \int \frac{dz}{\left(3 + \frac{4}{3}\right)^2 - \frac{25}{9}} \quad u = z + \frac{4}{3}$$

$$= \frac{1}{3} \int \frac{du}{u^2 - \left(\frac{5}{3}\right)^2} \quad \left\{ \int \frac{du}{u^2 - a^2} = \right.$$

$$= \frac{1}{3} \frac{1}{\frac{25}{9}} \ln \left| \frac{u - \frac{5}{3}}{u + \frac{5}{3}} \right| = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right|$$

$$= \frac{1}{1} \ln \left| \frac{z + \frac{4}{3} - \frac{5}{3}}{z + \frac{4}{3} + \frac{5}{3}} \right|$$

$$= \frac{1}{1} \ln \left| \frac{z - \frac{1}{3}}{z + 3} \right|$$

$$= \frac{1}{1} \ln \left| \frac{z - \frac{1}{3}}{z + 3} \right| + C //$$

Cálculo B - Lista 6

1. $\int \frac{3dz}{8+7 \cos z}$
2. $\int \frac{dx}{1+\sin x}$
3. $\int \frac{3dx}{7+8 \cos x}$
4. $\int \frac{dx}{\sin x - \cos x + 2}$
5. $\int \frac{dx}{\sin x + \tan x}$
6. $\int \frac{dx}{5+4 \cos x}$
7. $\int \frac{dx}{3-5 \sin x}$
8. $\int \frac{dx}{4 \sin x - 3 \cos x} dx$

Respostas

1. $\frac{6}{\sqrt{15}} \arctan\left(\frac{1}{\sqrt{15}} \tan \frac{x}{2}\right) + C$
2. $\frac{-2}{1+\tan \frac{x}{2}} + C$
3. $\frac{3}{\sqrt{15}} \ln \left| \frac{\sqrt{15} + \tan \frac{x}{2}}{-\sqrt{15} + \tan \frac{x}{2}} \right| + C$
4. $\sqrt{2} \arctan\left(\frac{1+3 \tan \frac{x}{2}}{\sqrt{2}}\right) + C$
5. $\frac{1}{2} \ln |\tan \frac{x}{2}| - \frac{1}{4} \tan^2 \frac{x}{2} + C$
6. $\frac{2}{3} \arctan\left(\frac{1}{3} \tan \frac{x}{2}\right) + C$
7. $\frac{1}{4} \ln \left| \frac{-3 + \tan \frac{x}{2}}{-\frac{1}{3} + \tan \frac{x}{2}} \right| + C$
8. $\frac{1}{5} \ln \left| \frac{-\frac{1}{3} + \tan \frac{x}{2}}{3 + \tan \frac{x}{2}} \right| + C$