

26. Cont.

b. . .

$$\cos\left(\frac{\pi}{12} - x\right) = \frac{\sqrt{2}(\sqrt{3}+1)}{4} \cos x + \frac{\sqrt{2}(\sqrt{3}-1)}{4} \sin x$$

$$\sec \frac{\pi}{12} = \frac{1}{\cos \frac{\pi}{12}} = \frac{4}{\sqrt{2}(\sqrt{3}+1)}$$

$$\begin{aligned} \sin\left(\frac{\pi}{12} - x\right) &= \sin \frac{\pi}{12} \cos x - \cos \frac{\pi}{12} \sin x \\ &= \frac{\sqrt{2}(\sqrt{3}-1)}{4} \cos x - \frac{\sqrt{2}(\sqrt{3}+1)}{4} \sin x \end{aligned}$$

$$\csc \frac{\pi}{12} = \frac{1}{\sin \frac{\pi}{12}} = \frac{4}{\sqrt{2}(\sqrt{3}-1)}$$

∴

$$\begin{aligned} \cos\left(\frac{\pi}{12} - x\right) \sec \frac{\pi}{12} - \sin\left(\frac{\pi}{12} - x\right) \csc \frac{\pi}{12} &= \\ &= \left(\frac{\sqrt{2}(\sqrt{3}+1)}{4} \cos x + \frac{\sqrt{2}(\sqrt{3}-1)}{4} \sin x\right) \frac{4}{\sqrt{2}(\sqrt{3}+1)} \\ &\quad - \left(\frac{\sqrt{2}(\sqrt{3}-1)}{4} \cos x - \frac{\sqrt{2}(\sqrt{3}+1)}{4} \sin x\right) \frac{4}{\sqrt{2}(\sqrt{3}-1)} \end{aligned}$$

$$\begin{aligned} &= \cancel{\cos x} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \sin x + \\ &\quad - \cancel{\cos x} + \frac{\sqrt{3}+1}{\sqrt{3}-1} \sin x \end{aligned}$$

$$\begin{aligned} &= \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}\right) \sin x \\ &= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{3-1} \sin x \\ &= \frac{3-2\sqrt{3}+1 + 3+2\sqrt{3}+1}{2} \sin x \\ &= \underline{\underline{4 \sin x}} \end{aligned}$$

c)

$$\begin{aligned} \frac{\sin(x-y)}{\cos x \cos y} &= \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y} \\ &= \tan x - \tan y \end{aligned}$$

$$\frac{\sin(3-x)}{\cos 3 \cos x} = \tan 3 - \tan x$$

$$\frac{\sin(4-3)}{\cos 4 \cos 3} = \tan 4 - \tan 3$$

$$\begin{aligned} \frac{\sin(x-y)}{\cos x \cos y} + \frac{\sin(3-x)}{\cos 3 \cos x} + \frac{\sin(4-3)}{\cos 4 \cos 3} &= \\ &= \cancel{\tan x} - \cancel{\tan y} + \tan 3 - \cancel{\tan x} + \cancel{\tan 4} - \cancel{\tan 3} \\ &= 0 \end{aligned}$$