20.

$$\operatorname{cosec}(x+y) =$$

$$= \frac{1}{\sin(x+y)}$$

$$= \frac{1}{\sin x \cos y + \sin y \cos x}$$

$$= \frac{1}{\frac{1}{\operatorname{cosec} x} \cdot \frac{1}{\sec y} + \frac{1}{\operatorname{cosec} y} \frac{1}{\sec x}}$$

$$= \frac{1}{\frac{\operatorname{cosec} y \sec x + \operatorname{cosec} x \sec y}{\operatorname{cosec} x \sec y \operatorname{cosec} y \sec x}}$$

$$\boxed{\operatorname{cosec}(x+y) = \frac{\operatorname{cosec} x \sec x \operatorname{cosec} y \sec y}{\operatorname{cosec} x \sec y + \operatorname{cosec} y \sec x}}$$

21.

$$\sin(x+y+z) = \sin[(x+y)+z]$$

$$= \sin(x+y)\cos z + \sin z \cos(x+y)$$

$$= (\sin x \cos y + \sin y \cos x)\cos z$$

$$+ \sin z(\cos x \cos y - \sin x \sin y)$$

$$= \sin x \cos y \cos z + \sin y \cos x \cos z$$

$$+ \sin z \cos x \cos y - \sin z \sin x \sin y$$

$\therefore$

$$\boxed{\begin{aligned} \sin(x+y+z) \equiv\ & \sin x \cos y \cos z + \\ & + \cos x \sin y \cos z + \\ & + \cos x \cos y \sin z + \\ & - \sin x \sin y \sin z \end{aligned}}$$

22.

$$x, y, z \in \left(\frac{\pi}{2}, \pi\right)$$

$$\cos x = -\frac{1}{3} \Rightarrow \sin x = +\sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sin y = \frac{1}{4} \Rightarrow \cos y = -\sqrt{1-\frac{1}{16}} = -\frac{\sqrt{15}}{4}$$

$$\sin z = \frac{1}{5} \Rightarrow \cos z = -\sqrt{1-\frac{1}{25}} = -\frac{2\sqrt{6}}{5}$$

$$\cos(x+y-z) =$$

$$= \cos(x+y)\cos z + \sin(x+y)\sin z$$

$$= (\cos x \cos y - \sin x \sin y)\cos z +$$

$$+ (\sin x \cos y + \sin y \cos x)\sin z$$

$$= \cos x \cos y \cos z - \sin x \sin y \cos z$$

$$+ \sin x \cos y \sin z + \cos x \sin y \sin z$$

$$= \frac{-1}{3}\,\frac{-\sqrt{15}}{4}\,\frac{-\cancel{2}\sqrt{6}}{5} - \frac{\cancel{2}\sqrt{2}}{3}\,\frac{1}{\cancel{4}}\,\frac{-2\sqrt{6}}{5}$$

$$+ \frac{\cancel{2}\sqrt{2}}{3}\,\frac{-\sqrt{15}}{\cancel{4}_2}\,\frac{1}{5} + \frac{-1}{3}\,\frac{1}{4}\,\frac{1}{5} =$$