

Cálculo B - Lista 4

1. $\int \frac{dx}{x^2-4}$
2. $\int \frac{x^2 dx}{x^2+x-6}$
3. $\int \frac{5x-2}{x^2-4} dx$
4. $\int \frac{(4x-2) dx}{x^3-x^2-2x}$
5. $\int \frac{4w-11}{2w^2+7w-4} dw$
6. $\int \frac{9t^2-26t-5}{3t^2-5t-2} dt$
7. $\int \frac{6x^2-2x-1}{4x^3-x} dx$
8. $\int \frac{x^2+x+2}{x^2-1} dx$
9. $\int \frac{dx}{x^3+3x^2}$
10. $\int \frac{x^2+4x-1}{x^3-x} dx$
11. $\int \frac{dx}{x^2(x+1)^2}$
12. $\int \frac{3x^2-x+1}{x^3-x^2} dx$
13. $\int \frac{x^2-3x-7}{(2x+3)(x+1)^2} dx$
14. $\int \frac{dt}{(t+2)^2(t+1)}$
15. $\int \frac{3z+1}{(z^2-4)^2} dz$
16. $\int \frac{(5x^2-11x+5)}{x^3-4x^2+5x-2} dx$
17. $\int \frac{x^4+3x^3-5x^2-4x+17}{x^3+x^2-5x+3} dx$
18. $\int \frac{2x^4-2x+1}{2x^5-x^4} dx$
19. $\int \frac{-24x^3+30x^2+52x+17}{9x^4-6x^3-11x^2+4x+4} dx$
20. $\int \frac{dx}{16x^4-8x^2+1}$

Respostas

1. $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C$
2. $x - \frac{9}{5} \ln |x + 3| + \frac{4}{5} \ln |x - 2| + C$
3. $2 \ln |x - 2| + 3 \ln |x + 2| + C$
4. $\ln |x| + \ln |x - 2| - 2 \ln |x + 1| + C$
5. $\ln \left| \frac{(w+4)^3}{2w-1} \right| + C$
6. $3t - 3 \ln |t - 2| - \frac{2}{3} \ln \left| t + \frac{1}{3} \right| + C$
7. $\frac{1}{4} \ln \left| \frac{x^4(2x+1)^3}{2x-1} \right| + C$
8. $x + 2 \ln |x - 1| - \ln |x + 1| + C$
9. $-\frac{1}{3x} + \frac{1}{9} \ln \left| \frac{x+3}{x} \right| + C$
10. $\ln \left| \frac{x(x-1)^2}{(x+1)^2} \right| + C$
11. $-\frac{1}{x} - \frac{1}{x+1} + 2 \ln \left| \frac{x+1}{x} \right| + C$
12. $\frac{1}{x} + 3 \ln |x - 1| + C$
13. $\frac{3}{x+1} + \ln |x + 1| - \frac{1}{2} \ln |2x + 3| + C$
14. $\ln \left| \frac{t+1}{t+2} \right| + \frac{1}{t+2} + C$
15. $\frac{1}{32} \ln \left| \frac{z+2}{z-2} \right| + \frac{5}{16} \frac{1}{z+2} - \frac{7}{16} \frac{1}{z-2} + C$
16. $\frac{1}{1-x} + 2 \ln |x - 1| + 3 \ln |x - 2| + C$
17. $\frac{1}{2}x^2 + 2x - \frac{3}{x-1} - \ln |x^2 + 2x - 3| + C$
18. $\frac{1}{3x^3} + \ln |2x - 1| + C$
19. $-\frac{3}{x-1} - \frac{1}{3(3x+2)} - \ln((3x + 2)^{2/3}(x - 1)^2) + C$
20. $-\frac{1}{8(2x+1)} - \frac{1}{8(2x-1)} + \frac{1}{8} \ln \left| \frac{2x+1}{2x-1} \right| + C$

Lista 4 - Calculo B

Integração dos quocientes racionais por
fracções parciais: Denominador tem somente
factores lineares

Leithold pg. 560

1. $\int \frac{dx}{x^2-4}$

$$\frac{P(x)}{Q(x)} = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$\frac{1}{(x-2)(x+2)} = \frac{A(x+2) + B(x-2)}{(x-2)(x+2)}$$

$$\begin{cases} 1 = A(x+2) + B(x-2) \end{cases}$$

$$x=2 : 1 = 4A \Rightarrow A = 1/4$$

$$x=-2 : 1 = -4B \Rightarrow B = -1/4$$

$$\frac{1}{x^2-4} = \frac{1}{4(x-2)} - \frac{1}{4(x+2)}$$

$$\int \frac{dx}{x^2-4} = \int \frac{dx}{4(x-2)} - \int \frac{dx}{4(x+2)}$$

$$= \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C$$

Polynom - Division

$$\int \frac{dx}{x^2-4} = \frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + C$$

2. $\int \frac{x^2}{x^2+x-6} dx =$

$$\frac{x^2}{x^2+x-6} \Rightarrow \begin{array}{r} x^2 \\ -x^2 - x + 6 \\ \hline -x + 6 \end{array} \Bigg| \frac{x^2+x-6}{1}$$

$\frac{p}{q} = M + \frac{r}{q}$

$$\therefore \frac{x^2}{x^2+x-6} = 1 + \frac{-x+6}{x^2+x-6} \quad (*)$$

Mos

$$\begin{aligned} x^2+x-6 &= 0 \\ x &= \frac{-1 \pm \sqrt{1+24}}{2} \\ &= \frac{-1 \pm 5}{2} \\ &= -3 \text{ oder } 2 \end{aligned}$$

$$\frac{-x+6}{x^2+x-6} = \frac{-x+6}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$\begin{aligned} -x+6 &= A(x-2) + B(x+3) \\ x=2: \quad -2+6 &= A \cdot 0 + B \cdot 5 \\ 4 &= 5B \Rightarrow B = \frac{4}{5} \\ x=-3: \quad -(-3)+6 &= -5A \\ 9 &= -5A \Rightarrow A = -\frac{9}{5} \end{aligned}$$

$$\frac{-x+6}{(x+3)(x-2)} = \frac{-\frac{9}{5}}{5(x+3)} + \frac{\frac{4}{5}}{5(x-2)} \quad (**)$$

⊗ → ⊗ :

$$\frac{x^2}{x^2+x-6} = 1 - \frac{9}{5(x+3)} + \frac{4}{5(x-2)}$$

$$\int \frac{x^2}{x^2+x-6} dx = \int dx - \frac{9}{5} \int \frac{dx}{x+3} + \frac{4}{5} \int \frac{dx}{x-2}$$

$$= x - \frac{9}{5} \ln|x+3| + \frac{4}{5} \ln|x-2| + C$$

$$\int \frac{x^2}{x^2+x-6} dx = x - \frac{9}{5} \ln|x+3| + \frac{4}{5} \ln|x-2| + C$$

3. $\int \frac{5x-2}{x^2-4} dx$

$$\frac{5x-2}{x^2-4} = \frac{5x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$5x-2 = A(x+2) + B(x-2)$$

$$x = -2: \quad -12 = -4B \Rightarrow B = 3$$

$$x = 2: \quad 8 = 4A \Rightarrow A = 2$$

$$\frac{5x-2}{(x-2)(x+2)} = \frac{2}{x-2} + \frac{3}{x+2} \Rightarrow$$

$$\int \frac{5x-2}{x^2-4} dx = \int \frac{2}{x-2} dx + \int \frac{3}{x+2} dx$$

$$= 2 \ln|x-2| + 3 \ln|x+2| + C //$$

$$= \ln|(x-2)^2(x+2)^3| + \ln C'$$

$$= \ln|(x-2)^2(x+2)^3 C'|$$

$$= \ln|(x-2)^2(x+2)^3 C'| //$$

4. $\int \frac{(4x-2)}{x^3-x^2-2x} dx$

$$\frac{4x-2}{x^3-x^2-2x} = \frac{4x-2}{x(x^2-x-2)}$$

$$x^2-x-2=0$$

$$x = \frac{1 \pm \sqrt{1+8}}{2}$$

$$= \frac{1 \pm 3}{2}$$

$$x=2, x=-1$$

$$= \frac{4x-2}{x(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$4x-2 = A(x-2)(x+1) + Bx(x+1) + Cx(x-2)$$

$$x=2 : 6 = B \cdot 2 \cdot 3 \Rightarrow B=1$$

$$x=-1 : -6 = C(-1)(-3) \Rightarrow C=-2$$

$$x=0 : -2 = A(-2) \cdot 1 \Rightarrow A=1$$

$$\frac{4x-2}{x(x-2)(x+1)} = \frac{1}{x} + \frac{1}{x-2} - \frac{2}{x+1}$$

$$\int \frac{4x-2}{x^3-x^2-2x} dx = \int \frac{4x-2}{x(x-2)(x+1)} dx$$

$$= \int \frac{1}{x} dx + \int \frac{1}{x-2} dx - \int \frac{2}{x+1} dx$$

$$\int \frac{4x-2}{x^3-x^2-2x} dx = \ln|x| + \ln|x-2| - 2\ln|x+1| + C$$

5. $\int \frac{4w-11}{2w^2+7w-4} dw$

$$\frac{4w-11}{2w^2+7w-4} = \frac{4w-11}{2(w+4)(w-\frac{1}{2})} = \frac{A}{w+4} + \frac{B}{w-\frac{1}{2}}$$

$$2w^2+7w-4=0$$

$$w = \frac{-7 \pm \sqrt{49+32}}{4}$$

$$= \frac{-7 \pm \sqrt{81}}{4}$$

$$= \frac{-7 \pm 9}{4}$$

$$= -4, \frac{1}{2}$$

$$4w-11 = 2A(w-\frac{1}{2}) + 2B(w+4)$$

$$w = \frac{1}{2} : -9 = 2B \frac{9}{2} \Rightarrow B = -1$$

$$w = -4 : -27 = 2A(-\frac{9}{2}) \Rightarrow A = 3$$

$$\therefore \frac{4w-11}{2(w+4)(w-\frac{1}{2})} = \frac{3}{w+4} - \frac{1}{w-\frac{1}{2}}$$

$$\int \frac{4w-11}{2w^2+7w-4} dw = \int \frac{3}{w+4} dw - \int \frac{1}{w-\frac{1}{2}} dw$$

$$= 3 \ln|w+4| - 1 \ln|w-\frac{1}{2}| + C //$$

$$|x||y| = |xy|$$

$$= \ln \left| \frac{(w+4)^3}{w-\frac{1}{2}} \right| + C$$

$$= \ln \left| \frac{(w+4)^3}{2w-1} \right| + C$$

$$= \ln \left| \frac{C(w+4)^3}{2w-1} \right| //$$

$$6. \int \frac{9x^2-26x-5}{3x^2-5x-2} dx$$

$$\begin{array}{r|l} 9x^2-26x-5 & 3x^2-5x-2 \\ -9x^2+15x+6 & \\ \hline & -11x+1 \end{array}$$

$$\frac{p}{q} = m + \frac{r}{q}$$

$$\therefore \frac{9x^2-26x-5}{3x^2-5x-2} = 3 + \frac{-11x+1}{3x^2-5x-2}$$

MoS

$$3x^2-5x-2=0$$

$$x = \frac{5 \pm \sqrt{25+24}}{6}$$

$$= \frac{5 \pm \sqrt{49}}{6}$$

$$= \frac{5 \pm 7}{6}$$

$$= 2, -\frac{1}{3}$$

$$3x^2-5x-2 = 3(x-2)(x+\frac{1}{3})$$

$$\frac{9x^2-26x-5}{3x^2-5x-2} = 3 + \frac{-11x+1}{3(x-2)(x+\frac{1}{3})} \quad (*)$$

Mo) $\frac{6x^2 - 26x - 5}{3(x-2)(x+\frac{1}{3})} = \frac{A}{x-2} + \frac{B}{x+\frac{1}{3}}$

$$\frac{-11x + 1}{3(x-2)(x+\frac{1}{3})} = \frac{A}{x-2} + \frac{B}{x+\frac{1}{3}}$$

$$\Leftrightarrow \frac{-11x + 1}{(x-2)(x+\frac{1}{3})} = \frac{3A}{x-2} + \frac{3B}{x+\frac{1}{3}}$$

$$-11x + 1 = 3(x+\frac{1}{3})A + 3(x-2)B$$

$$t = -\frac{1}{3} : \frac{11}{3} + 1 = (3(-\frac{1}{3}-2))B$$

$$\frac{14}{3} = 3(-\frac{7}{3})B \Rightarrow \parallel B = -\frac{2}{3} \parallel$$

$$t = 2 : -22 + 1 = 3(2+\frac{1}{3})A$$

$$-21 = 3 \cdot \frac{7}{3} A \Rightarrow \parallel A = -3 \parallel$$

$$\frac{-11x + 1}{3(x-2)(x+\frac{1}{3})} = (-3) \frac{1-3}{x-2} = -3 \frac{-2}{3(x+\frac{1}{3})} \quad (**)$$

(**) \rightarrow (*) :

$$\left\{ \frac{9x^2 - 26x - 5}{3x^2 - 5x - 2} = 3 - \frac{3}{x-2} - \frac{2}{3(x+\frac{1}{3})} \right.$$

$$\therefore \int \frac{9x^2 - 26x - 5}{3x^2 - 5x - 2} dx = \int 3 dx - 3 \int \frac{dx}{x-2} - \frac{2}{3} \int \frac{dx}{x+\frac{1}{3}}$$

$$\int \frac{9x^2 - 26x - 5}{3x^2 - 5x - 2} dx = 3x - 3 \ln|x-2| - \frac{2}{3} \ln|x+\frac{1}{3}| + C$$

$$7. \int \frac{6x^2 - 2x - 1}{4x^3 - x} dx$$

$$\frac{6x^2 - 2x - 1}{4x^3 - x} = \frac{6x^2 - 2x - 1}{x(4x^2 - 1)}$$

$$\begin{aligned} |x| &= 1 \\ 4x^2 - 1 &= 0 \\ 4x^2 &= 1 \\ x &= \pm \frac{1}{2} \end{aligned}$$

$$= \frac{6x^2 - 2x - 1}{x(4(x + \frac{1}{2})(x - \frac{1}{2}))} = \frac{A}{x} + \frac{B}{x + \frac{1}{2}} + \frac{C}{x - \frac{1}{2}}$$

$$6x^2 - 2x - 1 = 4A(x + \frac{1}{2})(x - \frac{1}{2}) + 4Bx(x - \frac{1}{2}) + 4Cx(x + \frac{1}{2})$$

$$x = -\frac{1}{2}: 6\frac{1}{4} - 2(-\frac{1}{2}) - 1 = 4B(-\frac{1}{2})(-1)$$

$$\frac{3}{2} + 1 - 1 = 4\frac{B}{2} \Rightarrow B = \frac{3}{4}$$

$$x = 0: -1 = 4A\frac{1}{2}(-\frac{1}{2}) \Rightarrow A = 1$$

$$x = \frac{1}{2}: 6\frac{1}{4} - 2\frac{1}{2} - 1 = 4C\frac{1}{2} \times 1$$

$$\frac{3}{2} - 2 = 4\frac{C}{2}$$

$$-\frac{1}{2} = 4\frac{C}{2} \Rightarrow C = -\frac{1}{4}$$

$$\int \frac{6x^2 - 2x - 1}{x(x + \frac{1}{2})(x - \frac{1}{2})} = \frac{1}{x} + \frac{3}{4(x + \frac{1}{2})} - \frac{1}{4(x - \frac{1}{2})}$$

$$9. \int \frac{6x^2 - 2x - 1}{x(x + \frac{1}{2})(x - \frac{1}{2})} dx = \int \frac{dx}{x} + \frac{3}{4} \int \frac{dx}{x + \frac{1}{2}} - \frac{1}{4} \int \frac{dx}{x - \frac{1}{2}}$$

$$= \ln|x| + \frac{3}{4} \ln|x + \frac{1}{2}| - \frac{1}{4} \ln|x - \frac{1}{2}| + C$$

or

$$= \frac{1}{4} (4 \ln|x| + 3 \ln|x + \frac{1}{2}| - \ln|x - \frac{1}{2}| + C)$$

$$= \frac{1}{4} \ln \left| \frac{x^4 (x + \frac{1}{2})^3}{x - \frac{1}{2}} \right| + C$$

$$= \frac{1}{4} \ln \left| \frac{x^4 (2x + 1)^3}{x - 1} \right| + C$$

$$8. \int \frac{x^2 + x + 2}{x^2 - 1} dx$$

$$\begin{array}{r} x^2 + x + 2 \\ -x^2 + 1 \\ \hline x + 3 \end{array} \bigg/ \frac{x^2 - 1}{1}$$

$$\therefore \frac{x^2 + x + 2}{x^2 - 1} = 1 + \frac{x + 3}{x^2 - 1}$$

Now

$$\frac{x + 3}{x^2 - 1} = \frac{x + 3}{(x + 1)(x - 1)} = \frac{A}{x + 1} + \frac{B}{x - 1}$$

$$\therefore x + 3 = A(x - 1) + B(x + 1)$$

$$x+3 = A(x-1) + B(x+1)$$

$$x=1 : 4 = 2B \Rightarrow B=2$$

$$x=-1 : 2 = A(-2) \Rightarrow A=-1$$

$$\frac{x+3}{x^2-1} = \frac{-1}{x+1} + \frac{2}{x-1}$$

~~xx~~ \rightarrow ~~x~~ :

$$\frac{x^2+x+2}{x^2-1} = 1 - \frac{1}{x+1} + \frac{2}{x-1}$$

$$\int \frac{x^2+x+2}{x^2-1} dx = \int dx - \int \frac{dx}{x+1} + 2 \int \frac{dx}{x-1}$$

$$\int \frac{x^2+x+2}{x^2-1} dx = x - \ln|x+1| + 2 \ln|x-1| + C$$

$$= x + \ln \left| \frac{(x-1)^2 c}{x+1} \right| //$$

$$9. \int \frac{dx}{x^3+3x^2} = \int \frac{1}{x^2(x+3)} dx = \frac{1}{x^2} = \frac{1-x+5x}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

$$\frac{1}{x^3+3x^2} = \frac{1-x+5x}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

\Leftrightarrow

$$1 = A(x+3) + B(x+3) + Cx^2$$

$$x=0 : 1 = B \cdot 3 \Rightarrow \| B = 1/3 \|$$

$$x=-3 : 1 = C \cdot 9 \Rightarrow \| C = 1/9 \|$$

$$x=1 : 1 = 4A + \frac{4}{3} + \frac{1}{9}$$

$$1 = 4A + \frac{4}{3} + \frac{1}{9}$$

$$-\frac{4}{9} = 4A \Rightarrow \| A = -\frac{1}{9} \|$$

$$1 - \frac{4}{3} - \frac{1}{9} =$$

$$= \frac{9-12-1}{9}$$

$$= -\frac{4}{9}$$

$$\frac{1}{x^3+3x^2} = -\frac{1}{9x} + \frac{1}{3x^2} + \frac{1}{9(x+3)}$$

$$\int \frac{dx}{x^3+3x^2} = -\frac{1}{9} \int \frac{dx}{x} + \frac{1}{3} \int \frac{dx}{x^2} + \frac{1}{9} \int \frac{dx}{x+3}$$

$$= -\frac{1}{9} \ln|x| + \frac{1}{3} \frac{x^{-1}}{-1} + \frac{1}{9} \ln|x+3| + C$$

$$\int \frac{dx}{x^3+3x^2} = -\frac{1}{9} \ln|x| - \frac{1}{3x} + \frac{1}{9} \ln|x+3| + C //$$

atau

$$= -\frac{1}{3x} + \frac{1}{9} \ln \left| \frac{x+3}{x} \right| + C //$$

$$10. \int \frac{x^2+4x-1}{x^3-x} dx + B(x+1) + \frac{Cx}{x+1} \cdot P$$

$$x=1: 4 = 2B \Rightarrow B=2$$

$$\frac{x^2+4x-1}{x^3-x} = \frac{x^2+4x-1}{x(x^2-1)}$$

$$= \frac{x^2+4x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

\Leftrightarrow

$$x^2+4x-1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$x=1: 4 = B \cdot 2 \Rightarrow B=2$$

$$x=-1: -4 = C(-1)(-2) \Rightarrow C=-2$$

$$x=0: -1 = A(-1) \cdot 1 \Rightarrow A=1$$

$$\therefore \frac{x^2+4x-1}{x(x-1)(x+1)} = \frac{1}{x} + \frac{2}{x-1} - \frac{2}{x+1}$$

$$\int \frac{x^2+4x-1}{x(x-1)(x+1)} dx = \int \frac{dx}{x} + 2 \int \frac{dx}{x-1} - 2 \int \frac{dx}{x+1}$$

$$\int \frac{x^2+4x-1}{x^3-x} dx = \ln|x| + 2 \ln|x-1| - 2 \ln|x+1| + C$$

$$= \ln \left| \frac{x(x-1)^2}{(x+1)^2} \right| + C$$

11. $\int \frac{dx}{x^2(x+1)^2}$

$$\frac{1}{x^2(x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

m.m.c.: $x^2(x+1)^2$

$$1 = Ax(x+1)^2 + B(x+1)^2 + Cx^2(x+1) + Dx^2$$

$x=0$: $1 = B$

$x=-1$: $1 = D$

$x=1$: $1 = 4A + 4B + 2C + D$

$1 = 4A + 4 + 2C + 1$

$-2 = 2A + C$

$x=-2$: $1 = A(-2) + B - 4C + 4D$

$1 = -2A + 1 - 4C + 4$

$-4 = -2A - 4C$

$-2 = -A - 2C$

i. 9.)

$$10. \quad -2 = 2A + C \Rightarrow C = -2 - 2A \quad ||$$

$$-2 = -A - 2C \Rightarrow \left. \begin{aligned} -2 &= -A - 2(-2 - 2A) \\ -2 &= -A + 4 + 4A \\ -6 &= 3A \Rightarrow A = -2 \end{aligned} \right\}$$

$$C = -2 - 2(-2)$$

$$C = +2$$

$$\frac{1}{x^2(x+1)^2} = \frac{-2}{x} + \frac{1}{x^2} + \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

$$\int \frac{dx}{x^2(x+1)^2} = -2 \int \frac{dx}{x} + \int \frac{dx}{x^2} + 2 \int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2}$$

$$= -2 \ln|x| + \frac{x^{-1}}{-1} + 2 \ln|x+1| + \frac{(x+1)^{-1}}{-1} + C$$

$$\boxed{\int \frac{dx}{x^2(x+1)^2} = -\frac{1}{x} - \frac{1}{x+1} + 2 \ln \left| \frac{x+1}{x} \right| + C}$$

$$12. \int \frac{3x^2 - x + 1}{x^3 - x^2} dx = \int \frac{3x^2 - x + 1}{x^2(x-1)} dx$$

$$\frac{3x^2 - x + 1}{x^2(x-1)} = \frac{3x^2 - x + 1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$(x-1) \cdot 0 + (1+x) \cdot B + 3x^2 - x + 1 = Ax(x-1) + B(x-1) + Cx^2$$

$$x=1 : 1 = A \quad 3 = C$$

$$x=0 : 1 = -C \quad 1 = -B$$

$$x=2 : 1 = A \quad 11 = 2A + B + 4C$$

$$1 = A \quad 11 = 2A - 1 + 12 \Rightarrow A = 0$$

$$\therefore B = -1 \quad C = 3$$

$$\frac{3x^2 - x + 1}{x^3 - x^2} = \frac{1}{x} - \frac{1}{x^2} + \frac{3}{x-1}$$

$$\int \frac{3x^2 - x + 1}{x^3 - x^2} dx = \int \frac{1}{x} dx - \int \frac{1}{x^2} dx + 3 \int \frac{1}{x-1} dx$$

$$= \ln|x| + \frac{1}{x} + 3 \ln|x-1| + C$$

$$\int \frac{3x^2 - x + 1}{x^3 - x^2} dx = \frac{1}{x} + 3 \ln|x-1| + C$$

$$13. \int \frac{x^2 - 3x - 7}{(2x+3)(x+1)^2} dx$$

$$\frac{x^2 - 3x - 7}{(2x+3)(x+1)^2} = \frac{A}{2x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x^2 - 3x - 7 = A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$$

$$x = -1: \quad -3 = C$$

$$x = -\frac{3}{2}: \quad \frac{9}{4} + \frac{9}{2} - 7 = A\left(-\frac{1}{2}\right)^2$$

$$\frac{9 + 18 - 28}{4} = A \frac{1}{4}$$

$$-\frac{1}{4} = A \frac{1}{4} \Rightarrow A = -1$$

$$x = 0: \quad -7 = A + 3B + 3C$$

$$-7 = -1 + 3B - 9$$

$$3 = 3B \Rightarrow B = 1$$

$$\frac{x^2 - 3x - 7}{(2x+3)(x+1)^2} = \frac{-1}{2x+3} + \frac{1}{x+1} - \frac{3}{(x+1)^2}$$

$$\int \frac{x^2 - 3x - 7}{(2x+3)(x+1)^2} dx = \int \frac{-dx}{2x+3} + \int \frac{dx}{x+1} - 3 \int \frac{dx}{(x+1)^2}$$

$$= -\frac{1}{2} \ln|2x+3| + \ln|x+1| - 3 \frac{(x+1)^{-1}}{-1}$$

$$\int \frac{x^2 - 3x - 7}{(2x+3)(x+1)^2} dx = \frac{3}{x+1} + \ln|x+1| - \frac{1}{2} \ln|2x+3| + C$$

$$14. \int \frac{dt}{(x+2)^2(x+1)}$$

$$\frac{1}{(x+2)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \quad \text{mmc: } (x+2)^2(x+1)$$

$$1 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$$

$$x=-1: \quad 1 = A$$

$$x=-2: \quad 1 = -C$$

$$x=0: \quad \begin{cases} 1 = A \cdot 4 + 2B + C \\ 1 = 4 + 2B - 1 \end{cases}$$

$$-2 = 2B \Rightarrow B = -1$$

$$\frac{1}{(x+2)^2(x+1)} = \frac{1}{x+1} - \frac{1}{x+2} - \frac{1}{(x+2)^2}$$

$$\int \frac{dt}{(x+2)^2(x+1)} = \int \frac{dt}{x+1} - \int \frac{dt}{x+2} - \int \frac{dt}{(x+2)^2}$$

$$= \ln|x+1| - \ln|x+2| - \frac{(x+2)^{-1}}{-1} + C$$

$$\int \frac{dt}{(x+2)^2(x+1)} = \ln \left| \frac{x+1}{x+2} \right| + \frac{1}{x+2} + C$$

$$15. \int \frac{3z+1}{(z^2-4)^2} dz$$

$$\frac{3z+1}{(z^2-4)^2} = \frac{3z+1}{(z-2)^2(z+2)^2} = \frac{A}{z-2} + \frac{B}{(z-2)^2} + \frac{C}{z+2} + \frac{D}{(z+2)^2}$$

$$3z+1 = A(z-2)(z+2)^2 + B(z+2)^2 + C(z-2)^2(z+2) + D(z-2)^2$$

$$z=2: 7 = B \cdot 16 \Rightarrow B = 7/16$$

$$z=-2: -5 = D \cdot 16 \Rightarrow D = -5/16$$

$$z=-1: \begin{cases} -2 = -3A + B + 9C + 9D \\ -2 = -3A + \frac{7}{16} + 9C - \frac{45}{16} \end{cases}$$

$$-2 - \frac{7}{16} + \frac{45}{16}$$

$$-2 + \frac{38}{16}$$

$$= \frac{-32+38}{16}$$

$$= \frac{6}{16} = \frac{3}{8}$$

$$\parallel \frac{3}{8} = -3A + 9C \parallel$$

$$z=0: \begin{cases} 1 = A(-2)4 + B \cdot 4 + C \cdot 4 \cdot 2 + D \cdot 4 \\ 1 = -8A + \frac{7}{4} + 8C - \frac{5}{4} \end{cases}$$

$$\parallel \frac{1}{2} = -8A + 8C \parallel$$

$$\begin{cases} \frac{3}{8} = -3A + 9C \\ \frac{1}{2} = -8A + 8C \end{cases} \Leftrightarrow \begin{cases} \frac{1}{8} = -A + 3C \\ \frac{1}{16} = -A + C \end{cases}$$

$$\begin{cases} \frac{1}{8} = -A + 3C \\ \frac{1}{16} = -A + C \end{cases}$$

$$\frac{1}{8} - \frac{1}{16} = 2C$$

$$\frac{1}{16} = 2C \Rightarrow C = \frac{1}{32}$$

$$\frac{1}{8} = -A + 3C$$

$$= -A + \frac{3}{32} \Rightarrow A = \frac{3}{32} - \frac{1}{8}$$

$$\int \frac{5x^2 - 11x + 5}{x^3 - 9x^2 + 14x - 2} dx = \int \frac{A}{x-2} + \int \frac{B}{x-2}^2 + \int \frac{C}{x+2} - \int \frac{D}{(x+2)^2}$$

$$\frac{3z + 1}{(z-2)^2(z+2)^2} = \frac{-1}{32(z-2)} + \frac{7}{16(z-2)^2} + \frac{1}{32(z+2)} - \frac{5}{16(z+2)^2}$$

$$\int \frac{3z + 1}{(z^2 - 4)^2} dz = -\frac{1}{32} \int \frac{dz}{z-2} + \frac{7}{16} \int \frac{dz}{(z-2)^2} + \frac{1}{32} \int \frac{dz}{z+2} - \frac{5}{16} \int \frac{dz}{(z+2)^2}$$

$$= -\frac{1}{32} \ln|z-2| + \frac{7}{16} \frac{-1}{z-2} + \frac{1}{32} \ln|z+2| - \frac{5}{16} \frac{-1}{z+2} + C$$

$$= \frac{1}{32} \ln \left| \frac{z+2}{z-2} \right| + \frac{5}{16} \frac{1}{z+2} - \frac{7}{16} \frac{1}{z-2} + C$$

$$\int \frac{3z + 1}{(z^2 - 4)^2} dz = \frac{1}{32} \ln \left| \frac{z+2}{z-2} \right| + \frac{5}{16} \frac{1}{z+2} - \frac{7}{16} \frac{1}{z-2} + C$$

$$x=0: 2 = 3A - 9B + C \Rightarrow 2 = 3A - 9B + C$$

$$x=1: 1 = A - B + C \Rightarrow 1 = A - B + C$$

$$x=2: 0 = A + C \Rightarrow A = -C$$

$$16. \int \frac{(5x^2 - 11x + 5) dx}{x^3 - 4x^2 + 5x - 2}$$

$$\frac{5x^2 - 11x + 5}{x^3 - 4x^2 + 5x - 2}$$

$$x^3 - 4x^2 + 5x - 2 \xrightarrow{x=1} (1 - 4 + 5 - 2) = 0 \quad \therefore x=1 \text{ is a root}$$

$$x^3 - 4x^2 + 5x - 2 \quad | \quad x-1$$

$$\underline{-x^3 + x^2} \quad | \quad x^2 - 3x + 2$$

$$\underline{-3x^2 + 5x - 2}$$

$$\underline{+3x^2 - 3x} \quad | \quad 9C + 9D$$

$$\underline{2x - 2}$$

$$\underline{-2x + 2} \quad | \quad -45$$

$$0$$

$$x^3 - 4x^2 + 5x - 2 = (x-1)(x^2 - 3x + 2)$$

$$= (x-1)(x-1)(x-2)$$

$$x^2 - 3x + 2 = 0$$

$$x = \frac{3 \pm \sqrt{9-8}}{2}$$

$$= \frac{3 \pm 1}{2}$$

$$\frac{5x^2 - 11x + 5}{x^3 - 4x^2 + 5x - 2} = \frac{5x^2 - 11x + 5}{(x-1)^2(x-2)}$$

$$\frac{5x^2 - 11x + 5}{x^3 - 4x^2 + 5x - 2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

$$\text{mmc } (x-1)^2(x-2)$$

$$5x^2 - 11x + 5 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$x=1 : -1 = B(-1) \Rightarrow B=1$$

$$x=2 : 3 = C \Rightarrow C=3$$

$$x=0 : 5 = 2A - 2B + C \Rightarrow 5 = 2A - 2 + 3 \Rightarrow A=2$$

$$\frac{5x^2 - 11x + 5}{x^3 - 4x^2 + 5x - 2} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{3}{x-2}$$

$$\int \frac{5x^2 - 11x + 5}{x^3 - 4x^2 + 5x - 2} dx = \int \frac{2 dx}{x-1} + \int \frac{1 dx}{(x-1)^2} + \int \frac{3 dx}{x-2}$$

$$= 2 \ln|x-1| + \frac{-1}{x-1} + 3 \ln|x-2| + C$$

$$\int \frac{5x^2 - 11x + 5}{x^3 - 4x^2 + 5x - 2} dx = \frac{1}{1-x} + 2 \ln|x-1| + 3 \ln|x-2| + C$$

$$\int \frac{x^4 + 3x^3 - 5x^2 - 4x + 17}{x^3 + x^2 - 5x + 3} dx$$

$\begin{array}{r} x^4 + 3x^3 - 5x^2 - 4x + 17 \\ -x^4 + x^3 + 5x^2 + 3x \\ \hline 0 + 2x^3 + 10x^2 - 7x + 17 \\ -2x^3 - 2x^2 + 10x - 6 \\ \hline -2x^2 + 3x + 11 \end{array}$	$\begin{array}{r} x^3 + x^2 - 5x + 3 \\ x + 2 \\ \hline 3 \end{array}$
---	--

$$\frac{x^4 + 3x^3 - 5x^2 - 4x + 17}{x^3 + x^2 - 5x + 3} = x + 2 + \frac{-2x^2 + 3x + 11}{x^3 + x^2 - 5x + 3}$$

$$x^3 + x^2 - 5x + 3 \xrightarrow{x=1} 1 + 1 - 5 + 3 = 0$$

$$\therefore x^3 + x^2 - 5x + 3 = (x-1)M(x)$$

Hilroy

$$\begin{array}{r}
 x^3 + x^2 - 5x + 3 \quad | \quad x-1 \\
 \underline{-x^3 + x^2} \\
 2x^2 - 5x + 3 \\
 \underline{-2x^2 + 2x} \\
 -3x + 3 \\
 \underline{-(-3x + 3)} \\
 0
 \end{array}$$

$$\frac{x^3 + x^2 - 5x + 3}{x-1} = x^2 + 2x - 3$$

$$x^3 + x^2 - 5x + 3 = (x-1)(x^2 + 2x - 3)$$

$$\begin{aligned}
 x^2 + 2x - 3 &= 0 \\
 x &= \frac{-2 \pm \sqrt{4 + 12}}{2} \\
 &= \frac{-2 \pm 4}{2} \\
 &= -3, 1
 \end{aligned}$$

$$\begin{aligned}
 x^3 + x^2 - 5x + 3 &= (x-1)(x+3)(x-1) \\
 &= (x-1)^2(x+3)
 \end{aligned}$$

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$$\frac{x^4 + 3x^3 - 5x^2 - 4x + 17}{x^3 + x^2 - 5x + 3} = x + 2 + \frac{-2x^2 + 3x + 11}{(x-1)^2(x+3)}$$

May:

$$\frac{-2x^2 + 3x + 11}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$$

$$-2x^2 + 3x + 11 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

$$x=1: \quad 12 = B \cdot 4 \Rightarrow B=3$$

$$x=-3: \quad -16 = C \cdot 16 \Rightarrow C=-1$$

$$x=0: \quad 11 = -3A + 3B + C$$

$$x=1: \quad -1 = -3A + 9 - 1 \quad (C=-1)$$

$$3 = -3A - 2B \Rightarrow A=-1$$

$$\frac{-2x^2 + 3x + 11}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} \quad (**)$$

$$(**) \rightarrow (*) : \quad 0 = (1-x)^2 B - B + (x+3)C \Rightarrow B=0$$

$$\frac{x^4 + 3x^3 - 5x^2 - 4x + 17}{x^3 + x^2 - 5x + 3} = x + 2 - \frac{1}{x-1} - \frac{1}{x+3} + \frac{3}{(x-1)^2}$$

$$\int \frac{x^4 + 3x^3 - 5x^2 - 4x + 17}{x^3 + x^2 - 5x + 3} dx =$$

$$= \int (x+2) dx - \int \frac{dx}{x-1} - \int \frac{dx}{x+3} + 3 \int \frac{dx}{(x-1)^2}$$

$$= \frac{x^2 + 2x}{2} - \ln|x-1| - \ln|x+3| + 3 \frac{-1}{x-1}$$

$$= \frac{x^2 + 2x}{2} - \ln|(x-1)(x+3)| - \frac{3}{x-1} + C$$

$$= \frac{1}{2}x^2 + 2x - \frac{3}{x-1} - \ln|x^2 + 2x - 3| + C$$

$$A=3$$

$$B=C=-1$$

$$\frac{x^2 + 2x}{2} - \frac{3}{x-1}$$

$$5x + 6 = 5x + 6$$

$$0 = 2B + A$$

$$0 = 2C + B$$

$$5 = 0C + C$$

$$1 = 1$$

18. $\int \frac{2x^4 - 2x + 1}{2x^5 - x^4} dx$

$$\frac{2x^4 - 2x + 1}{2x^5 - x^4} = \frac{2x^4 - 2x + 1}{x^4(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{2x-1}$$

$$2x^4 - 2x + 1 = Ax^3(2x-1) + Bx^2(2x-1) + Cx(2x-1) + D(2x-1) + Ex^4$$

$$x = \frac{1}{2} : \frac{2}{16} - 1 + 1 = E \frac{1}{16} \Rightarrow E = 2$$

$$x = 0 : 1 = -D \Rightarrow D = -1$$

$$x = 1 : \begin{cases} 1 = A + B + C + D + E \\ = A + B + C - 1 + 2 \\ A = A + B + C + 1 \\ 0 = A + B + C \end{cases}$$

outra maneira:

$$\begin{aligned} 2x^4 - 2x + 1 &= Ax^3(2x-1) + Bx^2(2x-1) + Cx(2x-1) + D(2x-1) + Ex^4 \\ &= 2Ax^4 - Ax^3 + 2Bx^3 - Bx^2 + 2Cx^2 - Cx + 2Dx - D + Ex^4 \end{aligned}$$

$$2x^4 - 2x + 1 = (2A+E)x^4 + (-A+2B)x^3 + (-B+2C)x^2 + (-C+2D)x - D$$

$$\Rightarrow \begin{cases} 2A+E = 2 \\ -A+2B = 0 \\ -B+2C = 0 \\ -C+2D = -2 \\ D = -1 \end{cases}$$

$$\begin{aligned}
 2A + E &= 2 \Rightarrow E = 2 \\
 -A + 2B &= 0 \Rightarrow -A + 2B = 0 \Rightarrow A = 0 \\
 -B + 2C &= 0 \Rightarrow -B + 2C = 0 \Rightarrow B = 0 \\
 -C + 2D &= -2 \Rightarrow -C - 2 = -2 \Rightarrow C = 0 \\
 D &= -1
 \end{aligned}$$

$$\frac{2x^4 - 2x + 1}{2x^5 - x^4} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{2x-1}$$

$$= -\frac{1}{x^4} + \frac{2}{2x-1}$$

$$\begin{aligned}
 \int \frac{2x^4 - 2x + 1}{2x^5 - x^4} dx &= \int \frac{-dx}{x^4} + 2 \int \frac{dx}{2x-1} \\
 &= -\frac{1}{3x^3} + 2 \ln|2x-1| + C
 \end{aligned}$$

$$\int \frac{2x^4 - 2x + 1}{2x^5 - x^4} dx = -\frac{1}{3x^3} + \ln|2x-1| + C$$

$$(2x) \cdot (x^2 + 1) = 2x^3 + 2x$$

19. $\int \frac{-24x^3 + 30x^2 + 52x + 17}{9x^4 - 6x^3 - 11x^2 + 4x + 4} dx$

$9x^4 - 6x^3 - 11x^2 + 4x + 4 \xrightarrow{x=1} 9 - 6 - 11 + 4 + 4 = 0$

$$\begin{array}{r} 9x^4 - 6x^3 - 11x^2 + 4x + 4 \\ -9x^4 + 9x^3 \\ \hline 3x^3 - 11x^2 + 4x + 4 \\ -3x^3 + 3x^2 \\ \hline \end{array} \quad \left| \begin{array}{r} x-1 \\ \hline 9x^3 + 3x^2 - 8x - 4 \end{array} \right.$$

$$\begin{array}{r} -8x^2 + 4x + 4 \\ +8x^2 - 8x \\ \hline \end{array}$$

$$\begin{array}{r} -4x + 4 \\ +4x - 4 \\ \hline \end{array}$$

$$\left\{ \begin{array}{l} 9x^4 - 6x^3 - 11x^2 + 4x + 4 = (x-1)(9x^3 + 3x^2 - 8x - 4) \quad (*) \end{array} \right.$$

Now, $9x^3 + 3x^2 - 8x - 4 \xrightarrow{x=1} 9 + 3 - 8 - 4 = 0$

$\Rightarrow (x-1)$ is a factor

$$\begin{array}{r} 9x^3 + 3x^2 - 8x - 4 \\ -9x^3 + 9x^2 \\ \hline 12x^2 - 8x - 4 \\ -12x^2 + 12x \\ \hline 4x - 4 \\ -4x + 4 \\ \hline 0 \end{array} \quad \left| \begin{array}{r} x-1 \\ \hline 9x^2 + 12x + 4 \end{array} \right.$$

$$\Rightarrow 9x^3 + 3x^2 - 8x - 4 = (x-1)(9x^2 + 12x + 4) \quad (**)$$

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$$9x^2 + 12x + 4 = 0$$

$$x = \frac{-12 \pm \sqrt{144 - 144}}{18}$$

$$= \frac{-12}{18} = -\frac{2}{3}$$

$$x = -\frac{2}{3}$$

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$$9x^2 + 12x + 4 = 9\left(x + \frac{2}{3}\right)^2 = (3x+2)^2$$

$$\textcircled{***} \rightarrow \textcircled{**} : \left\{ \begin{array}{l} 9x^3 + 3x^2 - 8x - 4 = (x-1)(3x+2)^2 \end{array} \right. \textcircled{****}$$

$$\textcircled{***} \rightarrow \textcircled{*} : \left\{ \begin{array}{l} 9x^4 - 6x^3 - 11x^2 + 4x + 4 = (x-1)^2(3x+2)^2 \end{array} \right. \textcircled{\star}$$

Darf:

$$\frac{-24x^3 + 30x^2 + 52x + 17}{9x^4 - 6x^3 - 11x^2 + 4x + 4} = \frac{-24x^3 + 30x^2 + 52x + 17}{(x-1)^2(3x+2)^2} =$$

$$\text{Nenner: } (x-1)^2(3x+2)^2 = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{3x+2} + \frac{D}{(3x+2)^2}$$

$$(-24x^3 + 30x^2 + 52x + 17) = A(x-1)(3x+2)^2 + B(3x+2)^2 + C(x-1)^2(3x+2) + D(x-1)^2$$

$$x=1 : -24 + 30 + 52 + 17 = 25B$$

$$75 = 25B \Rightarrow B = 3$$

$$x = -\frac{2}{3} : -24\left(-\frac{2}{3}\right)^3 + 30\left(-\frac{2}{3}\right)^2 + 52\left(-\frac{2}{3}\right) + 17 = D\left(-\frac{2}{3}-1\right)^2$$

$$+24 \cdot \frac{8}{27} + 30 \cdot \frac{4}{9} - \frac{104}{3} + 17 = D\left(-\frac{5}{3}\right)^2$$

$$\frac{64}{9} + \frac{120}{9} - \frac{104}{3} + 17 = D \frac{25}{9}$$

$$\frac{64 + 120 - 312 + 17}{9}$$

$$\frac{-128}{9} + 17 = D \frac{25}{9}$$

$$\frac{25}{9} = D \frac{25}{9} \Rightarrow D = 1$$

$$x=0 : 17 = -4A + 4B + 2C + D$$

$$17 = -4A + 12 + 2C + 1$$

$$4 = -4A + 2C \Rightarrow \parallel 2 = -2A + C \parallel$$

$$x=-1 : 24 + 30 - 52 + 17 = -2A + B - 4C + 4D$$

$$19 = -2A + 3 - 4C + 4$$

$$12 = -2A - 4C$$

$$\parallel 6 = -A - 2C \parallel$$

$$\begin{aligned} \therefore 1) \quad 2 &= -2A + C \Rightarrow C = 2 + 2A \\ 6 &= -A - 2C \Rightarrow 6 = -A - 2(2 + 2A) \\ &= -A - 4 - 4A \\ &= -5A - 4 \\ 6 &= -5A - 4 \Rightarrow A = -2 \end{aligned}$$

$$\begin{aligned} \therefore 2) \quad 1 &= 3A + B + 9C \\ &= 3(-2) + B + 9(-2) \\ &= -6 + B - 18 \\ &= B - 24 \\ 1 &= B - 24 \Rightarrow B = 25 \end{aligned}$$

$$\frac{-24x^3 + 30x^2 + 52x + 17}{9x^4 - 6x^3 - 11x^2 + 4x + 4} = \frac{-2}{x-1} + \frac{3}{(x+1)^2} - \frac{2}{3x+2} + \frac{1}{(3x+2)^2}$$

$$\int \frac{-24x^3 + 30x^2 + 52x + 17}{9x^4 - 6x^3 - 11x^2 + 4x + 4} dx = -2 \int \frac{dx}{x-1} + 3 \int \frac{dx}{(x+1)^2} - 2 \int \frac{dx}{3x+2} + \int \frac{dx}{(3x+2)^2}$$

$$= -2 \ln|x-1| + 3 \frac{-1}{x+1} - 2 \frac{\ln|3x+2|}{3}$$

$$= -\frac{3}{x-1} - \frac{1}{3(3x+2)} - \ln|(x-1)^2| - \ln|(3x+2)^{2/3}| + C$$

$$\int \frac{-24x^3 + 30x^2 + 52x + 17}{9x^4 - 6x^3 - 11x^2 + 4x + 4} dx = -\frac{3}{x-1} - \frac{1}{3(3x+2)} - \ln \left[(3x+2)^{2/3} (x-1)^2 \right] + C$$

20. $\int \frac{dx}{16x^4 - 8x^2 + 1}$

$16x^4 - 8x^2 + 1 = 16y^2 - 8y + 1 \quad (y = x^2)$

$16y^2 - 8y + 1 = 0$
 $y = \frac{8 \pm \sqrt{64 - 64}}{32} = \frac{8}{32} = \frac{1}{4}$

$y = x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}$

$16x^4 - 8x^2 + 1 = 16(x + \frac{1}{2})^2(x - \frac{1}{2})^2 = (2x+1)^2(2x-1)^2$

$\frac{1}{16x^4 - 8x^2 + 1} = \frac{1}{(2x+1)^2(2x-1)^2}$

$= \frac{A}{2x+1} + \frac{B}{(2x+1)^2} + \frac{C}{2x-1} + \frac{D}{(2x-1)^2}$

$1 = A(2x+1)(2x-1)^2 + B(2x-1)^2 + C(2x+1)^2(2x-1) + D(2x+1)^2$

$x=0 : 1 = A + B - C + D$

$x = \frac{1}{2} : 1 = 4D \Rightarrow D = \frac{1}{4}$

$x = -\frac{1}{2} : 1 = 4B \Rightarrow B = \frac{1}{4}$

$x=1 : 1 = 3A + B + 9C + 9D$

$$1 = A + B - C + D$$

$$= A + \frac{1}{4} - C + \frac{1}{4}$$

$$= A - C + \frac{1}{2}$$

$$\left\| \frac{1}{2} = A - C \right\|$$

$$1 = 3A + B + 9C + 9D$$

$$= 3A + \frac{1}{4} + 9C + \frac{9}{4}$$

$$1 = 3A + 9C + \frac{5}{2}$$

$$-\frac{3}{2} = 3A + 9C \rightarrow \left\| -\frac{1}{2} = A + 3C \right\|$$

$$\therefore \begin{cases} \frac{1}{2} = A - C \\ -\frac{1}{2} = A + 3C \end{cases} \Rightarrow 1 = -4C \Rightarrow C = -\frac{1}{4}$$

$$\frac{1}{2} = A - C$$

$$x = -2 : 1 = -4C = A + \frac{1}{4} \Rightarrow A = \frac{1}{2} - \frac{1}{4}$$

$$A = \frac{1}{4}$$

$$\frac{1}{16x^4 - 8x^2 + 1} = \frac{1}{4(2x+1)} + \frac{1}{4(2x+1)^2} - \frac{1}{4(2x-1)} + \frac{1}{4(2x-1)^2}$$

$$\int \frac{dx}{16x^4 - 8x^2 + 1} = \frac{1}{9} \int \frac{dx}{2x+1} + \frac{1}{9} \int \frac{dx}{(2x+1)^2} - \frac{1}{9} \int \frac{dx}{2x-1}$$

$$+ \frac{1}{9} \int \frac{dx}{(2x-1)^2}$$

$$= \frac{1}{9} \frac{\ln|2x+1|}{2-4-64} + \frac{1}{9} \frac{-1}{2(2x+1)} - \frac{1}{9} \frac{\ln|2x-1|}{2}$$

$$+ \frac{1}{9} \frac{-1}{2(2x-1)} + C$$

$$= \frac{1}{8} \ln|2x+1| - \frac{1}{8(2x+1)} - \frac{1}{8} \ln|2x-1| - \frac{1}{8(2x-1)} + C$$

$$\int \frac{dx}{16x^4 - 8x^2 + 1} = \frac{1}{8} \ln \left| \frac{2x+1}{2x-1} \right| - \frac{1}{8(2x+1)} - \frac{1}{8(2x-1)} + C$$

$$\frac{1}{8} \ln \left| \frac{x+\frac{1}{2}}{x-\frac{1}{2}} \right| - \frac{1}{10(x-\frac{1}{2})} - \frac{1}{10(x+\frac{1}{2})} + C$$

$$1 = A(x+1)(x-1)^2 + B(x+1)^2 + C(x-1)^2 + D(x+1)^2$$

$$1 = 1A + B + C + D$$

$$1 = 4D \Rightarrow D = 1/4$$

$$1 = 4B \Rightarrow B = 1/4$$

$$1 = 3A + D + C + 1/4$$