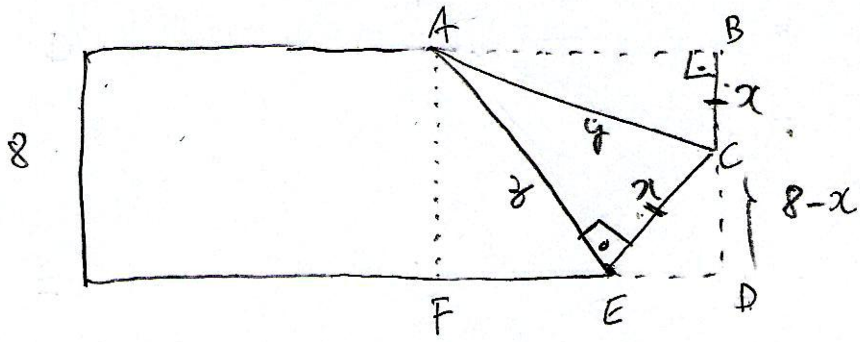
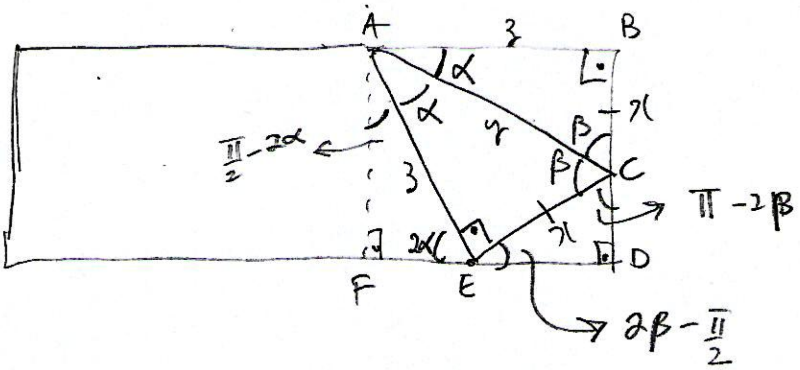
13.



Notemos



$\pi - 2\beta + \frac{\pi}{6} + \varphi = \pi$

Mas, do ΔABC temos: $\alpha + \beta = \frac{\pi}{2}$

$\therefore \beta = \frac{\pi}{2} - \alpha$

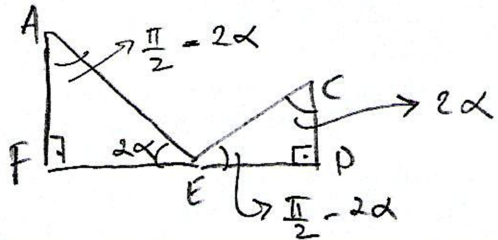
Daí, no ΔEDC temos:

$$\begin{cases} 2\beta - \frac{\pi}{2} = 2\left(\frac{\pi}{2} - \alpha\right) - \frac{\pi}{2} \\ \qquad = \pi - 2\alpha - \frac{\pi}{2} \\ \qquad = \frac{\pi}{2} - 2\alpha \end{cases}$$

e

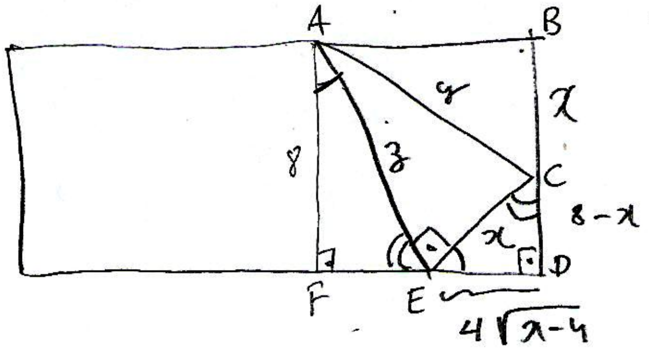
$$\begin{cases} \pi - 2\beta = \pi - 2\left(\frac{\pi}{2} - \alpha\right) \\ \qquad = \pi - \pi + 2\alpha \\ \qquad = 2\alpha \end{cases}$$

isto é



$\therefore$ $\Delta AFE \sim \Delta EDC$.

$\Delta AFE \approx \Delta EDC$ (Semelhantes)



Daí tem-se:

$$\frac{AE}{AF} = \frac{EC}{ED}$$

$$\therefore \quad \frac{z}{8} = \frac{x}{4\sqrt{x-4}}$$

$$\therefore \quad z = \frac{\cancel{8}x}{\cancel{4}\sqrt{x-4}} = \frac{2x}{\sqrt{x-4}} \quad (*)$$

Mas, usando Pitágoras no triângulo retângulo ΔAEC temos:

$$y^2 = \underbrace{z^2}_{(*)} + x^2$$

$$\therefore \quad y^2 = \frac{4x^2}{x-4} + x^2 \quad \text{com } 4 < x < 8$$

$$y^2 = \frac{\cancel{4x^2} + x^3 - \cancel{4x^2}}{x-4} = \frac{x^3}{x-4}$$

$$\therefore \quad y = \frac{x^{3/2}}{\sqrt{x-4}} \quad ; \quad 4 < x < 8$$

Obs.: Tecnicamente esta relação entre y e x deve ser entendida tendo em vista o problema em questão.

Não podemos tomar $x = 4$ pois
$y = \dfrac{x^{3/2}}{\sqrt{x-4}}$ não está definido para
$x = 4$, contudo para valores $x \to 4^+$
temos que $y \to +\infty$, o que também
não é algo aceitável no problema
pois sabemos que y é finito.

Resta então admitir que o mínimo
de y se existir ocorrerá no ponto
crítico.

Daí $y' = \left(\dfrac{x^{3/2}}{\sqrt{x-4}}\right)' = \dfrac{\frac{3}{2} x^{1/2}\sqrt{x-4} - x^{3/2}\frac{1}{2\sqrt{x-4}}}{(x-4)}$

$$= \frac{1}{2}\frac{\left(3x^{1/2}(x-4) - x^{3/2}\right)}{(x-4)^{3/2}}$$

$$= \frac{1}{2}\frac{\left(3x^{3/2} - 12x^{1/2} - x^{3/2}\right)}{(x-4)^{3/2}}$$

$$= \frac{1}{2}\frac{\left(2x^{3/2} - 12x^{1/2}\right)}{(x-4)^{3/2}}$$

$y'(x) = 0 \implies \cancel{2}x^{3/2} - \cancel{12}_{6}\, x^{1/2} = 0 \quad \therefore$

$x^{1/2}(x-6) = 0$

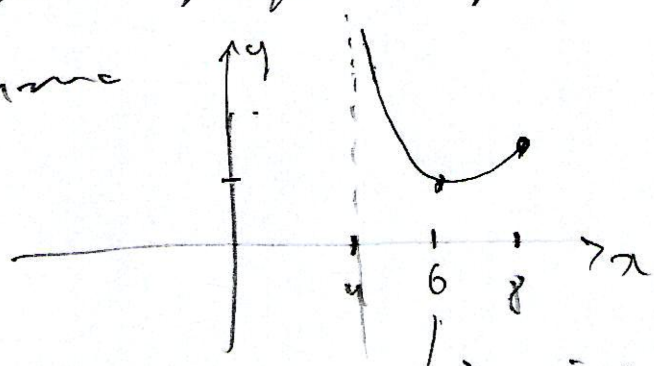
$\therefore$ $\cancel{x = 0}$ (. não está no domínio)

$x = 6$

↳ é o valor de x que produz o menor valor de y.

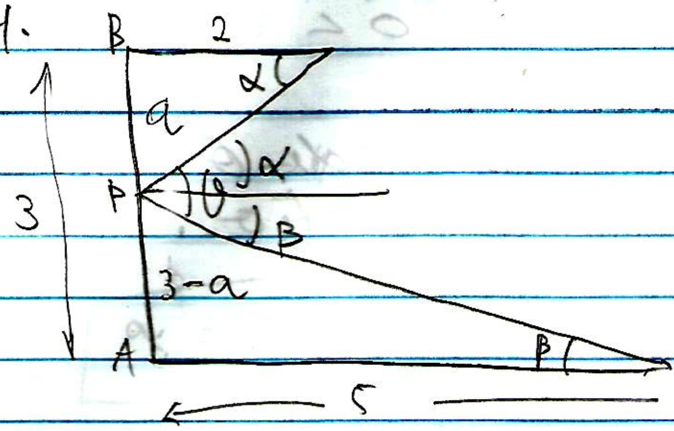
obs. Note que $y(x) = \dfrac{x^{3/2}}{\sqrt{x-4}}$, $4 < x < 8$

tem um gráfico aproximadamente na forma



↳ mínimo em $x = 6$.

14.



$$\theta = \alpha + \beta$$

Da figura temos:

$$\begin{cases} tg\,\alpha = \dfrac{a}{2} \\ tg\,\beta = \dfrac{3-a}{5} \end{cases}$$

Além disso sabemos que:

$$tg\,\underbrace{(\alpha+\beta)}_{\theta} = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\, tg\,\beta}$$

$$\left\| \; tg\,\theta = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\, tg\,\beta} \; \right\|$$

$$= \frac{\frac{a}{2} + \frac{3-a}{5}}{1 - \frac{a}{2} \cdot \frac{3-a}{5}} = \frac{\frac{5a + 6 - 2a}{10}}{1 - \frac{3a - a^2}{10}}$$

$$= \frac{\frac{3a+6}{10}}{\frac{10 - 3a + a^2}{10}} = \frac{3a+6}{a^2 - 3a + 10}$$

$$\left\| \; tg\,\theta = \frac{3a+6}{a^2-3a+10} \; \right\|$$

/ /

Da figura vemos que $0 < \theta < \frac{\pi}{2}$

Logo, o valor máximo de θ coincidirá com o valor máximo de $tg\,\theta$:

Podemos então maximizar $tg\,\theta$:

$$tg\,\theta = \frac{3a+6}{a^2-3a+10}$$

$$\downarrow$$

$$\frac{d\,tg\,\theta}{da} = \frac{3}{a^2-3a+10} - \frac{3a+6}{(a^2-3a+10)^2}(2a-3)$$

$$0 = \frac{3(a^2-3a+10)-(3a+6)(2a-3)}{(a^2-3a+10)^2}$$

$$= \frac{3a^2-9a+30-(6a^2-9a+12a-18)}{(a^2-3a+10)^2}$$

$$= \frac{3a^2-9a+30-(6a^2+3a-18)}{(a^2-3a+10)^2}$$

$$= \frac{-3a^2-12a+48}{(a^2-3a+10)^2}$$

$$\Rightarrow \quad -3a^2-12a+48=0$$

$$a^2+4a-16=0$$

$$a = -4 \pm \sqrt{16+64}\,/2$$

$$= -4 \pm \sqrt{80}\,/2$$

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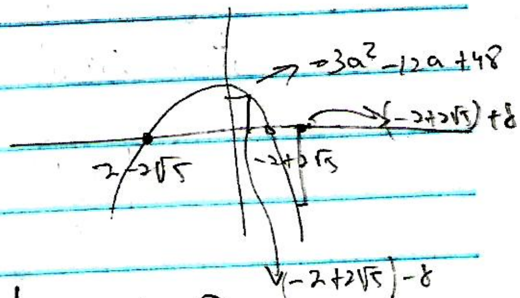
14- Cont.

$$a = \frac{-4 \pm 4\sqrt{5}}{2}$$

80	2
40	2
20	2
10	2
5	5
1	

$$\boxed{a = -2 + 2\sqrt{5}}$$



$$\left.\frac{d\,tg\theta}{da}\right|_{(-2+2\sqrt{5})-\delta} = \left.\frac{\overbrace{-3a^2-12a+48}^{>0}}{\underbrace{(a^2-3a+10)^2}_{>0}}\right|_{(-2+2\sqrt{5})-\delta} > 0$$

$$\left.\frac{d\,tg\theta}{da}\right|_{(-2+2\sqrt{5})+\delta} = \left.\frac{\overbrace{-3a^2-12a+48}^{<0}}{\underbrace{(a^2-3a+10)^2}_{>0}}\right|_{(-2+2\sqrt{5})+\delta} < 0$$

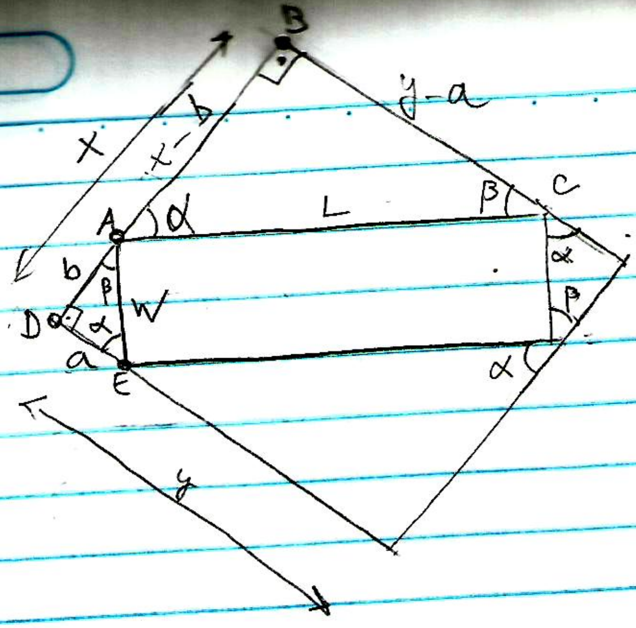
$\therefore$ $(-2+2\sqrt{5})$ é pto. de máximo.

Daí: $b = 3 - a = 3 - (-2 + 2\sqrt{5})$

$= \underline{5 - 2\sqrt{5}}$

Resposta: A fim de maximizar o ângulo θ, o ponto P deve estar à uma distância $5-2\sqrt{5}$ do ponto A.

15.



$S = xy \longrightarrow$ Maximizar S.

Relação:

$$\begin{cases} a^2 + b^2 = W^2 \\ \Delta_{ABC} \approx \Delta_{EDA} \Rightarrow \begin{cases} \dfrac{L}{W} = \dfrac{y-a}{b} \\ \dfrac{L}{W} = \dfrac{x-b}{a} \end{cases} \end{cases}$$

$$\frac{L}{W} = \frac{y-a}{b} \Rightarrow \quad y = \frac{Lb}{W} + a \quad (*)$$

$$\frac{L}{W} = \frac{x-b}{a} \Rightarrow \quad x = \frac{La}{W} + b \quad (**)$$

$$\therefore \quad S = xy = \left(\frac{Lb}{W} + a\right)\left(\frac{La}{W} + b\right)$$

tilibra

$$S = \frac{L^2 ab}{W^2} + \frac{Lb^2}{W} + \frac{La^2}{W} + ab$$

$$= ab\left(\frac{L^2}{W^2}+1\right) + \frac{L}{W}\left(a^2+b^2\right)$$

$$= ab\left(\frac{L^2}{W^2}+1\right) + \frac{L}{\cancel{W}}\, W^{\cancel{2}}$$

$$S = ab\left(\frac{L^2}{W^2}+1\right) + LW$$

$$\boxed{S = a\sqrt{W^2-a^2}\left(\frac{L^2}{W^2}+1\right) + LW}$$

$$\frac{dS}{da} = \sqrt{W^2-a^2}\left(\frac{L^2}{W^2}+1\right) + \frac{a}{\cancel{2}\sqrt{W^2-a^2}}(-\cancel{2}a)\left(\frac{L^2}{W^2}+1\right)$$

$$0 = \left(\frac{L^2}{W^2}+1\right)\left(\sqrt{W^2-a^2} - \frac{a^2}{\sqrt{W^2-a^2}}\right)$$

$$\Rightarrow \quad \sqrt{W^2-a^2} - \frac{a^2}{\sqrt{W^2-a^2}} = 0$$

$$\frac{W^2-a^2-a^2}{\sqrt{W^2-a^2}} = 0 \qquad (a \neq W)$$

$$W^2 = 2a^2 \Rightarrow \boxed{a = \frac{W}{\sqrt{2}}}$$

$$a^2 + b^2 = w^2 \Rightarrow b^2 = w^2 - a^2$$

$$= w^2 - \frac{w^2}{2}$$

$$= \frac{w^2}{2}$$

$$\boxed{b = \frac{w}{\sqrt{2}}}$$

Daí, substituindo $a = b = \frac{w}{\sqrt{2}}$ em (*) e (**) obtém-se

$$\begin{cases} y = \frac{L}{\cancel{w}} \frac{\cancel{w}}{\sqrt{2}} + \frac{w}{\sqrt{2}} = \frac{L+w}{\sqrt{2}} \\ x = \frac{L}{\cancel{w}} \frac{\cancel{w}}{\sqrt{2}} + \frac{w}{\sqrt{2}} = \frac{L+w}{\sqrt{2}} \end{cases}$$

$\therefore$

$$S = xy = \frac{L+w}{\sqrt{2}} \cdot \frac{L+w}{\sqrt{2}}$$

$$\boxed{S = \frac{(L+w)^2}{2}}$$