13. $\begin{cases} x \in \left(\frac{\pi}{2}, \pi\right) \\ y \in \left(\pi, \frac{3\pi}{2}\right) \end{cases}$

$\begin{cases} \cos x = -\frac{5}{13} \\ \operatorname{tg} y = \frac{4}{3} \end{cases}$

• $\sin^2 x = 1 - \cos^2 x = 1 - \left(\frac{-5}{13}\right)^2$

$= 1 - \frac{25}{169} = \frac{144}{169}$

$\sin x = \pm\sqrt{\frac{144}{169}} = \pm \frac{12}{13}$

$x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \sin x > 0.$

$\therefore \ // \sin x = \frac{12}{13} //$

• $1 + \operatorname{tg}^2 y = \sec^2 y$

$1 + \left(\frac{4}{3}\right)^2 = \sec^2 y$

$1 + \frac{16}{9} = \sec^2 y$

$\therefore \ \sec y = \pm\sqrt{\frac{25}{9}} = \pm \frac{5}{3}$

$y \in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow \sec y < 0$

$\therefore \ // \sec y = -\frac{5}{3} //$

Agora,

$// \cos y = \frac{1}{\sec y} = -\frac{3}{5} //$

$\sin y = -\sqrt{1 - \cos^2 y}$ ($\sin y < 0$ para $y \in \left(\pi, \frac{3\pi}{2}\right)$)

$= -\sqrt{1 - \frac{9}{25}}$

$\therefore \ \sin y = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$

$// \sin y = -\frac{4}{5} //$

isto é:

$\begin{cases} \cos x = -\frac{5}{13} \\ \sin x = \frac{12}{13} \\ \operatorname{tg} x = \frac{\sin x}{\cos x} = -\frac{12}{5} \end{cases}$

$\begin{cases} \cos y = -\frac{3}{5} \\ \sin y = \frac{-4}{5} \\ \operatorname{tg} y = \frac{4}{3} \end{cases}$

Daí

a) $\sin(x+y) = \sin x \cos y + \sin y \cos x$

$= \frac{12}{13}\left(-\frac{3}{5}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-36}{65} + \frac{20}{65}$

$= \frac{-16}{65} //$

b) $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \frac{-5}{13}\left(\frac{-3}{5}\right) + \frac{12}{13}\left(\frac{-4}{5}\right)$

$= \frac{15}{65} - \frac{48}{65} = \frac{-33}{65} //$